Which Conic Section Is Represented By The Following Equation? 5x2 10xy 5y2 2x Y 4 = 0 Circle Ellipse

Which Conic Section Is Represented By The Following Equation? 5x2 10xy 5y2 2x Y 4 = 0 Circle Ellipse

Understanding conic sections is fundamental in algebra and geometry, as they form the basis for many advanced mathematical concepts and applications in physics, engineering, and computer graphics. The given equation, although seemingly complex, can be analyzed systematically to identify which conic section it represents. In this article, we will explore the process of analyzing the equation, simplifying and classifying it, and understanding the properties of the conic section it depicts.

Analyzing the Given Equation

The equation provided is:

$$5x^2 + 10xy + 5y^2 + 2x y + 4 = 0$$

At first glance, this appears to be a quadratic equation in two variables, involving both x^2, y^2, and the mixed term xy. This suggests that the conic might be rotated or skewed, requiring a thorough analysis to classify it.

Step 1: Recognizing the General Form of a Conic

A general second-degree equation in x and y is:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

In our case:

- $A = 5$
- $B = 10$
- $C = 5$
- $D = 0$ (since no x term)

- $E = 0$ (since no y term)
- $F = 4$

The presence of the xy term indicates that the conic may be rotated relative to the coordinate axes, which affects classification.

Step 2: Computing the Discriminant

The discriminant for classifying conic sections is:

$$ \Delta = B^2 - 4AC $$

Calculating:

$$ \Delta = (10)^2 - 4 \times 5 \times 5 = 100 - 100 = 0 $$

The discriminant being zero is significant—it suggests the conic is a parabola, a degenerate conic, or a special case like a paraboloid in three dimensions. But further analysis is necessary because of the xy term.

Step 3: Forming the Quadratic Form and Diagonalization

Since the equation contains a mixed term, it can be diagonalized via rotation of axes to eliminate the xy term. This process involves:

- Constructing the matrix form of the quadratic part
- Rotating axes to remove the cross-term

The quadratic form matrix:

$$
Q = \begin{bmatrix}
A & B/2 \\
B/2 & C
\end{bmatrix} =
\begin{bmatrix}
5 & 5 \\
5 & 5
\end{bmatrix}
$$

Note that $B/2 = 10/2 = 5$.

Diagonalizing the Quadratic Form

To classify the conic, we need to find the eigenvalues and eigenvectors of matrix Q. This will help us identify the principal axes and the type of conic.

Eigenvalues Calculation

The characteristic equation:

$$
\det(Q - \lambda I) = 0
$$

$$
\det \left(\begin{bmatrix} 5 - \lambda & 5 \\ 5 & 5 - \lambda \end{bmatrix} \right) = 0
$$

Calculating the determinant:

$$
(5 - \lambda)(5 - \lambda) - 25 = 0
$$

$$
(5 - \lambda)^2 - 25 = 0
$$

$$
(5 - \lambda)^2 = 25
$$

$$
5 - \lambda = \pm 5
$$

Therefore:

- When $5 - \lambda = 5$, $\lambda = 0$
- When $5 - \lambda = -5$, $\lambda = 10$

Eigenvalues are:

$$
\lambda_1 = 0, \quad \lambda_2 = 10
$$

Eigenvectors

- For $\lambda=0$:

$$
(Q - 0 \cdot I) \mathbf{v} = 0
$$

$$
\begin{bmatrix}
5 & 5 \\
5 & 5
\end{bmatrix} \mathbf{v} = 0
$$

This implies:

$$
5 v_x + 5 v_y = 0 \Rightarrow v_x = -v_y
$$

Eigenvector:

$$
\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}
$$

- For $\lambda=10$:

$$
\begin{bmatrix}
-5 & 5 \\
5 & -5

\end{bmatrix} \mathbf{v} = 0
\]

\[
-5 v_x + 5 v_y = 0 \Rightarrow v_y = v_x
\]

Eigenvector:

\[
\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

Reclassification of the Conic

Given the eigenvalues:

- One is zero (\(\lambda=0\))
- One is positive (\(\lambda=10\))

The presence of a zero eigenvalue indicates a degenerate case or a conic that can be classified as a parabola, hyperbola, or ellipse depending on the context.

But since the discriminant is zero, and the quadratic form matrix has eigenvalues with different signs, the conic is a parabola or a degenerate conic.

However, the classification depends on the full quadratic form, including linear and constant terms, and the nature of the conic.

Completing the Square and Rotation

To better understand the shape, we can perform a rotation to eliminate the \(xy\) term and rewrite the equation in a standard form.

Step 1: Rotation Angle

The rotation angle θ is given by:

$$
\tan 2\theta = \frac{B}{A - C}
$$

In this case:

$$
\tan 2\theta = \frac{10}{5 - 5} = \frac{10}{0}
$$

Since denominator is zero, $\tan 2\theta$ is undefined, implying $2\theta = 90^\circ$ or $\theta = 45^\circ$.

This means rotating axes by 45° will eliminate the xy term.

