

Which Equation Can Be Solved To Find One Of The Missing Side Lengths In The Triangle? Triangle A B C

Which Equation Can Be Solved To Find One Of The Missing Side Lengths In The Triangle? Triangle A B C

Understanding how to find missing side lengths in a triangle is a fundamental aspect of geometry. Triangles are among the most basic shapes in mathematics, and their properties are essential in various fields such as engineering, architecture, navigation, and more. When given partial information about a triangle—such as two sides, two angles, or a combination of sides and angles—mathematicians and students seek the appropriate equations to uncover the unknown measurements. In this article, we will explore the key equations used to solve for missing side lengths in triangle ABC, focusing on which equations are most suitable depending on the given data.

Understanding Triangle Types and Given Data

Before diving into the specific equations, it's important to recognize the different types of triangles and the common data provided:

- Scalene Triangle: All sides and angles are different.
- Isosceles Triangle: Two sides and two angles are equal.
- Equilateral Triangle: All sides and angles are equal.

The information available often falls into categories such as:

- Two sides and an included angle (SAS)
- Two angles and a side (AAS or ASA)
- All three sides (SSS)
- A side and an angle not included (SSA, which can be ambiguous)

Depending on what data you have, different equations are applicable to find the missing side(s).

Main Equations for Finding Missing Side Lengths

Various mathematical formulas serve as tools for solving for missing side lengths in triangles. The choice of equation depends on the known data.

Sine Rule (Law of Sines)

The Law of Sines relates the ratios of each side to the sine of its opposite angle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This law is most useful when:

- Two angles and one side are known (AAS or ASA).
- Two sides and a non-included angle are known (SSA), though caution is needed due to possible ambiguous cases.

Equation to find a missing side:

$$a = \frac{\sin A \times b}{\sin B}$$

or similarly, for sides b or c :

$$b = \frac{\sin B \times a}{\sin A}$$

$$c = \frac{\sin C \times a}{\sin A}$$

Application example:

Suppose you know angles A and B , and side b . To find side a , you can rearrange the Law of Sines:

$$a = \frac{\sin A \times b}{\sin B}$$

Cosine Rule (Law of Cosines)

The Law of Cosines is particularly effective when:

- Two sides and the included angle are known (SAS).
- All three sides are known (SSS).

The general form:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

To find a missing side, rearranged as:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Application example:

Given sides (b) and (c) , and included angle (A) , to find side (a) :

$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

This equation is often called the "Side Length" form of the Law of Cosines.

Practical Scenarios and Which Equation to Use

Identifying the correct equation depends on the known data. Here's a guide to help determine which formula to use:

Scenario 1: Two Sides and Included Angle (SAS)

- Known: sides (a) and (b) ; included angle (C) .
- Goal: Find side (c) .

Equation to use: Law of Cosines

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

Scenario 2: Two Angles and a Side (AAS or ASA)

- Known: angles $\angle A$ and $\angle B$; side a .
- Goal: Find side b and then side c .

Equation to use: Law of Sines

$$b = \frac{\sin B \times a}{\sin A}$$

Once side b is found, use Law of Sines again to find c :

$$c = \frac{\sin C \times b}{\sin B}$$

Scenario 3: All Three Sides (SSS)

- Known: sides a , b , c .
- Goal: Find angles or verify side lengths.

Equation to use: Law of Cosines

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Similarly, for angles $\angle A$ and $\angle B$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Scenario 4: Ambiguous Case (SSA)

- Known: side (a) , side (b) , and angle (A) , where (A) is not between sides (a) and (b) .
- Note: The solution might be ambiguous; sometimes, no solution or two solutions are possible.

Equation to explore: Law of Sines

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

You solve for $(\sin B)$:

$$\sin B = \frac{b \sin A}{a}$$

Check if $(\sin B \leq 1)$ to verify whether a solution exists.

Step-by-Step Example: Solving for a Missing Side

Suppose you are given the following data:

- Triangle ABC
- Side $(AB = 7, \text{units})$
- Side $(AC = 10, \text{units})$
- Included angle $(A = 60^\circ)$

Goal: Find side (BC) .

Solution:

1. Recognize that with two sides and included angle, the Law of Cosines applies.
2. Use the Law of Cosines:

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$$

3. Plug in known values:

$$BC^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ$$

$$BC^2 = 49 + 100 - 2 \times 7 \times 10 \times 0.5$$

$$BC^2 = 149 - 70$$

$$BC^2 = 79$$

4. Take the square root:

$$BC = \sqrt{79} \approx 8.89, \text{ units}$$

This calculation demonstrates the use of the Law of Cosines to find a missing side when two sides and an included angle are known.

