What Values Of A And B Make F(x) = X3 + Ax2 +bx Have:a) A Local Max At X = -1 And A Local Max At X =

What Values Of A And B Make F(x) = X3 + Ax2 + Bx Have: a) A Local Max At X = -1 And A Local Max At X =

Understanding the behavior of cubic functions, especially when analyzing their critical points such as local maxima and minima, is fundamental in calculus and algebra. In particular, determining the specific values of parameters A and B in the cubic function

$$F(x) = x^3 + A x^2 + B x$$

that produce local maxima at given points provides insight into the function's shape and critical behaviors. This article aims to explore the conditions under which the function $F(x)$ exhibits local maxima at $x = -1$ and another specified point, elucidate the derivation process, and present comprehensive solutions.

Understanding the Function and Critical Points

Before delving into the specific conditions for A and B, it's essential to revisit some foundational concepts related to cubic functions and critical points.

Definition of Critical Points and Local Maxima

A critical point of a function occurs where its first derivative is zero or undefined. For the cubic function

$$F(x) = x^3 + A x^2 + B x,$$

the critical points will be found by solving

$$F'(x) = 0.$$

A critical point is a local maximum if the second derivative at that point is negative, indicating a concave downward curvature.

Derivative Calculations

Calculating the first and second derivatives of $F(x)$:

$$F'(x) = 3x^2 + 2A x + B.$$

$$F''(x) = 6x + 2A.$$

These derivatives are pivotal in determining the critical points and their nature.

Finding Critical Points of $F(x)$

To analyze the maxima, we begin by solving the first derivative equation.

First Derivative Equation

Set

$$F'(x) = 3x^2 + 2A x + B = 0.$$

This quadratic in x determines the critical points.

Conditions for Critical Points

Suppose the function has critical points at $x = c$. Then,

$$3c^2 + 2A c + B = 0.$$

Given the problem states that $F(x)$ has a local maximum at $x = -1$, we can substitute $c = -1$:

$$3(-1)^2 + 2A(-1) + B = 0 \Rightarrow 3 - 2A + B = 0.$$

This yields the first relation:

$$B = 2A - 3.$$

Condition for a Local Maximum at $x = -1$

To confirm that $x = -1$ corresponds to a local maximum, we examine the second derivative at this point.

Second Derivative at $x = -1$

$$F''(-1) = 6(-1) + 2A = -6 + 2A.$$

For a local maximum, the second derivative must be negative:

$$F''(-1) < 0 \Rightarrow -6 + 2A < 0 \Rightarrow 2A < 6 \Rightarrow A < 3.$$

Thus, the parameter A must be less than 3 for the critical point at $x = -1$ to be a maximum.

Determining the Second Critical Point and Corresponding Max

The derivative quadratic

$$3x^2 + 2A x + B = 0$$

has two roots, corresponding to potential maxima or minima.

Since one critical point is at $x = -1$, the other critical point, say $x = c$, is found via quadratic formula:

$$x = \frac{-2A \pm \sqrt{(2A)^2 - 4 \times 3 \times B}}{2 \times 3} = \frac{-2A \pm \sqrt{4A^2 - 12B}}{6}.$$

Recall from earlier that

$$B = 2A - 3.$$

Substitute into the discriminant:

$$4A^2 - 12(2A - 3) = 4A^2 - 24A + 36.$$

For real critical points, the discriminant must be non-negative:

$$4A^2 - 24A + 36 \geq 0.$$

Divide through by 4:

$$A^2 - 6A + 9 \geq 0.$$

Note that

$$A^2 - 6A + 9 = (A - 3)^2.$$

Since squares are always non-negative,

$$ (A - 3)^2 \geq 0, $$

which is always true. Therefore, the quadratic always has real roots for any real A.

But the nature (max or min) depends on the second derivative at those points.

