Write An Equation Of An Ellipse In Standard Form With Center At The Origin And With The Given Vertex

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Understanding the equation of an ellipse is fundamental in the study of conic sections in mathematics. When the ellipse is centered at the origin and one of its vertices is known, it becomes straightforward to derive its standard form equation. This article provides a comprehensive guide on how to write the equation of such an ellipse, including definitions, step-by-step instructions, and practice examples.

What Is an Ellipse?

Before delving into the specifics of deriving the equation, it's essential to understand what an ellipse is.

Definition

An ellipse is a set of all points in a plane such that the sum of the distances from two fixed points (called foci) is constant.

Key Features of an Ellipse

- Center: The midpoint between the two vertices and foci.
- Vertices: The points on the ellipse that lie along the major axis, at the maximum distance from the center.
- Foci: Two fixed points located along the major axis, used to define the shape.
- Major Axis: The longest diameter passing through the vertices and foci.
- Minor Axis: The shortest diameter, perpendicular to the major axis at the center.

Standard Form Equation of an Ellipse

The standard form of an ellipse depends on the orientation of its major axis:

Horizontal Major Axis

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
$$

- Center at (0, 0)
- Vertex at (±a, 0)
- Foci at (±c, 0), where $c^2 = a^2 - b^2$

Vertical Major Axis

$$
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
$$

- Center at (0, 0)
- Vertex at (0, ±a)
- Foci at (0, ±c), where $c^2 = a^2 - b^2$

In this guide, we focus on the case where the center is at the origin, and the ellipse has a vertex with a known coordinate.

Given Data and Assumptions

Suppose you are provided with the following:

- The center of the ellipse is at the origin (0, 0).
- One vertex of the ellipse is given, for example, at point $(a, 0)$ or $(0, a)$.

Your goal: To write the standard form equation of the ellipse based on this information.

Step-by-Step Guide to Derive the Equation

Step 1: Identify the Orientation of the Ellipse

Determine if the vertex lies along the x-axis or y-axis:

- If the vertex is at $(a, 0)$, the ellipse has a horizontal major axis.
- If the vertex is at $(0, a)$, the ellipse has a vertical major axis.

Step 2: Determine the Values of a

Since the vertex is at a known point, the distance from the center to the vertex gives a:

- If vertex at $(a, 0)$, then a is the x-coordinate of the vertex.
- If vertex at $(0, a)$, then a is the y-coordinate of the vertex.

Step 3: Write the Equation Based on Orientation

Depending on the orientation:

- Horizontal: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Vertical: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Step 4: Find c, the Foci Distance

The foci are located at a distance c from the center along the major axis, where:

$$
c^2 = a^2 - b^2
$$

To find c, additional information is needed, such as the location of the foci or another point on the ellipse.

Step 5: Determine b

If the foci are known, or if the length of the minor axis is given, you can find b.

- If no additional data is provided, b remains a variable, and the equation will be expressed in terms of b^2.

Special Case: When Only the Vertex Is Given

Often, only one vertex is provided, and no other points or foci are given. In such cases:

- The value of a is directly obtained from the vertex.
- Without further data, b remains unknown, so the equation cannot be fully specified.
- If the problem specifies the length of the minor axis or the distance to the foci, you can compute b and c.

Example: Deriving the Equation Step-by-Step

Suppose you are given:

- The ellipse has its center at the origin.

- The vertex is at $(5, 0)$.

Step 1: Since the vertex is at $(5, 0)$, the major axis is horizontal.

Step 2: The value of a is 5.

Step 3: Write the initial equation:

$$
\frac{x^2}{25} + \frac{y^2}{b^2} = 1
$$

Step 4: To find b, additional data such as the position of the foci, the length of the minor axis, or a point on the ellipse is needed.

If the foci are at $(\pm c, 0)$, and the foci are at $(\pm 4, 0)$, then:

$$
c = 4
$$

and

$$
c^2 = a^2 - b^2 \Rightarrow 16 = 25 - b^2 \Rightarrow b^2 = 9
$$

Final Equation:

$$
\frac{x^2}{25} + \frac{y^2}{9} = 1
$$

Additional Tips and Considerations

- Always verify the orientation: Check if the major axis is horizontal or vertical based on the vertex location.
- Use symmetry: The ellipse is symmetric about its axes, which simplifies understanding its structure.
- Additional points: When multiple points on the ellipse are known, they help determine b and c.
- Foci location: Knowledge of the foci can help find c directly, completing the equation.