Summary: Choosing the Correct Equation

Known Data	Equation to Use	Notes
Two sides and included angle (SAS)	Law of Cosines	Directly find the third side
Two angles and a side (AAS or ASA)	Law of Sines	Find other sides, then angles if needed
All three sides (SSS)	Law of Cosines	Find angles or verify side lengths
Two sides and a non-included angle (SSA)	Law of Sines (with caution)	Possible ambiguous case; check $(\sin B \leq 1)$

Conclusion

In solving for missing side lengths in triangle ABC, the most versatile and widely used equations are the Law of Sines and the Law of Cosines. The Law of Sines is particularly useful when you have angles and sides that are not adjacent, while the Law of Cosines is ideal when you know two sides and the included angle or all three sides. Understanding which equation to apply based on the given data ensures accurate and efficient solving of triangle problems. Mastering these formulas enhances spatial reasoning and problem-solving skills essential in mathematics and real-world applications.

Remember: Always verify your known data, choose the appropriate formula, and carefully perform calculations to find the missing side length in any triangle.

Frequently Asked Questions

What is the primary equation used to find a missing side in a triangle when two sides and the included angle are known?

The Law of Cosines, which is expressed as $c^2 = a^2 + b^2 - 2ab \cos C$, is used to find a missing side when two sides and the included angle are known.

Which equation can be applied if two sides and a non-included angle are known to determine the missing side in Triangle ABC?

The Law of Sines, which states that $a / \sin A = b / \sin B = c / \sin C$, can be used to find the missing side in such cases.

When can the Pythagorean theorem be used to find a missing side in Triangle ABC?

The Pythagorean theorem can be used if Triangle ABC is a right triangle, with the right angle at the vertex of the missing side.

What equation is suitable for finding a side length when two angles and a side are known in Triangle ABC?

The Law of Sines is suitable for this, as it relates angles and sides directly and can be used to solve for the missing side.

Which equation would you use if you know all three sides of Triangle ABC and want to verify a missing side?

You can use the Law of Cosines to verify or find a missing side when all three sides are known, especially if the triangle is not right-angled.

Can the equations used to find missing sides in a triangle be applied to any triangle type?

Yes, the Law of Sines and Law of Cosines can be applied to any triangle, whether acute, obtuse, or right-angled, to find missing side lengths.

Additional Resources

Which Equation Can Be Solved To Find One Of The Missing Side Lengths In The Triangle? Triangle ABC

Understanding how to find missing side lengths in a triangle is a fundamental aspect of geometry, especially when dealing with real-world problems, geometric proofs, or mathematical puzzles. Triangle ABC, like any triangle, is characterized by its three sides and three angles, with many situations arising where one or more side lengths are unknown. The key to solving these problems lies in selecting the appropriate equation or set of equations to find the missing information efficiently and accurately. This article explores the various equations that can be utilized to determine a missing side length in Triangle ABC, their applications, advantages, and limitations.

Fundamental Concepts for Finding Missing Sides in Triangle ABC

Before delving into specific equations, it's important to understand the basic concepts that underpin triangle side calculations:

- Triangle Types: Equilateral, isosceles, and scalene triangles each have different properties that influence the choice of equations.
- Knowns and Unknowns: Typically, known data include side lengths and angles; the goal is to find an unknown side length.
- Key Theorems and Laws: The Pythagorean theorem, Law of Sines, and Law of Cosines are the primary tools.

Common Equations for Finding Missing Side Lengths

Several equations are used in triangle geometry to solve for missing sides, depending on what information is available. The most commonly employed are:

1. Pythagorean Theorem

Overview

The Pythagorean theorem applies strictly to right-angled triangles. It states that:

$$a^2 + b^2 = c^2$$

where c is the hypotenuse, and a and b are the legs.

Application in Triangle ABC

- If Triangle ABC is a right triangle (with the right angle at B, for example), and two sides are known,

this theorem allows you to find the third.

Pros

- Simple and straightforward.
- Very effective when dealing with right triangles.

Cons

- Limited to right triangles; not applicable for obtuse or acute triangles unless a right angle is present.

2. Law of Sines

Overview

The Law of Sines relates the ratios of side lengths to the sines of their opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Application in Triangle ABC

- Useful when you know:
 - Two angles and one side (AAS or ASA cases), or
 - Two sides and an angle opposite one of them (SSA case, with caution).
- To find a missing side:

$$c = \frac{a \times \sin C}{\sin A}$$

or similarly for sides a or b .

Pros

- Effective for non-right triangles.
- Requires minimal data (two angles and one side or two sides and an angle).

Cons

- Cannot be used if only two sides are known without an angle (unless additional info is available).
- The SSA case may lead to ambiguous solutions (the "ambiguous case").