Ensuring the Second Critical Point Is a Local Minimum

Given $F''(x) = 6x + 2A$, evaluate at the second critical point c:

$$ c = \frac{-2A + \sqrt{(A - 3)^2}}{6} \text{ or } \frac{-2A - \sqrt{(A - 3)^2}}{6}. $$

Since $\sqrt{(A - 3)^2} = |A - 3|$, the two critical points are:

1. $c_1 = \frac{-2A + |A - 3|}{6}$,
2. $c_2 = \frac{-2A - |A - 3|}{6}$.

To determine whether these are maxima or minima, evaluate $F''(c)$.

Summary of Conditions for A and B

Bringing all pieces together, we derive the conditions:

- For a local maximum at $x = -1$:

$$ B = 2A - 3, $$

with

$$ A < 3 $$ (to ensure $F''(-1) < 0$).

- The second critical point's nature depends on the value of A:

- When $A < 3$, the second critical point is a minimum.
- When $A > 3$, the second critical point's second derivative is positive, indicating a local minimum or maximum depending on the curvature.

Given the above, for the function to have two local maxima at specified points, additional conditions are necessary. However, the original question seems to specify only the existence of a local maximum at $x = -1$ and another at a specified point.

Conclusion: Final Conditions on A and B

Based on the detailed analysis, the key findings are:

- Value of B in terms of A:

$$ \boxed{B = 2A - 3}. $$

- Range of A for a local maximum at $x = -1$:

$$ \boxed{A < 3}. $$

- Second critical point's nature:

- At $x = \frac{-2A + |A - 3|}{6}$, the second derivative determines whether it's a maximum or minimum.

- Additional considerations:

- To specify the exact second maximum at a particular point, set the critical point accordingly and analyze the second derivative there.

Practical Example

Suppose we want the function to have a local maximum at $x = -1$. Choose $A = 2$, which satisfies $A < 3$.

Calculate B:

$$ B = 2(2) - 3 = 4 - 3 = 1. $$

Thus, the function becomes:

$$ F(x) = x^3 + 2x^2 + x. $$

Verify the critical point at $x = -1$:

$$ F'(-1) = 3(-1)^2 + 2(2)(-1) + 1 = 3 - 4 + 1 = 0. $$

Second derivative at $x = -1$:

$$ F''(-1) = 6(-1) + 2(2) = -6 + 4 = -2 < 0, $$

confirming a local maximum.

The other critical point:

$$ c = \frac{-2A + |A - 3|}{6} = \frac{-4 + 1}{6} = \frac{-3}{6} = -0.5. $$

Evaluate $F''(-0.5)$:

$$ F''(-0.5) = 6(-0.5) + 4 = -3 + 4 = 1 > 0, $$

indicating a local minimum at $x = -0.5$.

Summary

In conclusion, to achieve a cubic function \(F(x

Frequently Asked Questions

What conditions on A and B ensure that the function F(x) = x^3 + Ax^2 + Bx has a local maximum at x = -1?

For $F(x) = x^3 + Ax^2 + Bx$, the critical points occur where $F'(x) = 3x^2 + 2A x + B = 0$. To have a local maximum at $x = -1$, the derivative must be zero at $x = -1$, so $3(-1)^2 + 2A(-1) + B = 0$, which simplifies to $3 - 2A + B = 0$. Additionally, the second derivative $F''(x) = 6x + 2A$ must be negative at $x = -1$ for a maximum: $F''(-1) = -6 + 2A < 0$, so $2A < 6$, or $A < 3$.

How do the values of A and B relate to the existence of a local maximum at x = 1 for the function F(x) = x^3 + Ax^2 + Bx?

To have a local maximum at $x = 1$, set $F'(1) = 0$: $3(1)^2 + 2A(1) + B = 0$, which simplifies to $3 + 2A + B = 0$. Also, the second derivative at $x = 1$, $F''(1) = 6(1) + 2A = 6 + 2A$, must be less than zero for a maximum: $6 + 2A < 0$, so $A < -3$. Combining these, $B = -3 - 2A$, with $A < -3$.