Step 2: Rotation of Coordinates

Define new variables:

$$
x' = x \cos \theta + y \sin \theta
$$
$$
y' = -x \sin \theta + y \cos \theta
$$

With $\theta=45^\circ$:

$$
x' = \frac{x + y}{\sqrt{2}}, \quad y' = \frac{-x + y}{\sqrt{2}}
$$

Substituting into the original equation and simplifying will give a clearer form to classify the conic.

Final Classification and Conclusion

Based on the analysis:

- The discriminant $\Delta = 0$
- Eigenvalues indicate a degenerate or parabolic form
- Rotation to eliminate the xy term suggests the conic is a parabola

Therefore, the conic section represented by the given equation is a parabola.

Summary of Key Points

- The quadratic form matrix's eigenvalues help classify conics
- A zero discriminant often indicates a parabola or degenerate conic
- Rotation of axes simplifies the equation, revealing its nature
- The analysis confirms that the given equation corresponds to a parabola

---

Additional Notes on Conic Sections

Conic Section	General Equation	Discriminant (Δ)	Key Features
Circle	$x^2 + y^2 + Dx + Ey + F = 0$	$\Delta < 0$	Equal coefficients for x^2 and y^2, no xy term
Ellipse	$Ax^2 + Cy^2 + Dx + Ey + F = 0$, $A \neq C$	$\Delta < 0$	Different coefficients, no xy term
Hyperbola	$Ax^2 - Cy^2 + Dx + Ey + F = 0$	$\Delta > 0$	Opposite signs in quadratic terms
Parabola	$A x^2 + D x + E y + F = 0$ or with rotated axes	$\Delta = 0$	One quadratic term dominates

---

Conclusion

The detailed algebraic analysis, including matrix diagonal

Frequently Asked Questions

What type of conic section is represented by the equation $5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$?

The given equation represents a rotated conic, specifically an ellipse, if it can be rewritten in standard form with a positive definite quadratic form.

How can I determine whether the conic section is a circle or an ellipse from its general quadratic equation?

Calculate the discriminant $B^2 - 4AC$; if it is less than zero, the conic is an ellipse (including a circle if $A = C$ and $B = 0$). For the given equation, if it simplifies to an ellipse, it is not a circle due to unequal coefficients.

Does the equation $5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$ represent a circle or an ellipse?

This equation represents an ellipse because the quadratic form is not a perfect circle (which requires equal coefficients for x^2 and y^2 and no xy term).

Can the equation $5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$ be rewritten in standard form? If so, what does it reveal about the conic?

Yes, by completing the square and rotating axes, the equation can be transformed into standard form, revealing it as an ellipse, not a circle, due to unequal coefficients and the presence of the xy term.

What is the significance of the xy term in the general conic equation regarding the conic's orientation?

The xy term indicates that the conic is rotated relative to the coordinate axes. Eliminating this term through rotation helps identify the conic's true shape—ellipse, circle, parabola, or hyperbola.

Based on the given options, is the conic represented by the equation a **circle or an ellipse?**

Given the coefficients and the presence of the xy term, the conic is an ellipse, not a circle.

Additional Resources

Which Conic Section Is Represented By The Following Equation? $5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$: Circle or Ellipse? An In-Depth Analysis

Conic sections are fundamental in the study of mathematics, especially within coordinate geometry, due to their wide-ranging applications in physics, engineering, astronomy, and computer graphics. Determining the specific conic represented by a given quadratic equation is a core skill that involves understanding the general form, discriminants, and the effects of rotation and translation on conic graphs. This article aims to thoroughly analyze the equation:

$5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$

and identify whether it depicts a circle or an ellipse, providing detailed insights into the methods and reasoning involved.

Understanding the General Equation of Conic Sections

Any conic section in Cartesian coordinates can be expressed as a second-degree polynomial:

$$ Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 $$

where:

- A, B, C, D, E, F are real coefficients.

The classification of the conic depends primarily on the coefficients A, B, C. The discriminant Δ for conic classification is given by:

$$ \Delta = B^2 - 4AC $$

- If $\Delta < 0$, the conic is an ellipse (which includes the circle as a special case).
- If $\Delta = 0$, the conic is a parabola.

- If $\Delta > 0$, the conic is a hyperbola.

In addition, the presence of the xy term indicates that the conic may be rotated relative to the axes, necessitating further analysis to classify it accurately.

Analyzing the Given Equation

The specific equation under consideration:

$$5x^2 + 10xy + 5y^2 + 2x + y - 4 = 0$$

has coefficients:

- $A = 5$
- $B = 10$
- $C = 5$
- $D = 2$
- $E = 1$
- $F = -4$

The first step is to compute the discriminant:

$$\Delta = B^2 - 4AC = (10)^2 - 4 \times 5 \times 5 = 100 - 100 = 0$$

Interpretation: The discriminant being zero indicates that the conic is either a parabola or a degenerate conic (e.g., a point, line, or pair of lines). However, because the quadratic form involves both x^2 and y^2 terms with equal coefficients, this warrants further investigation.

Identifying the Conic Type: Is It a Circle or an Ellipse?

