A Jar Consists Of Blue, Red And Green Marbles. The Probability Of Each Of Them Is 2/7 , 3/k And 5/2k.

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Understanding probabilities is fundamental in statistics and everyday decision-making. When dealing with a jar containing different colored marbles, calculating the probability of drawing a particular color helps in predicting outcomes, making informed choices, and solving related mathematical problems. In this article, we explore a scenario where a jar contains blue, red, and green marbles with specified probabilities. We will analyze the given probability expressions, find the value of the unknown parameter k, and discuss the implications of these probabilities in practical contexts.

Overview of the Marble Probabilities

In our scenario, a jar contains marbles of three different colors: blue, red, and green. The probabilities associated with drawing each color are provided as follows:

- Blue marbles: 2/7
- Red marbles: 3/k
- Green marbles: 5/2k

These probabilities are essential because they describe how likely it is to randomly select a marble of a specific color from the jar.

Understanding Probability

Probability measures the chance of an event occurring, expressed as a number between 0 and 1. When dealing with multiple mutually exclusive outcomes, the sum of their probabilities must equal 1, representing the certainty that one of these outcomes will occur.

In this context:

- The probability of drawing a blue marble is 2/7.
- The probability of drawing a red marble is 3/k.

- The probability of drawing a green marble is 5/2k.

Since these are the only possible outcomes (assuming the jar contains only blue, red, and green marbles), the sum of their probabilities must be 1.

Deriving the Value of k

Given that the probabilities sum to 1, we can set up an equation:

(Probability of Blue) + (Probability of Red) + (Probability of Green) = 1

Substituting the given values:

2/7 + 3/k + 5/2k = 1

Our goal is to find the value of k that satisfies this equation.

Step-by-Step Solution

Step 1: Find a common denominator

To combine the fractions, we need a common denominator. The denominators are 7, k, and 2k. The least common denominator (LCD) is 14k.

Step 2: Rewrite each fraction with the LCD

- 2/7 becomes (2 2k)/(14k) = (4k)/(14k)
- 3/k becomes (3 14)/(14k) = (42)/(14k)
- 5/2k becomes (5 7)/(14k) = (35)/(14k)

Step 3: Write the sum with common denominator

$$
\frac{4k}{14k} + \frac{42}{14k} + \frac{35}{14k} = 1
$$

Simplify numerator:

$$
\frac{4k + 42 + 35}{14k} = 1
$$

Step 4: Cross-multiplied equation

Multiply both sides by 14k:

$$
4k + 42 + 35 = 14k
$$

Combine like terms:

$$
4k + 77 = 14k
$$

Step 5: Solve for k

Subtract 4k from both sides:

$$
77 = 14k - 4k
$$
$$
77 = 10k
$$

Divide both sides by 10:

$$
k = \frac{77}{10} = 7.7
$$

Result: The value of k is 7.7.

Implications of the Calculated Probabilities

Knowing the value of k allows us to compute the exact probabilities of red and green marbles:

- Red: $3/k = 3/7.7 \approx 0.3896$
- Green: $5/2k = 5/(2\ 7.7) = 5/15.4 \approx 0.3247$

And the blue probability remains:

- Blue: $2/7 \approx 0.2857$

Adding these probabilities:

$$
0.2857 + 0.3896 + 0.3247 \approx 0.9999
$$

Due to rounding, this is essentially 1, confirming our calculations are consistent.

Practical implications:

- Predicting outcomes: If you randomly pick a marble from the jar, there's approximately a 28.6% chance it will be blue, 38.9% chance it will be red, and 32.5% chance it will be green.
- Designing experiments: These probabilities can help in designing experiments or games involving marbles.
- Quality control: Understanding the distribution helps in manufacturing or quality assessments if marbles are produced in specific ratios.

Further Analysis and Applications

Understanding probability distributions like this has broad applications beyond just marbles. Here are some areas where similar concepts are employed:

1. Quality Control in Manufacturing

Manufacturers often analyze the proportion of defective, acceptable, and excellent products in a batch. Probabilities help in assessing the quality and making decisions about process improvements.

2. Gambling and Gaming

In games of chance, probabilities determine payout ratios and game fairness. For example, understanding the odds of drawing certain cards or symbols guides both players and game designers.

3. Risk Assessment and Management

Industries assess the likelihood of various risks (e.g., machine failure, financial loss) to develop mitigation strategies. Probabilities inform policy decisions and resource allocation.

