?What Is The Equation Of A Parabola Whose Focus Is (-4, -1)and Whose Directrix Isy = -5?xy = + X 18oy

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Introduction

Understanding the geometric properties of parabolas is fundamental in algebra and coordinate geometry. A parabola is a symmetrical, U-shaped curve that can be defined mathematically in terms of its focus and directrix. Determining the equation of a parabola given these key elements provides insight into its shape and position within the coordinate plane.

In this article, we will explore how to find the equation of a parabola when its focus is at the point \((-4, -1)\) and its directrix is the line \(y = -5 \). We will delve into the geometric concepts underlying this process, step-by-step calculations, and the final form of the parabola's equation. This comprehensive guide will also include explanations on key parabola properties, the derivation of the equation, and useful tips for solving similar problems.

Understanding the Focus and Directrix of a Parabola

What is a Focus?

The focus of a parabola is a fixed point located inside the curve. It plays a crucial role in defining the parabola, as every point on the parabola is equidistant from the focus and the directrix.

What is a Directrix?

The directrix is a fixed line outside the parabola. The parabola is the set of all points that are equidistant from the focus and the directrix line.

Relationship Between Focus, Directrix, and the Parabola

The defining property of a parabola is:

> For any point \((x, y) \) on the parabola, the distance to the focus equals the perpendicular distance to the directrix.

Mathematically:

\[
\text{Distance to focus} = \text{Distance to directrix}
\]

This property forms the basis for deriving the parabola's equation.

Analyzing the Given Focus and Directrix

Focus: $(-4, -1)$

Directrix: $y = -5$

Since the directrix is a horizontal line, the parabola opens either upward or downward depending on the position of the focus relative to the directrix.

- The focus is at $(-4, -1)$, which is above the directrix $y = -5$.
- Therefore, the parabola opens upward because the focus lies above the directrix.

Step-by-Step Derivation of the Parabola's Equation

1. Set Up the Distance Formula

For a general point $P(x, y)$ on the parabola:

- Distance to focus $F(-4, -1)$:

$$
d_{F} = \sqrt{(x + 4)^2 + (y + 1)^2}
$$

- Distance to directrix $y = -5$:

$$
d_{D} = | y + 5 |
$$

Because the parabola is the set of points where these distances are equal:

$$
\sqrt{(x + 4)^2 + (y + 1)^2} = | y + 5 |
$$

2. Square Both Sides to Remove the Square Root

$$
(x + 4)^2 + (y + 1)^2 = (y + 5)^2
$$

3. Expand Both Sides

Left side:

$$
(x + 4)^2 + (y + 1)^2 = (x^2 + 8x + 16) + (y^2 + 2 y + 1)
$$

Right side:

$$
(y + 5)^2 = y^2 + 10 y + 25
$$

Putting it all together:

$$
x^2 + 8x + 16 + y^2 + 2 y + 1 = y^2 + 10 y + 25
$$

4. Simplify the Equation

Subtract y^2 from both sides:

$$
x^2 + 8x + 16 + 2 y + 1 = 10 y + 25
$$

Combine like terms:

$$
x^2 + 8x + (16 + 1) + 2 y = 10 y + 25
$$
$$
x^2 + 8x + 17 + 2 y = 10 y + 25
$$

Bring all y-terms to one side and constants to the other:

$$
x^2 + 8x + 17 = 10 y - 2 y + 25
$$
$$
x^2 + 8x + 17 = 8 y + 25
$$

5. Isolate y

Subtract 25 from both sides:

$$
x^2 + 8x + 17 - 25 = 8 y
$$
$$
x^2 + 8x - 8 = 8 y

\]

Divide both sides by 8:

$$
y = \frac{1}{8} (x^2 + 8x - 8)
$$

6. Write in Vertex or Standard Form

It’s often helpful to write the equation in vertex form by completing the square.

Completing the Square to Find the Vertex Form

Starting with:

$$
y = \frac{1}{8} (x^2 + 8x - 8)
$$

Factor out $\frac{1}{8}$:

$$
y = \frac{1}{8} x^2 + \frac{1}{8} \cdot 8 x - \frac{1}{8} \cdot 8
$$

$$
y = \frac{1}{8} x^2 + x - 1
$$

Focus on completing the square for the quadratic part:

$$
\frac{1}{8} x^2 + x
$$

Rewrite as:

$$
\frac{1}{8} x^2 + x = \frac{1}{8} x^2 + \frac{8}{8} x
$$

Factor out $\frac{1}{8}$:

$$
= \frac{1}{8} (x^2 + 8 x)
$$

Complete the square inside:

$$
x^2 + 8 x = (x + 4)^2 - 16
$$

So,

$$
\frac{1}{8} (x + 4)^2 - \frac{1}{8} \times 16 = \frac{1}{8} (x + 4)^2 - 2
$$

Therefore, the equation becomes:

$$
y = \frac{1}{8} (x + 4)^2 - 2 - 1
$$
$$
y = \frac{1}{8} (x + 4)^2 - 3
$$

Final Equation of the Parabola

$$
\boxed{
\boxed{ y = \frac{1}{8} (x + 4)^2 - 3 }
}
$$

This is the standard form of the parabola's equation, opening upward, with vertex at $(-4, -3)$.

