A Double-concave Lens Has Equal Radii Of Curvature Of 15.1 Cm. An Object Placed 14.2 Cm From The Lens

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Understanding how lenses form images is fundamental in optics, and the properties of double-concave lenses, in particular, play a vital role in various optical devices. In this article, we delve into the specifics of a double-concave lens with equal radii of curvature, examine the principles behind image formation, and explore how to calculate key parameters such as the focal length, image position, and magnification when an object is placed at a certain distance from the lens. Whether you are a student, educator, or an enthusiast looking to deepen your understanding of optical physics, this detailed discussion aims to clarify the concepts involved.

Introduction to Double-Concave Lenses

A double-concave lens, also known as a diverging lens, is characterized by two inward-curving surfaces. Unlike converging lenses (like convex lenses), diverging lenses cause incident rays to spread out or diverge after passing through the lens. They are commonly used in applications such as eyeglasses for nearsightedness, optical instruments, and laser beam expanders.

Key Features of Double-Concave Lenses

- Shape and Structure: Both surfaces are concave, with equal or different radii of curvature.

- Light Behavior: Parallel rays incident on a diverging lens spread out as if they originated from a focal point on the same side as the object.
- Applications: Corrective eyewear, optical devices, and beam expansion systems.

Understanding the Radii of Curvature in the Given Lens

The given double-concave lens has equal radii of curvature, each measuring 15.1 cm. This symmetry simplifies the calculations and allows us to determine the focal length using lensmaker's formula.

What Does Equal Radii of Curvature Imply?

- The surfaces are mirror images in terms of curvature.
- The lens's optical properties are symmetric.
- The calculations for focal length become straightforward due to symmetry.

Calculating the Focal Length of the Lens

The focal length (f) of a lens is an essential parameter dictating how it converges or diverges light. For a lens with known radii and refractive index, the lensmaker's formula provides an accurate way to determine the focal length.

Lensmaker's Formula

$$
\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)
$$

Where:

- n is the refractive index of the lens material (commonly glass, approximately 1.5).
- R_1 and R_2 are the radii of curvature of the two surfaces. For a double-concave lens, both are negative as per sign convention.

Applying the Formula

Given:

- $R_1 = -15.1\, \text{cm}$ (since the surface curves inward)
- $R_2 = -15.1\, \text{cm}$ (same as above)
- $n \approx 1.5$

Calculating:

$$
\frac{1}{f} = (1.5 - 1) \left(\frac{1}{-15.1\, \text{cm}} - \frac{1}{-15.1\, \text{cm}} \right) = 0.5 \times (0) = 0
$$

But this result indicates that the difference is zero, which suggests infinite focal length—implying a flat lens, which is physically impossible for a diverging lens with equal radii. This indicates a need to consider the sign convention carefully.

Sign Convention Recap:

- For lenses, the radii are positive if the center of curvature is on the outgoing side, negative if on the incoming side.
- For a double-concave lens, both surfaces are concave toward the incoming light, so both R_1 and R_2 are negative.

Thus, substituting:

$$
\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = (0.5) \left(\frac{1}{-15.1} - \frac{1}{-15.1} \right) = 0.5 \times 0 = 0
$$

This suggests an infinitely large focal length, which contradicts the expectation for a diverging lens. The issue arises because for double-concave lenses, the sign convention leads to both radii being negative, but the formula involves the difference.

Alternatively, the standard form for double-concave lenses with equal radii simplifies to:

$$
\frac{1}{f} = - (n - 1) \times \frac{2}{R}
$$

because both surfaces are concave with the same radius R.

Applying:

$$
\frac{1}{f} = - (1.5 - 1) \times \frac{2}{15.1\, \text{cm}} = - 0.5 \times \frac{2}{15.1} = - 0.5 \times 0.13245 \approx -0.0662\, \text{cm}^{-1}
$$

Therefore,

$$
f \approx - \frac{1}{0.0662} \approx -15.1\, \text{cm}
$$

Result:

The focal length of the lens is approximately -15.1 cm, indicating a diverging lens with focal length of 15.1 cm.

Image Formation for an Object Placed 14.2 Cm from the Lens

Now that we know the focal length, we can analyze the image formed when an object is placed 14.2 cm from the lens, on the same side as the object.

Using the Lens Formula

$$
\frac{1}{f} = \frac{1}{v} - \frac{1}{u}
$$

Where:

- u is the object distance (negative if object is on the same side as incoming light)
- v is the image distance (positive if the image is real and on the opposite side, negative if virtual and on the same side as the object)

Given:

- $u = -14.2\, \text{cm}$
- $f = -15.1\, \text{cm}$

Calculating:

$$
\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{15.1} + \frac{1}{-14.2} = -0.0662 - 0.0704 = -0.1366\, \text{cm}^{-1}
$$

Thus:

$$
v = - \frac{1}{0.1366} \approx -7.32\, \text{cm}
$$

Interpretation:

- The negative image distance indicates a virtual image formed on the same side as the object.
- The image is virtual, upright, and magnified.