4. Statistical Sampling and Surveys

Sampling techniques rely on probabilities to ensure representative data collection, leading to accurate inferences about larger populations.

Additional Considerations in Probability Problems

When working with probabilities, especially involving unknown parameters like k, several key points should be kept in mind:

- Sum of probabilities: Always verify that the total probability sums to 1, which serves as a consistency check.
- Validity of probabilities: Each probability must be between 0 and 1.
- Parameter constraints: When probabilities involve parameters like k, solving for their values must produce valid probabilities within the [0,1] interval.
- Real-world relevance: Consider whether the theoretical probabilities reflect realistic scenarios.

Conclusion

Analyzing the problem of a jar containing blue, red, and green marbles with given probabilities demonstrates the importance of algebraic techniques and probability theory in solving real-world problems. By setting up equations based on the total probability rule, we derived the value of k as 7.7, enabling us to determine the exact probabilities of drawing each color. These insights can be applied in various contexts, from manufacturing quality control to game design and risk assessment. Mastery of such probability calculations enhances analytical skills and supports informed decision-making across numerous fields.

Summary of Key Points

- The probabilities of blue, red, and green marbles are 2/7, 3/k, and 5/2k respectively.
- Assuming these are the only outcomes, their sum must be 1, leading to

the equation: 2/7 + 3/k + 5/2k = 1.

- Solving for k yields k = 7.7.
- Calculated probabilities are approximately 28.6% (blue), 38.9% (red), and 32.5% (green).
- Understanding these probabilities allows for better predictions and decision-making in relevant scenarios.

By mastering the process of deriving unknown parameters from probability distributions, students and professionals can confidently analyze similar problems and apply these concepts in practical situations.

Frequently Asked Questions

What are the probabilities of drawing a blue, red, and green marble from the jar?

The probability of drawing a blue marble is 2/7, red marble is 3/k, and green marble is 5/2k.

How can we find the value of k based on the total probability in the jar?

Since the total probability must sum to 1, we set 2/7 + 3/k + 5/2k = 1 and solve for k.

What is the value of k when the probabilities sum to 1?

By solving the equation 2/7 + 3/k + 5/2k = 1, we find that k = 14.

Are the probabilities of red and green marbles valid for k = 14?

Yes, substituting k=14 gives the probabilities of red as 3/14 and green as 5/28, both valid and less than 1.

What is the probability of drawing a marble that is either blue, red, or green?

The probability is 1, since the jar contains only these three marbles and the probabilities sum to 1.

If a marble is drawn at random, what is the chance it is green?

The probability of drawing a green marble is 5/2k; with k=14, this is 5/28.

How does changing the value of k affect the probabilities of red and green marbles?

Altering k changes the probabilities of red and green marbles proportionally, since they depend on k in the denominators.

Can the probabilities of red and green marbles be greater than 1/2 for some value of k?

No, because probabilities must be less than or equal to 1; for the given expressions, k must be sufficiently large to keep probabilities between 0 and 1.

Additional Resources

A Jar Consists Of Blue, Red And Green Marbles. The Probability Of Each Of Them Is 2/7, 3/k And 5/2k.

This intriguing problem presents a fascinating scenario in probability theory, involving a jar filled with marbles of different colors—blue, red, and green—and the associated probabilities of drawing each color. The probabilities given are fractional expressions involving an unknown constant k, which adds an element of algebraic reasoning to the problem. Analyzing such a problem requires understanding the fundamental principles of probability, the constraints of total probability, and the algebraic manipulation needed to determine the value of k. This review aims to explore the problem comprehensively, breaking down the concepts involved, examining possible solutions, and discussing the implications of the probability assignments.

Understanding the Problem Setup

The problem states that a jar contains marbles of three distinct colors: blue, red, and green. The probabilities of drawing each color are given as:

- Probability of blue marble: $\frac{2}{7}$
- Probability of red marble: $\frac{3}{k}$
- Probability of green marble: $\frac{5}{2k}$

The key question is: What is the value of k, given these probabilities, and do these probabilities form a valid probability distribution?

In probability, the sum of the probabilities of all mutually exclusive outcomes must equal 1. Therefore, for the three colors:

$$
P(\text{Blue}) + P(\text{Red}) + P(\text{Green}) = 1
$$

Substituting the given values:

$$
\frac{2}{7} + \frac{3}{k} + \frac{5}{2k} = 1
$$

This equation forms the foundation for solving for k.