Can A and B be determined uniquely if the function F(x) has both a local maximum at x = -1 and at x = 1?

Yes, by setting the derivatives to zero at both points: For $x = -1$, $3 - 2A + B = 0$; for $x = 1$, $3 + 2A + B = 0$. Solving these simultaneously, subtract the second from the first: $(3 - 2A + B) - (3 + 2A + B) = 0 - 0$, which simplifies to $-4A = 0$, so $A = 0$. Plug $A = 0$ into either equation to find B: $3 - 0 + B = 0$, so $B = -3$. Therefore, $A = 0$ and $B = -3$ satisfy both conditions.

What is the significance of the second derivative test in determining the nature of critical points for $F(x) = x^3 + Ax^2 + Bx$?

The second derivative test helps determine whether a critical point is a maximum or minimum. For $F(x) = x^3 + Ax^2 + Bx$, the second derivative is $F''(x) = 6x + 2A$. At a critical point where $F'(x) = 0$, if $F''(x) < 0$, the point is a local maximum; if $F''(x) > 0$, it's a local minimum. This test is essential for confirming the nature of critical points at specified x-values.

Additional Resources

What Values Of A And B Make $F(x) = x^3 + Ax^2 + Bx$ Have: a) A Local Max At $x = -1$ and a Local Max At x = ?

Introduction

In the realm of calculus and mathematical analysis, understanding the critical points of a function—particularly local maxima and minima—is fundamental. These points reveal the behavior of the function, such as peaks and valleys, which are crucial in optimization problems, physics, economics, and various applied sciences.

This article investigates the specific question: What values of A and B in the cubic function $F(x) = x^3 + A x^2 + B x$ produce a local maximum at $x = -1$ and another local maximum at some $x = c$?

While the problem as stated involves two local maxima, the initial focus is to analyze the conditions under which the function exhibits a local maximum at a specified point, notably at $x = -1$, and then extend this to find conditions for a second maximum at an arbitrary $x = c$. This detailed exploration not only reveals the relationships between the coefficients A and B but also deepens our understanding of cubic functions' critical point structure.

Fundamental Concepts and Approach

Before delving into the specifics, it’s essential to recall some core principles:

- Critical Points: Values of x where $F'(x) = 0$. These are potential locations of local maxima, minima, or saddle points.
- Second Derivative Test: If $F''(x) < 0$ at a critical point, it’s a local maximum; if $F''(x) > 0$, it’s a local minimum.
- Given Function: $F(x) = x^3 + A x^2 + B x$

The derivatives are:

$$
F'(x) = 3x^2 + 2A x + B
$$

\]

\[
F''(x) = 6x + 2A
\]

The critical points satisfy \(F'(x) = 0\). The nature (maxima or minima) depends on the second derivative at these points.

Part 1: Conditions for a Local Maximum at \(x = -1\)

Step 1: Find necessary conditions at \(x = -1\)

Since we want a local maximum at \(x = -1\), then:

- \(F'(-1) = 0\)
- \(F''(-1) < 0\)

Calculating \(F'(-1)\):

\[
F'(-1) = 3(-1)^2 + 2A(-1) + B = 3 - 2A + B
\]

Set this to zero:

\[
3 - 2A + B = 0 \quad \Rightarrow \quad B = 2A - 3
\]

This relation links B and A directly.

Next, examine the second derivative at \(x = -1\):

\[
F''(-1) = 6(-1) + 2A = -6 + 2A
\]

For a local maximum:

\[
F''(-1) < 0 \quad \Rightarrow \quad -6 + 2A < 0 \quad \Rightarrow \quad 2A < 6 \quad \Rightarrow \quad A < 3
\]

Thus, the coefficient A must satisfy:

\[
A < 3

and B is related to A via:

$$
B = 2A - 3
$$

Summary of Part 1:

Condition	Equation / Inequality	Explanation
Critical point at $x = -1$	$3 - 2A + B = 0$	Ensures $F'(-1) = 0$
Local maximum at $x = -1$	$A < 3$	Ensures $F''(-1) < 0$

Part 2: Determining the Second Local Maximum at $x = c$

The question hints at the possibility of a second local maximum at some $x = c$. For the function to have two local maxima, the critical points must satisfy:

$$
F'(x) = 3x^2 + 2A x + B = 0
$$

which is quadratic in x. Since the critical points are roots of $F'(x)$, the quadratic must have two real roots for two critical points.