Practice Problems

1. The vertex of an ellipse is at \((3, 0) \), and the foci are at \((\pm 4, 0) \). Write the standard form equation.

2. An ellipse has a vertex at \((0, 7) \) and foci at \((0, \pm 8) \). Find its equation in standard form.

3. Given the vertex at \((-2, 0) \) and the minor axis length of 6, determine the equation of the ellipse.

Conclusion

Writing the equation of an ellipse in standard form with the center at the origin and a given vertex involves understanding the key features of the ellipse, such as its axes, vertices, and foci. By identifying the major axis orientation, determining the value of \(a \), and using additional data to find \(b \) and \(c \), you can accurately derive the standard form equation. Mastery of this process is essential for solving geometric problems involving ellipses and for applications across science, engineering, and mathematics.

Summary of Steps

- Identify the orientation of the ellipse based on the vertex.
- Use the vertex coordinate to find \(a \).
- Determine if additional data (foci, other points) are available to find \(b \).
- Use the relation \(c^2 = a^2 - b^2 \) to find \(b \) if possible.
- Write the equation in the standard form based on the orientation.

By following these steps carefully, you can confidently write the equation of an ellipse with the given vertex and center at the origin, enhancing your understanding of conic sections and their properties.

Frequently Asked Questions

How do you write the equation of an ellipse centered at the origin with a given vertex?

To write the equation of an ellipse centered at the origin with a vertex, identify the semi-major axis length 'a' from the vertex, then use the standard form: ○ x^2 / a^2 + ○ y^2 / b^2 = 1, where the vertex lies along the major axis.

What is the standard form of an ellipse with center at the

origin and a vertex at (a, 0)?

The standard form is $x^2 / a^2 + y^2 / b^2 = 1$, where the vertex at (a, 0) indicates the length of the semi-major axis 'a'.

If an ellipse has a vertex at (5, 0) and is centered at the origin, what is its equation?

Since the vertex at (5, 0) indicates a = 5, the equation is $x^2 / 25 + y^2 / b^2 = 1$. The value of 'b' depends on the minor axis length, which must be provided or determined.

How do you determine the value of 'b' in the ellipse equation when given only a vertex?

You need additional information, such as the length of the minor axis or a point on the ellipse, to determine 'b'. Without that, only 'a' can be specified from the vertex.

Can the vertex be used to find the equation of an ellipse centered at the origin? If so, how?

Yes. The vertex provides the length of the semi-major axis 'a'. Using the vertex coordinate, you can set a = |x-coordinate| (if along x-axis), then write the standard form as $x^2 / a^2 + y^2 / b^2 = 1$, with 'b' determined from additional points or information.

What is the difference between the vertices and foci in an ellipse, and how does it affect the equation?

Vertices are the points where the ellipse reaches its maximum or minimum along the major axis, defining 'a'. Foci are points used to define the ellipse's shape via the sum of distances. The foci influence the relationship between 'a' and 'b', with $c^2 = a^2 - b^2$, impacting the equation.

Given a vertex at (0, 3) and the ellipse centered at the origin, what is the equation?

Since the vertex at (0, 3) lies along the y-axis, the semi-vertical axis 'a' is 3. The equation is $x^2 / b^2 + y^2 / 9 = 1$. The value of 'b' depends on additional information.

How do you write the equation of an ellipse with a vertex at (0, -4) and center at the origin?

The vertex at (0, -4) indicates a = 4 along the y-axis. The equation becomes $x^2 / b^2 + y^2 / 16 = 1$, with 'b' determined from other details or points.

What steps are involved in writing the standard form of an

ellipse given a vertex and the center at the origin?

First, identify the location of the vertex to determine 'a'. Next, decide if the major axis is horizontal or vertical based on the vertex's position. Then, write the standard form: x^2 / a^2 + y^2 / b^2 = 1 if horizontal, or y^2 / a^2 + x^2 / b^2 = 1 if vertical. Finally, find 'b' using additional points or information.

Why is knowing the vertex important when writing the equation of an ellipse with center at the origin?

The vertex helps determine the length of the semi-major axis 'a', which is essential for writing the standard form of the ellipse's equation. It defines the maximum extent of the ellipse along its major axis.

Additional Resources

Ellipse Equation Derivation and Standard Form with Center at the Origin and Given Vertex

Understanding the geometric beauty and algebraic elegance of ellipses is fundamental in many fields, from astronomy and physics to engineering and computer graphics. One common scenario involves determining the equation of an ellipse when the center is at the origin and a vertex is given. This article dives into the detailed process of deriving the standard form of such an ellipse, exploring the core concepts, mathematical derivations, and practical applications.

Understanding the Ellipse: Basic Concepts and Terminology

Before delving into the derivation process, it’s important to revisit the fundamental properties and terminology associated with ellipses.

What Is an Ellipse?

An ellipse is a conic section formed by the intersection of a plane and a cone, where the plane cuts through the cone at an angle inclined to the base, but not parallel to the side (which would produce a parabola) or perpendicular (which would produce a circle). Geometrically, an ellipse can be viewed as the set of all points such that the sum of the distances to two fixed points (called foci) is constant.

