Y(2)=4 5. . Xyy' = 2y2 + 4x?; Ans. = Solve The Following Differential Equations (IVP) 1. Xy = X + Y;

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Introduction

Differential equations are fundamental in mathematical modeling across various scientific and engineering disciplines. They describe how a quantity changes concerning another, often time or space. Solving differential equations, especially initial value problems (IVPs), enables us to understand dynamic systems' behavior effectively. In this article, we will explore the solution process for a specific differential equation:

$$ X y = X + Y $$

along with the initial condition $Y(2) = 4$, and analyze how to approach such equations systematically. Additionally, we will delve into a more complex differential equation:

$$ X y y' = 2 y^2 + 4 x $$

and demonstrate step-by-step solutions, emphasizing methods like separation of variables, substitution, and integrating factors.

Understanding the Differential Equation $X y = X + Y$

1. Recognizing the Type of Equation

The given equation:

$$X y = X + Y$$

can be rewritten as:

$$X y - Y = X$$

or

$$\frac{dY}{dX}$$

but note that in its current form, it appears to relate X, Y, and possibly derivatives. However, to clarify, suppose the notation implies that y is a function of x, and the equation is:

$$x y = x + y$$

which is a common form in differential equations involving functions $y(x)$.

2. Rewriting the Equation

Rearranged as:

$$x y - y = x$$

or

$$ y (x - 1) = x $$

which allows us to solve for y:

$$ y = \frac{x}{x - 1} $$

This indicates that the solution is a straightforward algebraic expression rather than a differential equation, assuming the initial condition $y(2) = 4$ is consistent.

3. Validating the Initial Condition

Substituting $x = 2$:

$$ y(2) = \frac{2}{2 - 1} = 2 $$

but the initial condition states $y(2) = 4$, which suggests a contradiction unless the initial condition refers to a different equation or is a typographical error.

Note: In case of an initial value problem (IVP), the differential equation might be different, or the notation may imply additional context. For demonstration, assume the initial condition aligns with the algebraic solution or explore alternative interpretations.

Solving the Differential Equation $X y y' = 2 y^2 + 4 x$

This is a more complex differential equation:

$$ x y y' = 2 y^2 + 4 x $$

which requires a systematic approach.

1. Recognizing the Equation Type

The structure suggests a Bernoulli or homogeneous differential equation. To analyze:

- y' denotes the derivative $\frac{dy}{dx}$.
- The equation involves quadratic terms in y.

2. Rearranging the Equation

Divide both sides by $x y$ (assuming $x \neq 0$, $y \neq 0$):

$$y' = \frac{2 y^2 + 4 x}{x y}$$

which simplifies to:

$$y' = \frac{2 y^2}{x y} + \frac{4 x}{x y}$$

$$y' = \frac{2 y}{x} + \frac{4}{y}$$

This form suggests the substitution approach.

3. Substitution Strategy

Let’s set:

$$v = y^2$$

then:

$$y = \sqrt{v}$$

$$y' = \frac{1}{2 \sqrt{v}} \frac{dv}{dx}$$

Substituting into the differential equation:

$$\frac{1}{2 \sqrt{v}} \frac{dv}{dx} = \frac{2 \sqrt{v}}{x} + \frac{4}{\sqrt{v}}$$

Multiply through by $2 \sqrt{v}$:

$$\frac{dv}{dx} = 2 \sqrt{v} \times \frac{2 \sqrt{v}}{x} + 2 \sqrt{v} \times \frac{4}{\sqrt{v}}$$

Simplify each term:

- $2 \sqrt{v} \times \frac{2 \sqrt{v}}{x} = \frac{4 v}{x}$
- $2 \sqrt{v} \times \frac{4}{\sqrt{v}} = 8$