Magnification of the Image

Magnification (m) is given by:

$$
m = \frac{v}{u}
$$

Substituting:

$$
m = \frac{-7.32}{-14.2} \approx 0.515
$$

- The positive value indicates the image is upright.
- The image is approximately 0.515 times the size of the object, i.e., reduced in size.

Practical Implications and Applications

Understanding the behavior of double-concave lenses with equal radii of curvature is crucial in designing optical systems where diverging lenses are used to manipulate light paths.

Applications include:

- Eyeglasses for Myopia: Correcting nearsightedness by diverging incoming light before it reaches the retina.
- Optical Instruments: Controlling light paths to achieve desired magnification and focus.
- Laser Systems: Expanding laser beams or controlling divergence.
- Imaging Systems: Correcting aberrations and managing virtual image formation.

Design Considerations:

- The focal length directly influences how close an object can be placed to the lens to produce a clear image.
- The virtual image formed at approximately 7.32 cm from the lens indicates the lens's divergence properties.
- The size and orientation of the image depend on object distance relative to the focal length.

Summary of Key Points

- The double-concave lens with equal radii of 15.1 cm has a focal length of approximately -15.1 cm.
- When an object is placed 14.2 cm from the lens, the resulting virtual image appears at about -7.32

cm from the lens (on the same side as the object).

- The image is upright, virtual, and reduced in size, with a magnification of approximately 0.515.
- The sign conventions are essential for

Frequently Asked Questions

What is the focal length of a double-concave lens with radii of curvature 15.1 cm?

Using the lens formula and the lensmaker's equation, the focal length (f) of a double-concave lens with radii of 15.1 cm is approximately -10.07 cm, indicating a diverging lens.

How do you determine the nature of the image formed by this lens for an object 14.2 cm away?

Since the object distance (14.2 cm) is greater than the focal length, the lens will produce a virtual, upright, and magnified image on the same side as the object.

What is the magnification of the image formed in this setup?

The magnification is approximately 1.25, indicating the image is about 25% larger than the object.

How can I calculate the image distance for the given object and lens?

Using the lens formula 1/f = 1/do + 1/di, with f □ -10.07 cm and do = 14.2 cm, the image distance di is approximately 6.0 cm on the same side as the object.

What does the negative focal length indicate about the lens's type?

A negative focal length indicates that the lens is diverging, characteristic of a double-concave lens.

If the object is moved closer to the lens, how does the image size change?

As the object moves closer to the lens (but beyond the focal point), the image becomes more magnified and remains virtual and upright.

Why is the radii of curvature important in determining the lens's properties?

The radii of curvature determine the lens's focal length and optical power, influencing how it converges or diverges light rays.

Can this lens form a real image for the given object distance?

No, because the object distance is greater than the focal length of a diverging lens, it only forms virtual images; real images occur with converging lenses at specific object distances.

How does the symmetry of the radii affect the lens's behavior?

Since the radii are equal, the lens is symmetric, which simplifies calculations and results in a predictable focal length based on the lensmaker's formula.

Additional Resources

A Double-Concave Lens Has Equal Radii Of Curvature Of 15.1 Cm. An Object Placed 14.2 Cm From The Lens

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Introduction: Understanding the Significance of Lens Geometry and Its Optical Properties

Double-concave lenses with equal radii of curvature of 15.1 cm serve as fundamental components in many optical systems, from simple magnifying glasses to complex imaging devices. The precise geometry of such lenses influences how light rays pass through them, determining their focal length, magnification capabilities, and imaging characteristics. When an object is placed at a specific distance from such a lens—here, 14.2 cm—the resulting image's properties can be predicted and analyzed through optical principles. This article delves into the physics behind the lens's behavior, explores the derivation of key parameters, and discusses practical applications and implications of these optical elements.

Understanding the Double-Concave Lens and Its Geometry

What Is a Double-Concave Lens?

A double-concave lens is a type of diverging lens characterized by both surfaces being concave inward. Unlike converging lenses (like convex lenses) that focus light rays to a point, diverging lenses spread light rays outward. This divergence causes virtual images that appear to originate from a point behind the lens.

Equal Radii of Curvature and Their Implications

The radii of curvature describe the degree of curvature of each lens surface. When both surfaces share the same radius—in this case, 15.1 cm—the lens exhibits symmetric geometry, which simplifies mathematical modeling and ensures uniform divergence properties.

- Radius of curvature (R): The radius of the sphere from which the lens surface segment is taken.
- Sign convention: For a double-concave lens, both surfaces are concave relative to incoming light, so their radii are negative when applying standard sign conventions in optics.

Lensmaker's Equation and Focal Length Calculation

The lensmaker's formula relates the radii of curvature and the refractive index (n) of the lens material to its focal length (f):

$$
\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)
$$

- For a symmetric double-concave lens:

$$
R_1 = -15.1\, \text{cm}, \quad R_2 = -15.1\, \text{cm}
$$

Assuming the lens material is made of standard optical glass with a refractive index $n \approx 1.52$, the focal length can be computed.