Part 3: Conditions for Two Critical Points

Recall the quadratic:

$$
3x^2 + 2A x + B = 0
$$

Discriminant:

$$
\Delta = (2A)^2 - 4 \times 3 \times B = 4A^2 - 12 B
$$

For two real critical points:

$$
\Delta > 0 \quad \Rightarrow \quad 4A^2 - 12 B > 0
$$

or equivalently:

$$
A^2 - 3 B > 0
$$

Using the relation from Part 1, $B = 2A - 3$, substitute into the discriminant:

$$
A^2 - 3(2A - 3) = A^2 - 6A + 9
$$

Factor:

$$
A^2 - 6A + 9 = (A - 3)^2
$$

Since square terms are always non-negative, the discriminant:

$$
\Delta = 4 (A - 3)^2
$$

which is always ≥ 0, with equality only when $A = 3$. For $\Delta > 0$:

$$
A \neq 3
$$

and for two distinct critical points, the condition is:

$$
A \neq 3
$$

Furthermore, at $A = 3$:

$$
\Delta = 0
$$

and the critical point is a repeated root, implying a possible inflection point.

Part 4: Nature of Critical Points and Local Maxima

To determine whether each critical point is a maximum or minimum, analyze the second derivative at each:

$$
F''(x) = 6x + 2A
$$

- At $x = -1$:

$$
F''(-1) = -6 + 2A
$$

which we already used to ensure a maximum when $A < 3$.

- At the other critical point(s), say at $x = c$:

$$
x_c = \frac{-2A \pm \sqrt{\Delta}}{6}
$$

Substituting $B = 2A - 3$ into the quadratic's roots:

$$
x_{1,2} = \frac{-2A \pm \sqrt{4A^2 - 12B}}{6}
$$

Given the previous substitution, the roots simplify to:

$$
x_{1,2} = \frac{-2A \pm 2|A - 3|}{6} = \frac{-A \pm |A - 3|}{3}
$$

Now, analyze the two cases:

1. When $A > 3$:

$$
|A - 3| = A - 3
$$
$$
x_1 = \frac{-A + (A - 3)}{3} = \frac{-3}{3} = -1
$$
$$
x_2 = \frac{-A - (A - 3)}{3} = \frac{-A - A + 3}{3} = \frac{-2A + 3}{3}
$$

Since $A > 3$, then:

$$
x_2 = \frac{-2A + 3}{3} < 0
$$

- At $x = -1$, the second derivative:

$$
F''(-1) = -6 + 2A
$$

which is positive if $A > 3$, indicating a local minimum at this point, conflicting with our initial goal for a maximum.

2. When $A < 3$:

$$
|A - 3| = 3 - A
$$
$$
x_1 = \frac{-A + (3 - A)}{3} = \frac{3 - 2A}{3}
$$
$$
x_2 = \frac{-A - (3 - A)}{3} = \frac{-A - 3 + A}{3} = \frac{-3}{3} = -1
$$

In this case, one critical point is at $x = -1$, and the other at $x = \frac{3 - 2A}{3}$.

- The second derivative at $x = -1$:

$$
F''(-1) = -6 + 2A < 0 \quad \text{(since $A<3$)},
$$

consistent with a maximum at $x = -1$.

- At the other critical point $x = \frac{3 - 2A}{3}$:

$$
F''(x) = 6x + 2A
$$

Plugging in \(x = \frac{3 -

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