Thus, the differential equation reduces to:

$$\frac{dv}{dx} = \frac{4 v}{x} + 8$$

4. Solving the Linear Differential Equation in v

This is a first-order linear ODE:

$$\frac{dv}{dx} - \frac{4 v}{x} = 8$$

Standard form:

$$\frac{dv}{dx} + P(x) v = Q(x)$$

where

$$P(x) = - \frac{4}{x}$$
$$Q(x) = 8$$

The integrating factor (IF):

$$ \mu(x) = e^{\int P(x) dx} = e^{\int -\frac{4}{x} dx} = e^{-4 \ln |x|} = |x|^{-4} $$

Assuming $ x > 0 $, we have:

$$ \mu(x) = x^{-4} $$

Multiply the entire differential equation by $ \mu(x) $:

$$ x^{-4} \frac{dv}{dx} - x^{-4} \frac{4 v}{x} = 8 x^{-4} $$

which simplifies to:

$$ \frac{d}{dx} \left(v x^{-4} \right) = 8 x^{-4} $$

Integrate both sides:

$$ v x^{-4} = \int 8 x^{-4} dx + C $$

Calculate the integral:

$$ \int 8 x^{-4} dx = 8 \int x^{-4} dx = 8 \times \frac{x^{-3}}{-3} = - \frac{8}{3} x^{-3} $$

Therefore,

$$ v x^{-4} = - \frac{8}{3} x^{-3} + C $$

Solve for $ v $:

$$ v = x^{4} \left(- \frac{8}{3} x^{-3} + C \right) $$

$$ v = - \frac{8}{3} x^{4} \times x^{-3} + C x^{4} $$

$$ v = - \frac{8}{3} x^{1} + C x^{4} $$

Recall $v = y^2$:

$$ y^2 = - \frac{8}{3} x + C x^{4} $$

Finally, take the square root:

$$ y = \pm \sqrt{ - \frac{8}{3} x + C x^{4} } $$

5. Applying Initial Conditions

Suppose an initial condition $y(x_0) = y_0$ is given; then, substitute to find C.

Summary of Solution Methods

In this article, we've demonstrated how to approach different types of differential equations:

- Algebraic equations: Simplified directly by algebraic manipulation.
- First-order linear ODEs: Solved using integrating factors.
- Bernoulli or homogeneous equations: Addressed via substitution to reduce to linear form.

Additional Tips for Solving Differential Equations

- Always identify the type of differential equation before choosing a solution method.

- Look for substitution opportunities—like $v = y^n$—to simplify nonlinear equations.
- Use integrating factors for linear first-order equations.
- Verify solutions by substituting back into the original differential equation.
- Pay attention to initial conditions to determine particular solutions.

Conclusion

Solving differential equations requires a systematic approach grounded in understanding the nature of each equation. Whether algebraic, linear, Bernoulli, or homogeneous, applying appropriate methods—such as substitution, separation of variables, or integrating factors—can lead to meaningful solutions. The example differential equations discussed in this article exemplify the importance of recognizing patterns and choosing strategic techniques to solve IVPs efficiently.

FAQs

Q1: What is an initial value problem (IVP)?
A1: An IVP specifies a differential equation along with initial conditions, typically the value of the function at a certain point, which helps in finding a particular solution.

Q2: How do I identify if a differential equation is linear?
A2: A first-order differential equation is linear if it can be written in the form $y' + P(x) y = Q(x)$, where $P(x)$ and $Q(x)$ are functions of x.

Q3: When should I use substitution methods?
A3: Substitution methods are useful when the differential equation has a nonlinear form that can be transformed into a

Frequently Asked Questions

What is the solution to the differential equation Xy = X + Y?

This differential equation can be rewritten as X dy/dX = X + y. Separating variables or using an integrating factor can help solve it. The general solution is y = C e^{X} - X - 1.

How do you solve the initial value problem Y(2) = 4 for the differential equation Xy = X + Y?

First, solve the differential equation to find the general solution. Then, substitute X=2 and Y=4 to find the constant C. This yields the particular solution y = 2e^{X} - X - 1.

What method is used to solve the differential equation Xy = X + Y?

The method involves rewriting the equation into a linear form and using an integrating factor or substitution methods suitable for first-order linear differential equations.

Can you explain the steps to solve the initial value problem involving Y(2) = 4?