Key Properties of the Parabola

Vertex

The vertex form indicates the vertex is at:

$$
(-4, -3)
$$

Axis of Symmetry

Since the parabola is expressed as $y = a (x - h)^2 + k$, the axis of symmetry is the vertical line:

$$
x = -4
$$

Focus and Directrix

- The focus is at $(-4, -1)$, which is 2 units above the vertex at $y = -3$.

- The directrix is a horizontal line below the vertex at:

$$
y = -3 - p
$$

where p is the distance from the vertex to the focus.

Distance p

From the focus at -1 to the vertex at -3:

$$
p = 2
$$

This confirms that the parabola opens upward, consistent with earlier observations.

Summary of the Parabola's Characteristics

Property	Description
Focus	$(-4, -1)$
Directrix	$y = -5$
Vertex	$(-4, -3)$
Equation	$y = \frac{1}{8} (x + 4)^2 - 3$
Opening	Upward
Axis of Symmetry	$x = -4$

Additional Tips for Solving Similar Problems

- Always identify the focus and directrix and understand their relative positions.
- Use the distance formula to set up the fundamental parabola property.
- When the directrix is horizontal, the parabola opens upward or downward depending on the focus position.
- Complete the square to convert the equation into vertex form for easier interpretation.
- Verify the vertex, focus, and directrix positions visually or algebraically to ensure accuracy.

Conclusion

Determining the equation of a parabola from its focus and directrix involves understanding the geometric definitions and applying algebraic techniques to derive

Frequently Asked Questions

What is the standard form equation of a parabola with focus at (-4, -1) and directrix y = -5?

The parabola opens upward with vertex at the midpoint between the focus and directrix, which is at (-4, -3). Its equation is $(x + 4)^2 = 4p(y + 3)$, where $p = 2$, so the equation is $(x + 4)^2 = 8(y + 3)$.

How do you determine the value of p for the parabola with focus at (-4, -1) and directrix y = -5?

p is the distance from the vertex to the focus (or directrix). The focus is at (-4, -1) and the vertex is at (-4, -3), so $p = |-1 - (-3)| = 2$.

What is the vertex of the parabola with focus (-4, -1) and directrix y = -5?

The vertex is midway between the focus and directrix along the axis of symmetry, at (-4, -3).

Does the parabola with focus (-4, -1) and directrix y = -5 open upward or downward?

Since the focus is above the directrix and p is positive, the parabola opens upward.

Can you express the parabola's equation in terms of x and y coordinates?

Yes, with the vertex at (-4, -3) and $p = 2$, the equation is $(x + 4)^2 = 8(y + 3)$.

Additional Resources

What Is The Equation Of A Parabola Whose Focus Is (-4, -1) and Whose Directrix Is y = -5?

Understanding the precise mathematical form of a parabola given its focus and directrix is a fundamental problem in conic sections. This inquiry not only illuminates core geometric principles but also demonstrates the elegance of algebraic methods in deriving equations that describe complex curves. In this detailed exploration, we will thoroughly investigate how to determine the equation of a parabola with the focus at (-4, -1) and the directrix y = -5, providing a comprehensive guide suitable for students, educators, and enthusiasts alike.

Introduction to Parabolas: Focus and Directrix

A parabola is a symmetric, U-shaped curve that can be defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. This geometric definition forms the foundation for deriving the algebraic equation of the parabola when the focus and directrix are known.

Key Concepts:

- Focus (F): A fixed point outside the parabola.
- Directrix (D): A fixed line outside the parabola.
- Parabola Definition: The set of all points $P(x, y)$ such that the distance from P to the focus F equals the perpendicular distance from P to the directrix D.

Mathematically, for any point $P(x, y)$ on the parabola:

$$\text{Distance}(P, F) = \text{Distance}(P, D)$$

Geometric Setup of the Problem

In our specific case, the focus is at $F(-4, -1)$, and the directrix is the horizontal line:

$$
y = -5
$$

This setup suggests a parabola that opens upward or downward, depending on the focus and directrix positioning.

Key observations:

- The focus is at $y = -1$, which is above the directrix $y = -5$.
- The parabola opens upward because the focus is above the directrix, consistent with the focus being "above" the directrix.

Step-by-Step Derivation of the Parabola Equation

To find the algebraic equation, we follow the fundamental geometric definition:

\[
\text{Distance}(P, F) = \text{Distance}(P, D)

$$

where:

- $P(x, y)$ is a generic point on the parabola,
- $F(-4, -1)$,
- $D: y = -5$.

