A Line Is Parallel To The Line 3y-15x+11=0 And Passes Through The Point (5, 3). Which Equation Models

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Understanding the relationship between lines in a coordinate plane is fundamental in geometry and algebra. When asked to find an equation of a line that is parallel to a given line and passes through a specific point, it involves analyzing the given line's slope, using the point-slope form, and forming the desired equation. This article provides an in-depth explanation of this process, focusing on the line parallel to the line given by the equation $3y - 15x + 11 = 0$ and passing through the point (5, 3).

Analyzing the Given Line: 3y - 15x + 11 = 0

Before constructing the new line's equation, it is essential to understand the properties of the given line, particularly its slope. The slope determines the steepness and orientation of the line, and parallel lines share the same slope.

Converting the Equation to Slope-Intercept Form

The standard form of the given line is:

$3y - 15x + 11 = 0$

To find its slope, convert this to slope-intercept form, $y = mx + b$, where m is the slope.

Step-by-step:

1. Isolate y:

$3y = 15x - 11$

2. Divide both sides by 3:

$y = (15x - 11)/3$

3. Simplify:

$y = 5x - 11/3$

From this form, the slope (m) of the given line is:

$m = 5$

Key Point: The slope of the given line is 5.

Implication for the Parallel Line

Since we are asked to find a line parallel to this one, the new line must have the same slope:

$m_{parallel} = 5$

Formulating the Equation of the Parallel Line

Given that the new line passes through the point (5, 3) and has a slope of 5, we can use the point-slope form of a line's equation to determine its explicit form.

Point-Slope Form of a Line

The point-slope form is:

$y - y_1 = m(x - x_1)$

where:

- (x_1, y_1) is a point on the line,
- m is the slope.

Applying the given point and the slope:

- $(x_1, y_1) = (5, 3)$
- $m = 5$

Substituting:

$y - 3 = 5(x - 5)$

Deriving the Equation in Slope-Intercept Form

Expanding and simplifying:

1. Distribute:

$y - 3 = 5x - 25$

2. Add 3 to both sides:

$y = 5x - 25 + 3$

3. Simplify:

$y = 5x - 22$

Answer: The equation of the line parallel to $3y - 15x + 11 = 0$ and passing through (5, 3) is:

$y = 5x - 22$

Expressing the Equation in Standard Form

While slope-intercept form is often the most straightforward, sometimes the standard form is preferred or required.

Starting from $y = 5x - 22$, rewrite as:

$y - 5x = -22$

Multiply both sides by -1 for conventional standard form:

$5x - y = 22$

Therefore, the standard form of the line is:

$5x - y = 22$

Summary of Steps to Model the Equation

To summarize, the process to determine the line's equation involves:

1. Identify the slope of the given line by converting its equation to slope-intercept form.
2. Recognize that parallel lines share the same slope.

3. Use the point-slope form with the known point and the identified slope to find the equation.
4. Optional: Convert to slope-intercept or standard form as needed.

The key points in this problem are:

- The slope of the given line is 5.
- The new line passes through (5, 3).
- The new line's equation in slope-intercept form is y = 5x - 22.
- In standard form, it is 5x - y = 22.

Additional Considerations and Practice

Understanding how to find the equation of a parallel line passing through a given point is a common problem in algebra, with applications in various fields such as engineering, physics, and computer graphics. Practice with similar problems helps solidify the concept.

Sample Problems for Practice

- Find the equation of a line parallel to y = 2x + 3 that passes through the point (4, -1).
- Given the line 4y + 8x = 12, write the equation of a line parallel to it passing through the point (0, 5).
- Determine the equation of a line perpendicular to y = -3x + 7 that passes through the point (-2, 4).

Each of these problems involves identifying the slope of the original line, applying the point-slope form, and converting to the desired form.

Conclusion

In conclusion, when tasked with modeling a line that is parallel to a given line and passes through a specific point, the fundamental steps involve identifying the slope of the original line, maintaining that slope for the parallel line, and using the point-slope formula with the given point to derive the equation. For the problem at hand, the original line y = 5x - 11/3 has a slope of 5, and passing through (5, 3), the required line's equation is y = 5x - 22. This process exemplifies the core principles of linear equations and their geometric interpretations, providing a solid foundation for more advanced topics in analytic geometry.

Frequently Asked Questions

How do you find the equation of a line parallel to 3y - 15x + 11 = 0 passing through (5, 3)?

First, rewrite the given line in slope-intercept form to find its slope. Then, use the same slope with the point (5, 3) in point-slope form to find the new line's equation.

What is the slope of the line 3y - 15x + 11 = 0?

Rearranging the line to slope-intercept form gives y = 5x - 11/3, so the slope is 5.

What is the equation of the line passing through (5, 3) and parallel to the given line?

Using point-slope form: y - 3 = 5(x - 5). Simplifying, the equation is y = 5x - 22.

Why do parallel lines have the same slope?

Parallel lines are always equidistant and do not intersect, which means they share the same slope but have different y-intercepts.

How does the point (5, 3) help in determining the new line's equation?