Yes. First, solve the differential equation to get the general solution. Next, apply the initial condition Y(2) = 4 to find the specific constant, resulting in the particular solution.

What is the significance of initial conditions like Y(2)=4 in solving differential equations?

Initial conditions help determine the specific solution out of the general solution by fixing the value of the constant of integration.

Are there multiple methods to solve the differential equation Xy = X + Y?

Yes, methods include substitution, integrating factors, or recognizing it as a linear differential equation to apply standard solution techniques.

What is the typical form of solutions for differential equations like Xy = X + Y?

Solutions often involve exponential functions combined with polynomial expressions, reflecting the nature of linear first-order differential equations.

How can I verify my solution to the differential equation Xy = X + Y?

Substitute the obtained solution back into the original differential equation to check if both sides are equal, confirming its correctness.

Additional Resources

Understanding and Solving Differential Equations: A Comprehensive Guide

Differential equations are fundamental tools in mathematics, physics, engineering, and many applied sciences. They describe the relationships involving functions and their derivatives, providing insight into how systems evolve over time or space. In this article, we will explore a specific differential equation: Y(2)=4 5. . Xyy' = 2y2 + 4x, and then proceed to solve an initial value problem (IVP): Xy = X + Y. Our goal is to demystify the process, provide step-by-step solutions, and highlight essential techniques used in solving such equations.

Understanding the Differential Equation: Y(2) = 4 5. . Xyy' = 2y2 + 4x

At first glance, this expression appears somewhat ambiguous. Let's clarify and interpret it correctly:

- $Y(2) = 4$: indicates the initial condition where the function $Y(x)$ equals 4 at $x = 2$.
- The other part, $Xyy' = 2y^2 + 4x$, is a differential equation involving x, y, and y'.

Rearranged, the differential equation is:

$$x\, y\, y' = 2\, y^2 + 4\, x$$

Dividing both sides by x (assuming $x \neq 0$):

$$y\, y' = (2\, y^2 + 4\, x) / x$$

which simplifies to:

$$y\, y' = 2\, y^2 / x + 4$$

Alternatively, it can be written as:

$$y\, y' = 2\, y^2 / x + 4$$

This is a nonlinear first-order differential equation, which can be approached through substitution or separation of variables, depending on its form.

Step-by-Step Solution Process

Step 1: Recognize the Type of Differential Equation

The equation:

$$y y' = 2 y^2 / x + 4$$

suggests a Bernoulli-type or a substitution to reduce it to a linear form. Since $y y'$ resembles $y\, dy/dx$, rewrite as:

$$y\, dy/dx = 2 y^2 / x + 4$$

or,

$$y\, dy/dx = 2 y^2 / x + 4$$

Step 2: Simplify and Recast the Equation

Divide both sides by y (assuming $y \neq 0$):

$$dy/dx = 2 y / x + 4 / y$$

Now, this form involves both y and $1/y$, indicating a substitution to handle the nonlinear term.

Step 3: Substitution to Simplify the Equation

Set:

$$z = y^2$$

then,

$$dz/dx = 2 y\, dy/dx$$

From the differential equation:

$dy/dx = (1/2)\, dz/dx / y$

But since $z = y^2$, then:

$dy/dx = (1/2)\, dz/dx / y$

Substituting back into the equation:

$dy/dx = (1/2)\, dz/dx / y$

So, rewriting the original differential equation:

$dy/dx = 2\, y / x + 4 / y$

Multiply both sides by y:

$y\, dy/dx = 2\, y^2 / x + 4$

But $y\, dy/dx = (1/2)\, dz/dx$, hence:

$(1/2)\, dz/dx = 2\, y^2 / x + 4$

Recall that $z = y^2$, so:

$(1/2)\, dz/dx = 2\, z / x + 4$

Multiply through by 2:

$dz/dx = 4\, z / x + 8$

This is a linear first-order differential equation in z.

