A Linear Sequence Starts A+3b A+7b A+11b The 2nd Term Has Value 19 The 5th Term Has Value 67 Work Out

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Understanding and solving linear sequences is a fundamental aspect of algebra and mathematical problem-solving. When given a sequence with specific terms and values, the key is to identify the pattern and derive the general formula to find unknown terms. In this guide, we will thoroughly analyze the sequence starting with terms like A+3b, A+7b, A+11b, and more, and use the provided information about certain term values to work out the unknowns, including the values of A and b, and any other missing elements. This comprehensive exploration aims to equip you with the skills needed to approach similar sequence problems confidently.

Understanding the Sequence: Structure and Pattern

Sequence Breakdown

The sequence provided is:

- 1st term: A + 3b
- 2nd term: A + 7b
- 3rd term: A + 11b

Notice the pattern in the sequence:

- The first term involves A + 3b.
- The second term involves A + 7b.
- The third term involves A + 11b.

The pattern suggests the sequence of the coefficients of b increases by 4 each time:

- 3, 7, 11,...

This indicates an arithmetic progression (AP) in the coefficients, with a common difference of 4.

General Term of the Sequence

Given the pattern, the nth term can be expressed as:

$$ T_n = A + (4n - 1) b $$

Here's why:
- For n=1: $T_1 = A + (41 - 1) b = A + 3b$
- For n=2: $T_2 = A + (42 - 1) b = A + 7b$
- For n=3: $T_3 = A + (43 - 1) b = A + 11b$

Thus, the general term formula captures the pattern succinctly.

Using Known Values to Find A and b

Given data points:
- The 2nd term is 19.
- The 5th term is 67.

We'll now plug these known values into our general term formula to form equations and solve for A and b.

Equation from the 2nd Term

Substituting n=2:

$$ T_2 = A + (42 - 1) b = A + 7b = 19 $$

This gives:

$$ A + 7b = 19 \quad \text{(Equation 1)} $$

Equation from the 5th Term

Substituting n=5:

$$ T_5 = A + (45 - 1) b = A + 19b = 67 $$

This gives:

$$ A + 19b = 67 \quad \text{(Equation 2)} $$

Solving the Equations

Now, we have a system of two equations:

1. $A + 7b = 19$
2. $A + 19b = 67$

Subtract Equation 1 from Equation 2:

$$ (A + 19b) - (A + 7b) = 67 - 19 $$
$$ A - A + 19b - 7b = 48 $$
$$ 12b = 48 $$

Solve for b:

$$ b = \frac{48}{12} = 4 $$

Now, substitute b=4 back into Equation 1 to find A:

$$ A + 7 \times 4 = 19 $$
$$ A + 28 = 19 $$
$$ A = 19 - 28 = -9 $$

Summary:

- $A = -9$
- $b = 4$

Final Expression for the Sequence

With A and b known, the nth term of the sequence can now be written as:

$$ T_n = A + (4n - 1)b $$
$$ T_n = -9 + (4n - 1) \times 4 $$

Simplify:

$$ T_n = -9 + 4(4n - 1) $$
$$ T_n = -9 + 16n - 4 $$
$$ T_n = 16n - 13 $$

Thus, the explicit formula for the sequence is:

$$ T_n = 16n - 13 $$

Verification of the Sequence

To ensure our formula is correct, verify with known terms:

- For n=2:

$$T_2 = 16 \times 2 - 13 = 32 - 13 = 19 \quad \checkmark$$

- For n=5:

$$T_5 = 16 \times 5 - 13 = 80 - 13 = 67 \quad \checkmark$$

The formula successfully reproduces the known terms, confirming its correctness.

Additional Insights and Applications

Understanding the Sequence Pattern

The sequence increases linearly with each term, with a common difference of:

$$T_{n+1} - T_n = (16(n+1) - 13) - (16n - 13) = 16n + 16 - 13 - 16n + 13 = 16$$

This confirms it's an arithmetic sequence with a common difference of 16.

Practical Applications of Such Sequences

Linear sequences and their formulas are widely used in various fields, such as:

- Finance: Calculating consistent growth or payments over time.
- Physics: Modeling uniform motion or constant acceleration.
- Computer Science: Analyzing algorithms with predictable step increases.
- Engineering: Designing systems with linear response characteristics.

Understanding how to derive the general term from specific data points enables professionals and students to model real-world scenarios accurately.

Problem-Solving Tips for Sequence Questions

When tackling similar problems, consider the following approach:

1. Identify the pattern: Look for the difference between successive terms.
2. Express the general term: Use the pattern to write a formula.
3. Plug in known values: Create equations with known term values.
4. Solve systematically: Use algebraic methods to find unknowns.
5. Verify your solution: Check the formula against known terms.

Summary and Key Takeaways

- The sequence begins with terms involving A and b, with a clear pattern in coefficients.
- The general term is $T_n = A + (4n - 1) b$.
- Using the given terms (second and fifth), we derived:
- $A = -9$
- $b = 4$
- The explicit sequence formula is $T_n = 16n - 13$.
- The sequence has a common difference of 16, confirming its arithmetic nature.
- Applying these methods helps solve a wide range of sequence problems effectively.