The key point in this analysis is the equality of the quadratic coefficients:

$$A = C = 5$$

and the presence of the xy term. Normally, a circle is characterized by:

- Equal quadratic coefficients for x^2 and y^2,
- No xy term (or, if present, can be eliminated via rotation).

Since our equation contains an xy term, the conic is rotated relative to the axes, which complicates classification.

Step 1: Remove the xy term via rotation.

Rotation of Axes to Eliminate the xy Term

The general approach to classify the conic involves rotating axes to eliminate the xy term. The rotation angle θ satisfies:

$$\tan 2\theta = \frac{B}{A - C}$$

Plugging in the values:

$$\tan 2\theta = \frac{10}{5 - 5} = \frac{10}{0}$$

which is undefined, indicating:

$$2\theta = 90^\circ \Rightarrow \theta = 45^\circ$$

This means rotating axes by 45° will eliminate the xy term.

Transforming the Equation via Rotation

Define new axes (x', y') rotated by 45°:

$$
\begin{cases}
x = x' \cos 45^\circ - y' \sin 45^\circ \\
y = x' \sin 45^\circ + y' \cos 45^\circ
\end{cases}
$$

with:

\[\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2} \]

Substituting into the quadratic form:

\[Q(x, y) = A x^2 + B xy + C y^2 \]

leads to a new quadratic form \(Q'(x', y') \), which, after algebraic manipulation, simplifies to a form without the \(xy \) term.

Result of rotation:

- The quadratic form becomes diagonal, with eigenvalues corresponding to principal axes.
- The eigenvalues are derived from the quadratic matrix:

\[
M = \begin{bmatrix}
A & B/2 \\
B/2 & C
\end{bmatrix}
= \begin{bmatrix}
5 & 5 \\
5 & 5
\end{bmatrix}
\]

Eigenvalues \(\lambda \) satisfy:

\[\det(M - \lambda I) = 0 \]

which yields:

\[
\det
\begin{bmatrix}
5 - \lambda & 5 \\
5 & 5 - \lambda
\end{bmatrix}
= (5 - \lambda)^2 - 25 = 0
\]

\[

$$
(5 - \lambda)^2 = 25
$$

$$
5 - \lambda = \pm 5
$$

Thus:

- For $+$: $5 - \lambda = 5 \Rightarrow \lambda = 0$
- For $-$: $5 - \lambda = -5 \Rightarrow \lambda = 10$

Eigenvalues:

$$ \boxed{\lambda_1 = 0, \quad \lambda_2 = 10} $$

This indicates a degenerate quadratic form, with one principal axis corresponding to zero eigenvalue.

Implications of Eigenvalues on the Conic's Nature

The eigenvalues suggest the quadratic form is degenerate in one direction, implying the conic is either a pair of lines or a degenerate conic.

However, the zero eigenvalue indicates that along the corresponding eigenvector, the quadratic form collapses, and the conic may reduce to a line or a pair of lines.

Completing the Square and Transforming the Equation

To confirm the conic's nature, we can attempt to complete the square or rotate the coordinate axes explicitly.

Alternatively, consider the quadratic portion:

$$ 5x^2 + 10xy + 5y^2 $$

Factor out 5:

$$5(x^2 + 2xy + y^2)$$

Note that:

$$x^2 + 2xy + y^2 = (x + y)^2$$

Thus,

$$5(x + y)^2$$

The original equation simplifies to:

$$5(x + y)^2 + 2x + y - 4 = 0$$

Now, set:

$$u = x + y$$

Express x in terms of u and y:

$$x = u - y$$

Substitute into the linear terms:

$$2x + y = 2(u - y) + y = 2u - 2y + y = 2u - y$$

The equation becomes:

$$5u^2 + 2u - y - 4 = 0$$

Solve for y:

$$- y = - 5u^2 - 2u + 4 \Rightarrow y = 5u^2 + 2u - 4$$

Recall that $x = u - y$:

$$x = u - (5u^2 + 2u - 4) = u - 5u^2 - 2u + 4 = -5u^2 - u + 4$$

Thus, the parametric form:

$$x = -5u^2 - u + 4$$
$$y = 5u^2 + 2u - 4$$

for all real u.

Interpreting the Result: Is It a Circle or an Ellipse?

The parametric equations describe a quadratic relation between x and y in terms of u, with quadratic terms in u. Since the quadratic forms are in u with different signs, the locus of points (x, y) traces a conic with a particular structure.

Given the earlier analysis:

- The quadratic form reduces to a perfect square $5(x + y)^2$.
- The equation simplifies to a quadratic in u, with the relation:

$$y = 5u$$

Which Conic Section Is Represented By The Following Equation 5x2 10xy 5y2 2x Y 4 0 Circle Ellipse

Which Conic Section Is Represented By The Following Equation 5x2 10xy 5y2 2x Y 4 0 Circle Ellipse

Related Articles

- 3 Mei Is A Park Ranger At Death Valley National Park In California.She Records The Elevation Relative
- Within The Beginning Stage Of The Data Warehouse Models Process, Which Of The Following Are Included
- What Were Some Economic, Political, And Societal Problems In The United States In The Late 1800s And

Back to Home