Step 4: Solve the Linear Differential Equation in z

The equation:

$$dz/dx - (4 / x) z = 8$$

is a linear differential equation of the form:

$$dz/dx + P(x) z = Q(x)$$

where:

- $P(x) = -4 / x$
- $Q(x) = 8$

Find the integrating factor (IF):

$$IF = \exp(\int P(x)\, dx) = \exp(\int -4 / x\, dx) = \exp(-4 \ln x) = x^{-4}$$

Step 5: Integrate and Find the General Solution

Multiply both sides of the differential equation by the integrating factor:

$$x^{-4}\, dz/dx - (4 / x) x^{-4} z = 8 x^{-4}$$

Simplify:

$$\frac{d}{dx} [x^{-4} z] = 8 x^{-4}$$

Integrate both sides with respect to x:

$$x^{-4} z = \int 8 x^{-4} \, dx + C$$

Calculate the integral:

$$\int 8 x^{-4} \, dx = 8 \int x^{-4} \, dx = 8 (-1/3) x^{-3} + C' = - (8/3) x^{-3} + C'$$

Thus,

$$x^{-4} z = - (8/3) x^{-3} + C$$

Multiply both sides by x^{4}:

$$z = - (8/3) x^{-3} x^{4} + C x^{4} = - (8/3) x^{1} + C x^{4}$$

Recall that $z = y^2$, so:

$$y^2 = - (8/3) x + C x^{4}$$

Step 6: Express the General Solution for y

Taking square roots:

$$y = \pm \sqrt{ - (8/3) x + C x^{4} }$$

This is the general solution of the differential equation, where C is an arbitrary constant.

Step 7: Apply Initial Conditions

Given $Y(2) = 4$, substitute $x = 2$ and $y = 4$:

$4 = \square[- (8/3)\ 2 + C\ 2^{4}]$

Calculate:

$4 = \square[- (16/3) + C\ 16]$

Square both sides:

$16 = - (16/3) + 16\ C$

Solve for C:

$16\ C = 16 + (16/3) = (48/3) + (16/3) = 64/3$

Therefore:

$C = (64/3) / 16 = (64/3)\ (1/16) = (64/16)\ (1/3) = 4\ (1/3) = 4/3$

Final Solution:

$y(x) = \pm\ \square[- (8/3)\ x + (4/3)\ x^{4}]$

with the constant $C = 4/3$.

Solving the Initial Value Problem: $Xy = X + Y$

Next, let's consider the initial value problem:

$X y = X + Y$

which can be rewritten as:

$x y = x + y$

Our goal is to solve for y as a function of x.

Step 1: Rearrange the Equation

Bring all to one side:

$x y - y = x$

Factor y:

$y (x - 1) = x$

Assuming $x \neq 1$, solve for y:

$y = x / (x - 1)$

This is a simple algebraic solution, which is valid for all $x \neq 1$.

Initial Conditions and Domain

If an initial condition, such as $Y(x_0) = y_0$, is provided, verify whether it satisfies the solution.

For example, at $x = 2$:

$$Y(2) = 2 / (2 - 1) = 2 / 1 = 2$$

If the initial condition matches, the solution is consistent.

Summary and Key Takeaways

- Differential equations can often be simplified using substitution techniques, especially when nonlinear terms are involved.
- Recognizing the type of differential equation (linear, Bernoulli, separable) guides the method of solution.
- Linear equations can be solved using integrating factors.
- Applying initial conditions helps determine particular solutions from the general form.
- Algebraic manipulation is often sufficient for straightforward differential equations like $x y = x + y$.

Final Remarks

Mastering the process of solving differential equations requires practice and familiarity with various techniques, including substitution, integrating factors, and algebraic rearrangements. Whether dealing with nonlinear equations or simple algebraic relationships, understanding the underlying structure is key. Armed with these methods, you are well-equipped to tackle a broad spectrum of differential equations encountered in academic and professional contexts.

Remember: Always check the domain of your solutions and initial conditions to ensure validity, and consider the physical or practical context when interpreting your solutions.

Happy solving! For further practice, explore more about Bernoulli equations, exact equations, and applications of differential equations in modeling real-world phenomena.

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