---

1. Expressing the Distance from $P(x, y)$ to the Focus $F(-4, -1)$:

$$
\text{Distance}(P, F) = \sqrt{(x + 4)^2 + (y + 1)^2}
$$

---

2. Expressing the Distance from $P(x, y)$ to the Directrix $y = -5$:

Since the directrix is a horizontal line, the perpendicular distance from P to the line is:

$$
| y - (-5) | = | y + 5 |
$$

---

3. Setting Up the Equation:

By the parabola definition:

$$
\sqrt{(x + 4)^2 + (y + 1)^2} = | y + 5 |
$$

Because the focus is above the directrix, and the parabola opens upward, y will be greater than -5, making $y + 5$ positive, allowing us to drop the absolute value:

$$
\sqrt{(x + 4)^2 + (y + 1)^2} = y + 5
$$

---

4. Solving for the Equation:

Square both sides:

$$
(x + 4)^2 + (y + 1)^2 = (y + 5)^2
$$

Expand both sides:

$$
(x + 4)^2 + y^2 + 2 y + 1 = y^2 + 10 y + 25
$$

Subtract y^2 from both sides:

$$
(x + 4)^2 + 2 y + 1 = 10 y + 25
$$

Bring all terms to one side:

$$
(x + 4)^2 + 2 y + 1 - 10 y - 25 = 0
$$

Combine like terms:

$$
(x + 4)^2 - 8 y - 24 = 0
$$

Finally, solve for y:

$$
(x + 4)^2 - 8 y = 24
$$
$$
-8 y = 24 - (x + 4)^2
$$
$$
y = -\frac{1}{8} \big(24 - (x + 4)^2 \big)
$$

Or, equivalently:

\[

\boxed{
y = -\frac{1}{8} (24 - (x + 4)^2)
}
\]

This is the standard form of the parabola's equation.

Analysis of the Derived Equation

The final equation:

\[
y = -\frac{1}{8} (24 - (x + 4)^2)
\]

can be rearranged to a more familiar form:

\[
(y + 3) = \frac{1}{8} (x + 4)^2
\]

by expanding and simplifying:

\[
y + 3 = \frac{1}{8} (x + 4)^2
\]

Properties of the Parabola:

- Vertex: The vertex occurs at the point where the derivative of the function is zero, or directly from the standard form, at \((h, k) \), where the parabola is written as \((y - k) = a(x - h)^2 \). From the form:

\[
y + 3 = \frac{1}{8} (x + 4)^2
\]

- The vertex is at \((-4, -3) \).

- Focus and directrix verification:

- The focus is at \((-4, -1) \), which is above the vertex at \(y = -3 \), consistent with the parabola opening upward.

- The directrix is at $y = -5$, which is below the vertex, confirming the symmetry.

Graphical Interpretation and Validation

Plotting the parabola based on the derived equation confirms its shape and the positions of the focus and directrix:

- The vertex at $(-4, -3)$,
- The focus at $(-4, -1)$,
- The directrix line at $y = -5$.

The parabola opens upward, with the vertex equidistant from focus and directrix, satisfying the fundamental geometric property.

Summary of Key Findings

- The parabola with focus at $(-4, -1)$ and directrix $y = -5$ is described algebraically by:

$$
\boxed{
y = -\frac{1}{8}(24 - (x + 4)^2)
}
$$

- Its vertex is at $(-4, -3)$,
- It opens upward,
- The focus is at $(-4, -1)$,
- The directrix is at $y = -5$.

Conclusion: Significance and Applications

This derivation exemplifies the synergy between geometric intuition and algebraic manipulation in understanding conic sections. Parabolas are pivotal in diverse fields such as physics (projectile motion), engineering (antenna design), and architecture. By mastering the method to derive equations from focus and directrix, students and professionals can analyze and design parabolic structures and systems with precision.

Understanding the explicit equation also facilitates computational modeling, simulation, and further analytical studies, fostering deeper insights into the properties and applications of parabolic curves.

Final Remarks

The process outlined demonstrates a systematic approach to solving a classic problem in conic sections. It emphasizes the importance of:

- Recognizing the geometric configuration,
- Applying the distance definition,
- Algebraically manipulating to reach a standard form,
- Validating the result through geometric properties.

Such thorough analysis ensures accurate comprehension and practical utility in both academic and real-world contexts.

In essence, the equation of the parabola with focus at $(-4, -1)$ and directrix $y = -5$ is:

$$
\boxed{
y = -\frac{1}{8} (24 - (x + 4)^2)
}
$$

This explicit form encapsulates the parabola's geometric essence, providing a foundation for further exploration and application.

What Is The Equation Of A Parabola Whose Focus Is 4 1 And Whose Directrix Isy 5 Xy X 18oy

What Is The Equation Of A Parabola Whose Focus Is 4 1 And Whose Directrix Isy 5 Xy X 18oy

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