Breaking Down the Probability Equation

Step 1: Combine the fractions involving k

Notice that the fractions $\frac{3}{k}$ and $\frac{5}{2k}$ involve k. To combine these, it's best to express everything over a common denominator.

$$
\frac{3}{k} + \frac{5}{2k} = \frac{6}{2k} + \frac{5}{2k} = \frac{6 + 5}{2k} = \frac{11}{2k}
$$

Thus, the probability sum simplifies to:

$$
\frac{2}{7} + \frac{11}{2k} = 1
$$

Step 2: Solve for k

Rewrite the equation:

$$
\frac{2}{7} + \frac{11}{2k} = 1
$$

Subtract $\frac{2}{7}$ from both sides:

$$
\frac{11}{2k} = 1 - \frac{2}{7} = \frac{7}{7} - \frac{2}{7} = \frac{5}{7}
$$

Now, solve for k:

$$
\frac{11}{2k} = \frac{5}{7}
$$

Cross-multiplied:

$$
11 \times 7 = 5 \times 2k
$$
$$
77 = 10k
$$

Finally:

$$
k = \frac{77}{10} = 7.7
$$

Result: The value of k is 7.7.

Verification of the Probabilities

Having found $k = 7.7$, we can now verify whether the probabilities make sense:

- Red: $\frac{3}{k} = \frac{3}{7.7} \approx 0.3896$
- Green: $\frac{5}{2k} = \frac{5}{2 \times 7.7} = \frac{5}{15.4} \approx 0.3247$

Adding all probabilities:

$$
\frac{2}{7} \approx 0.2857, \quad 0.3896, \quad 0.3247
$$

Sum:

$$

0.2857 + 0.3896 + 0.3247 \approx 1.0000
\]

This confirms that the probabilities sum to 1, satisfying the fundamental axiom of probability.

Features and Insights of the Probabilistic Setup

Features

- Algebraic Integration with Probability: The problem showcases how algebraic techniques are essential in solving probability problems involving unknown parameters.
- Parameter Determination: It emphasizes the importance of the total probability rule, which helps determine unknown constants in fractional probabilities.
- Real-world Applicability: Such problems mirror realistic scenarios where proportions or probabilities depend on adjustable parameters, allowing for flexible modeling.

Pros

- Demonstrates the importance of normalization in probability distributions.
- Reinforces algebraic manipulation skills.
- Encourages understanding of how probabilities relate to each other through constraints.

Cons

- Assumes the probabilities are valid without explicitly addressing possible constraints on \(k\), such as ensuring each probability is between 0 and 1.
- The value of \(k\) being non-integer might be less intuitive in practical settings, where counts are discrete.

Validity and Constraints

While the calculations provide a consistent value for \(k\), it's essential to verify that the probabilities are valid:

- Probability of red: \(\frac{3}{k} = \frac{3}{7.7} \approx 0.3896\) (valid, between 0 and 1)
- Probability of green: \(\frac{5}{2k} = \approx 0.3247\) (valid)
- Probability of blue: \(\frac{2}{7} \approx 0.2857\) (valid)

Since all are positive and less than 1, the distribution is valid.

Note: In practical contexts, if the probabilities involved counts of discrete marbles, \(k\) should be an integer to represent actual counts. However, in the purely probabilistic sense, fractional \(k\) values are acceptable.

Implications and Applications

The problem illustrates a typical scenario in probability where parameters are not directly given but can be inferred through the total probability rule. Applications include:

- Quality Control: Estimating proportions of defect types in a batch based on probabilities.
- Sampling and Surveys: Determining composition proportions when some parameters are unknown.
- Educational Tools: Teaching algebraic techniques for solving probability problems.

Conclusion

The problem of determining the probability distribution in a jar with blue, red, and green marbles, with probabilities expressed in terms of an unknown \(k\), offers valuable insight into the interplay of algebra and probability theory. By setting the total probability to 1, solving for \(k\), and verifying the validity of the derived probabilities, one can ensure a consistent and meaningful probabilistic model. The key takeaways include understanding how to manipulate fractional expressions, the importance of normalization, and the real-world relevance of such mathematical principles. These concepts form the foundation for analyzing complex probability scenarios, whether in theoretical exercises or practical applications.

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In summary:

- The value of \(k\) is 7.7.
- Probabilities are consistent and sum to 1.
- The problem underscores the importance of algebraic methods in probability.
- It provides a framework for understanding how parameters influence
probability distributions.

This comprehensive analysis not only solves the problem but also highlights
critical features of probability modeling, making it an instructive example
for students and practitioners alike.
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