Key Elements of an Ellipse

- Center (at the origin): The point around which the ellipse is symmetric. In this case, at (0,0).
- Vertices: The points on the ellipse that are farthest from the center along the major axis.
- Foci: Two fixed points inside the ellipse such that the sum of the distances from any point on the

ellipse to these foci is constant.
- Major axis: The longest diameter passing through the vertices and foci.
- Minor axis: The shortest diameter perpendicular to the major axis, passing through the center.

Standard Form of the Ellipse Equation

The algebraic equation of an ellipse centered at the origin takes a specific standard form, which simplifies the process of understanding and graphing the shape.

Horizontal and Vertical Ellipses

- Horizontal ellipse: The major axis is along the x-axis, and the equation is:
$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
$$
- Vertical ellipse: The major axis is along the y-axis, and the equation is:
$$
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
$$

Here:
- a is the length of the semi-major axis (half the length of the major axis),
- b is the length of the semi-minor axis (half the length of the minor axis),
- $a > b$.

Given Vertex and Center at the Origin: Deriving the Equation

When the center is at the origin and a vertex is given, the process for deriving the ellipse's equation involves understanding the relationship between the vertex and the axes, and then translating that into the standard algebraic form.

Step 1: Identify the Given Vertex

Suppose the vertex is given as (x_v, y_v).

- For an ellipse with the center at the origin, the vertex lies along the major axis.
- If the vertex is at $(a, 0)$, the major axis is along the x-axis.
- If the vertex is at $(0, a)$, the major axis is along the y-axis.

The key is to determine along which axis the given vertex lies, as this influences the form of the equation.

Step 2: Determine the Length of the Semi-Major Axis (a)

- If the vertex is at $(a, 0)$: then a directly gives the length of the semi-major axis.
- If the vertex is at $(0, a)$: similarly, a is the length along the y-axis.

Example:
Given vertex at $(3, 0)$, then $a = 3$.

Step 3: Establish the Equation Based on the Orientation

Depending on the position of the vertex:

- Horizontal major axis:

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
$$

with vertices at $(\pm a, 0)$.

- Vertical major axis:

$$
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
$$

with vertices at $(0, \pm a)$.

Step 4: Find the Length of the Minor Axis (b)

To fully specify the ellipse, the length of the semi-minor axis, b, must be known or deduced.

- When only the vertex is known:
Additional information such as the distance to the co-vertices, foci, or the eccentricity is necessary to determine b.

- Using the relationship between a, b, and the focal distance c:

$$
c^2 = a^2 - b^2
$$

where c is the distance from the center to each focus along the major axis.

In the absence of additional data, b remains a variable, and the equation is expressed in terms of a and b.

Illustrative Example: Deriving the Equation with a Given Vertex

Let's walk through a concrete example for clarity.

Given:
- Center at origin $(0,0)$,
- Vertex at $(5, 0)$,
- No additional information.

Objective:
- Derive the standard form of the ellipse's equation.

Step 1: Identify the major axis

Since the vertex is at $(5, 0)$, along the x-axis, the major axis is horizontal.

Step 2: Determine a

The vertex at $(5, 0)$ indicates:
$$
a = 5
$$

Step 3: Establish the general form

The standard form for a horizontal ellipse centered at the origin:
$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
$$
which simplifies here to:
$$
\frac{x^2}{25} + \frac{y^2}{b^2} = 1
$$

Step 4: Find b

Without additional data, b remains unknown. For a complete equation, suppose we are given the location of the co-vertex or the foci, or the eccentricity.

Suppose:
The foci are at $(\pm c, 0)$.
If c is known, then:

$$
b^2 = a^2 - c^2
$$
For illustration, assume the foci are at $(\pm 4, 0)$, implying:
$$
c = 4
$$
then:
$$
b^2 = 25 - 16 = 9
$$
and the equation becomes:
$$
\frac{x^2}{25} + \frac{y^2}{9} = 1
$$

Extension: Incorporating Additional Data and Variations

While the above covers the core derivation, real-world problems often involve more complex data.

1. If the Vertex is Along the y-Axis

- The process mirrors the above, but the standard form switches to:
$$
\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1
$$
- The vertex at $(0, a)$ gives a, and similar steps follow.

2. Using Eccentricity to Determine b

Eccentricity e relates the foci and axes:
$$
e = \frac{c}{a}
$$
- For a known eccentricity, $c = e \times a$, and then:
$$
b^2 = a^2 - c^2
$$
- This method is valuable when the shape’s "elongation" is specified.

3. General Equation with Translations

If the center is shifted away from the origin, the standard form becomes:

$$
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
$$

where (h,k) is the center's coordinate.

Practical Applications and Significance

The ability to derive the equation of an ellipse from a given vertex is foundational in numerous practical applications:

- Astronomy: Modeling planetary orbits, which are often elliptical.
- Engineering: Designing reflective surfaces and antennas based on elliptical shapes.
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