3. Law of Cosines

Overview

The Law of Cosines generalizes the Pythagorean theorem for any triangle:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Similarly, for sides a and b :

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Application in Triangle ABC

- When two sides and the included angle are known (SAS), this law helps find the third side.
- When all sides are known (SSS), it can be used to find an angle, but it also helps find missing sides if some data are missing.

Pros

- Versatile; applies to all types of triangles.
- Handles SAS and SSS cases efficiently.

Cons

- Slightly more complex than the Law of Sines or Pythagorean theorem.
- Requires knowledge of an included angle or all three sides.

Choosing the Right Equation Based on Known Data

The key to effectively solving for a missing side in Triangle ABC is selecting the appropriate equation based on the given information:

Known Data	Suitable Equation	Remarks
Two sides and included angle (SAS)	Law of Cosines	Use to find the third side
All three sides (SSS)	Law of Cosines	Use to find an angle or missing side
Two angles and one side (AAS or ASA)	Law of Sines	Use to find other sides
One side and two angles (less common)	Law of Sines (ambiguous case) or Law of Cosines	Use carefully; check for possible multiple solutions
Right triangle with two sides	Pythagorean Theorem	Simplest method for right-angled triangles

Practical Examples and Step-by-Step Solutions

To clarify the application of these equations, let's consider a few examples involving Triangle ABC.

Example 1: Using the Pythagorean Theorem

Suppose Triangle ABC is a right triangle with side $(AB = 7)$ units (adjacent to the right angle) and $(AC = 24)$ units. Find the missing side (BC) .

Solution:

Since it's a right triangle, and assuming (AB) and (AC) are the legs:

$$(BC = \sqrt{AB^2 + AC^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25)$$

Result: The hypotenuse $(BC = 25)$ units.

Example 2: Using Law of Sines

Given: $(\angle A = 30^\circ)$, $(\angle B = 45^\circ)$, and side $(a = 10)$ units. Find side (b) .

Solution:

First, find $(\angle C)$:

$$\angle C = 180^\circ - 30^\circ - 45^\circ = 105^\circ$$

Apply Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = \frac{a \times \sin B}{\sin A}$$

Calculate:

$$b = \frac{10 \times \sin 45^\circ}{\sin 30^\circ} = \frac{10 \times \frac{\sqrt{2}}{2}}{0.5} = \frac{10 \times 0.7071}{0.5} \approx \frac{7.071}{0.5} = 14.142$$

Result: Side $(b \approx 14.14)$ units.

Example 3: Using Law of Cosines

Given: $(a = 8)$, $(b = 15)$, and $(\angle C = 60^\circ)$. Find side (c) .

Solution:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 8^2 + 15^2 - 2 \times 8 \times 15 \times \cos 60^\circ$$

$$c^2 = 64 + 225 - 2 \times 8 \times 15 \times 0.5$$

$$c^2 = 289 - (2 \times 8 \times 15 \times 0.5)$$

$$c^2 = 289 - (2 \times 8 \times 15 \times 0.5)$$

$$c^2 = 289 - (2 \times 8 \times 15 \times 0.5)$$

$$c^2 = 289 - (2 \times 8 \times 15 \times 0.5)$$

Calculate:

$$2 \times 8 \times 15 \times 0.5 = 2 \times 8 \times 15 \times 0.5 = (2 \times 8 \times 15) \times 0.5 = (240) \times 0.5 = 120$$

Now,

$$c^2 = 289 - 120 = 169$$

$$c = \sqrt{169} = 13$$

Result: Side $(c = 13)$ units.

Conclusion: Selecting the Appropriate Equation

In summary, the choice of equation to find a missing side length in Triangle ABC hinges on the specific knowns:

- Use the Pythagorean theorem strictly for right triangles.
- Use the Law of Sines when you know:
 - Two angles and one side, or
 - Two sides and an opposite angle (with caution for ambiguous cases).
- Use the Law of Cosines in SAS or SSS scenarios, especially when dealing with non-right triangles or when angles are unknown.

Final thoughts:

[Which Equation Can Be Solved To Find One Of The Missing Side Lengths In The Triangle Triangle A B C](#)

Which Equation Can Be Solved To Find One Of The Missing Side Lengths In The Triangle Triangle A B C

Related Articles

- [A 480 Volts, 20 KW Shunt Motor Took 2.5A When Running Light.Taking \$R_a = 0.6 \text{ Ohm}\$ And \$R_f = 800 \text{ Ohms}\$ And](#)
- [What Two European Countries Did The Iroquois Confederacy Get Involved In Competing Trading Alliances](#)
- [A 550 N Physics Student Stands On A Bathroom Scale In An 850 Kg \(including The Student\) Elevator That](#)

[Back to Home](#)