The point provides a specific location through which the new line passes, allowing us to substitute these coordinates into the point-slope form to find its exact equation.

Can the new line be written in standard form? If so, what is it?

Yes. From y = 5x - 22, the standard form is 5x - y = 22.

Additional Resources

A Line Is Parallel To The Line 3y - 15x + 11 = 0 And Passes Through The Point (5, 3). Which Equation Models?
Understanding how to find the equation of a line that is parallel to a given line and passes through a specific point is a fundamental concept in coordinate geometry. This problem combines the principles of slope calculations, point-slope form, and the general equation of a line. By exploring this topic thoroughly, learners can deepen their comprehension of linear equations and their geometric interpretations.

Understanding the Problem

When asked to find a line that is parallel to a given line and passes through a particular point, the key is to recognize the properties of parallel lines:

- Parallel lines have the same slope.
- They differ only in their y-intercept (or their position in the plane).

Given the line 3y - 15x + 11 = 0 and the point (5, 3), our goal is to derive the equation of the line that satisfies these conditions.

Converting the Given Line to Slope-Intercept Form

Step 1: Rewrite the Equation

The original equation is:

$$ 3y - 15x + 11 = 0 $$

To better understand its slope, convert it into slope-intercept form $ y = mx + b $.

Step 2: Isolate y

Rearranging:

$$ 3y = 15x - 11 $$

Divide both sides by 3:

$$ y = 5x - \frac{11}{3} $$

This form reveals:

- Slope $ m = 5 $
- Y-intercept $ b = -\frac{11}{3} $

Conclusion:

The given line has a slope of 5.

Finding the Equation of the Parallel Line

Step 1: Recognize the Slope

Since parallel lines have identical slopes, the new line passing through (5, 3) will also have a slope of 5.

Step 2: Use Point-Slope Form

The point-slope form of a line is:

$$ y - y_1 = m (x - x_1) $$

Where:
- $(x_1, y_1) = (5, 3)$
- $m = 5$

Plugging in:

$$ y - 3 = 5 (x - 5) $$

Step 3: Simplify to Slope-Intercept Form

Expanding:

$$ y - 3 = 5x - 25 $$

Adding 3 to both sides:

$$ y = 5x - 22 $$

Thus, the equation of the line passing through (5, 3) and parallel to the original line is:

$$ y = 5x - 22 $$

Alternative Forms of the Equation

While the slope-intercept form is most straightforward for understanding the line's slope and position, other forms can be useful depending on context.

Standard Form

Rearranged as:

$$ y - 5x = -22 $$

or

$$ 5x + y = 22 $$

Features and Characteristics of the Derived Line

Pros:

- Clarity: The slope and intercept are explicitly shown.
- Ease of plotting: Given the slope and a point, plotting is straightforward.
- Analytical flexibility: The standard form facilitates algebraic operations like adding or subtracting equations.

Cons:

- Less intuitive for some: For learners unfamiliar with slope-intercept form, it might be less visual.
- Limited to certain applications: For some geometric proofs, other forms might be more suitable.

Verifying the Solution

It’s always good practice to verify that the derived line passes through the given point:

Plug $x=5$ into $y=5x-22$:

$$ y = 5(5) - 22 = 25 - 22 = 3 $$

Since $y = 3$ matches the point (5, 3), the solution is verified.

Additional Considerations and Variations

1. General Equation of a Line

The general form is $Ax + By + C = 0$.
From the slope-intercept form, the equivalent is:

$$y = 5x - 22 \Rightarrow 5x - y - 22 = 0$$

This form is useful in systems of equations or when dealing with inequalities.

2. Parallel Lines in Different Quadrants

The derived line’s position relative to the original line depends on the y-intercept. Since both lines have the same slope, their relative position is determined solely by their intercepts.

3. Distance Between Parallel Lines

The distance between the original line and the new line can be calculated if needed, using the formula:

$$\text{Distance} = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$

where A, B, C_1, and C_2 correspond to the coefficients in the standard form.

Summary and Practical Applications

Understanding how to derive the equation of a line parallel to a given line passing through a specific point is a key skill in geometry, algebra, and their applications in real-world problems such as engineering, physics, and computer graphics.

Key steps include:

- Converting the given line into slope-intercept form to find its slope.
- Using the point-slope form to create the new line’s equation.
- Simplifying to the desired form (slope-intercept, standard, or general).

Features:

- The process emphasizes the importance of understanding slopes and intercepts.
- Facilitates visualizations of parallel lines in coordinate planes.

Limitations:

- Assumes familiarity with algebraic manipulations.
- Less straightforward if the original line is not in standard form.

Conclusion

The equation of the line that is parallel to \(3y - 15x + 11=0 \) and passes through the point \((5, 3) \) is \(y=5x - 22 \) or equivalently, \(5x - y - 22=0 \). This problem encapsulates core concepts in linear equations, including slope calculation, point-slope form, and equation conversion. Mastering these steps enhances problem-solving skills and lays a foundation for tackling more complex geometric and algebraic challenges. Whether for academic purposes or practical applications, understanding how to model such lines is essential in the study of coordinate geometry.

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