Calculating the Focal Length of the Lens

Applying the Lensmaker's Equation

Given:

$$
R_1 = R_2 = -15.1\, \text{cm}
$$

and

$$
n = 1.52
$$

we substitute into the formula:

$$
\frac{1}{f} = (1.52 - 1) \left(\frac{1}{-15.1} - \frac{1}{-15.1} \right)
$$

Notice that:

$$
\frac{1}{-15.1} - \frac{1}{-15.1} = 0
$$

which suggests the initial straightforward approach yields zero. However, because both radii are equal and negative, the difference is zero, indicating that the lens acts as a planar or infinite focal length lens in the idealized scenario. But in reality, the divergence of the lens is characterized by its power, and the focal length can be derived considering the lens's optical power (P):

$$
P = \frac{(n - 1)}{R_1} + \frac{(n - 1)}{R_2}
$$

Since $R_1 = R_2$:

$$
P = 2 \times \frac{(n - 1)}{R}
$$

Plugging in the numbers:

$$
P = 2 \times \frac{0.52}{-15.1\, \text{cm}} = 2 \times \frac{0.52}{-15.1} \approx 2 \times (-0.0344) = -0.0688\, \text{cm}^{-1}
$$

The focal length (f) in centimeters is:

$$
f = \frac{1}{P} = \frac{1}{-0.0688} \approx -14.53\, \text{cm}
$$

Interpretation: The negative focal length confirms the diverging nature of the lens, with a focal length approximately $-14.53\, \text{cm}$.

Object Position and Image Formation: Applying the Lens Formula

The Lens Formula and Its Significance

The lens formula links the object distance (u), the image distance (v), and the focal length (f):

$$
\frac{1}{f} = \frac{1}{v} + \frac{1}{u}
$$

Where:

- u: Object distance from the lens (positive if object is on the same side as incoming light).
- v: Image distance from the lens (positive if real image on the opposite side; negative if virtual image on the same side as the object).

Given:

$$
u = 14.2\, \text{cm}
$$

$$
f \approx -14.53\, \text{cm}
$$

we solve for v:

$$
\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{14.53} - \frac{1}{14.2}
$$

Calculations:

$$
\frac{1}{v} \approx -0.0688 - 0.0704 = -0.1392\, \text{cm}^{-1}
$$

Thus:

$$
v \approx -\frac{1}{0.1392} \approx -7.19\, \text{cm}
$$

Interpretation: The negative value indicates that the image is virtual and located approximately 7.19 cm on the same side of the lens as the object.

Image Characteristics and Magnification Analysis

Nature and Position of the Image

- Type: Virtual, upright, and diminished.
- Location: Approximately 7.19 cm in front of the lens (on the same side as the object).
- Implication: Such a lens can be used in applications where virtual images are desirable, such as in eyepieces or certain optical instruments.

Magnification

The magnification (M) is given by:

$$
M = \frac{v}{u} \approx \frac{-7.19}{14.2} \approx -0.506
$$

- The negative sign indicates an upright image.
- The magnitude (~0.506) indicates the image is roughly half the size of the object (diminished).

Practical implication: For an object placed just beyond the focal length, the diverging lens creates a virtual, upright, and reduced image, useful in applications like virtual magnifiers or beam expanders.

Practical Applications and Real-World Relevance

Optical Devices Utilizing Double-Concave Lenses

- Eyewear for myopia: Diverging lenses help correct nearsightedness by diverging incoming light before it reaches the retina.
- Laser beam expanders: Double-concave lenses are used to expand laser beams, increasing the divergence for various applications.
- Optical instruments: Used in microscopes and telescopes to control light paths and image formation.

Design Considerations in Optical Engineering

Understanding the precise geometry and optical properties of lenses aids engineers and designers in:

- Optimizing image quality: Ensuring virtual images are suitably magnified or diminished.
- Minimizing aberrations: Symmetric design reduces optical distortions.
- Tailoring focal lengths: Adjusting radii and material properties to

achieve desired divergence or convergence.

Conclusion: The Interplay of Geometry and Optics in Lens Design

The case of a double-concave lens with equal radii of 15.1 cm exemplifies the delicate relationship between physical geometry and optical performance. By applying fundamental principles—such as the lensmaker's equation and the lens formula—optical scientists can predict how such a lens influences light rays and images. When an object is placed 14.2 cm from this lens, the resulting virtual, upright, and reduced image illustrates the diverging nature of the lens, which has broad applications in correction, imaging, and laser optics. Mastery of these concepts enables the design of sophisticated optical systems tailored to precise requirements, highlighting the importance of theoretical understanding in practical innovations.

In summary, this analysis underscores the importance of geometrical symmetry, material properties, and precise calculations in understanding and leveraging the behavior of double-concave lenses. As optical technology continues to evolve, such foundational

[A Double Concave Lens Has Equal Radii Of Curvature Of 15 1 Cm An Object Placed 14 2 Cm From The Lens]()

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