Conclusion

Mastering the process of analyzing linear sequences involves recognizing patterns, formulating general terms, and applying algebraic techniques to solve for unknowns. The example sequence starting with A+3b, A+7b, A+11b, and so on, illustrates these principles clearly. With practice, you'll be able to handle more complex sequences with confidence, whether for academic purposes or real-world applications. Remember to verify your answers and understand the underlying pattern to ensure accuracy and deepen your mathematical understanding.

Frequently Asked Questions

What is the common difference in the linear sequence A + 3b, A + 7b, A + 11b?

The common difference is 4b, since each term increases by 4b from the previous one.

Given that the second term of the sequence is 19, how can we express A in terms of b?

The second term is A + 7b = 19, so A = 19 - 7b.

Using the fifth term value of 67, how can we find the value of b?

The fifth term is A + 19b. Substituting A = 19 - 7b, we get (19 - 7b) + 19b = 67, which simplifies to 19 + 12b = 67; solving gives 12b = 48, so b = 4.

After finding b, what is the value of A in the sequence?

Substituting b = 4 into A = 19 - 7b gives A = 19 - 28 = -9.

What is the explicit formula for the nth term of the sequence?

The nth term is A + (2n - 1) 4b. Substituting A = -9 and b = 4, the nth term is -9 + (2n - 1) 16.

Additional Resources

Linear Sequence Analysis

Understanding the Nature of a Linear Sequence

In the realm of mathematics, sequences are fundamental in understanding patterns, growth, and relationships between numbers. Among these, linear sequences—also known as arithmetic sequences—are among the simplest and most widely studied. They are characterized by a constant difference between consecutive terms, which makes their analysis both accessible and insightful.

This article delves into a specific linear sequence described as:

$$ A + 3b, \quad A + 7b, \quad \text{and so forth} $$

with the goal to determine particular unknowns based on given conditions. We will explore the sequence's structure, identify the first term and common difference, and systematically work through the problem to find the solution.

Deciphering the Sequence Structure

Identifying the Pattern

The sequence provided is:

$$ A + 3b, \quad A + 7b, \quad \dots $$

At first glance, the terms suggest a pattern involving variables A and b. The terms are expressed as:

- First term (T_1): $A + 3b$
- Second term (T_2): $A + 7b$

Given this, the sequence seems to be:

$$T_1 = A + 3b$$
$$T_2 = A + 7b$$
$$T_3 = A + 11b$$
$$T_4 = A + 15b$$
$$T_5 = A + 19b$$
and so on.

Notice the pattern in the coefficients of b: 3, 7, 11, 15, 19, ... Each increases by 4 for each subsequent term.

Recognizing the Arithmetic Progression

The sequence of coefficients (3, 7, 11, 15, 19, ...) forms an arithmetic sequence with:

- First term: 3
- Common difference: 4

This means the general term for the n^{th} term of the sequence can be expressed as:

$$T_n = A + \left[3 + (n-1) \times 4 \right] b$$

Simplifying this:

$$T_n = A + (4n - 1) b$$

since:

$$3 + (n-1) \times 4 = 3 + 4n - 4 = 4n - 1$$

Thus, the general term of the sequence is:

$$T_n = A + (4n - 1) b$$

This formula is essential for solving for A and b given specific values.

Applying the Given Conditions

The problem provides two key details:

- The second term has a value of 19:

$$T_2 = 19$$

- The fifth term has a value of 67:

$$ T_5 = 67 $$

Using the general formula, these conditions translate to:

$$ T_2 = A + (4 \times 2 - 1) b = A + (8 - 1) b = A + 7b = 19 $$

$$ T_5 = A + (4 \times 5 - 1) b = A + (20 - 1) b = A + 19b = 67 $$

This yields two equations:

1. $A + 7b = 19$ (Equation 1)
2. $A + 19b = 67$ (Equation 2)

Solving for A and b

Step 1: Subtracting the equations

Subtract Equation 1 from Equation 2 to eliminate A:

$$
(A + 19b) - (A + 7b) = 67 - 19
$$
$$
A - A + 19b - 7b = 48
$$
$$
12b = 48
$$

Step 2: Finding b

Divide both sides by 12:

$$
b = \frac{48}{12} = 4
$$

Step 3: Finding A

Substitute $b = 4$ into Equation 1:

$$
A + 7 \times 4 = 19
$$
$$
A + 28 = 19
$$
$$
A = 19 - 28 = -9
$$

Final Sequence and Verification

With the values of A and b determined, the explicit formula for the sequence becomes:

$$
T_n = -9 + (4n - 1) \times 4
$$

Simplify:

$$
T_n = -9 + (4n - 1) \times 4 = -9 + 16n - 4
$$
$$
T_n = 16n - 13
$$

Verification:

- For $n=2$:

$$
T_2 = 16 \times 2 - 13 = 32 - 13 = 19 \quad \checkmark
$$

- For $n=5$:

$$
T_5 = 16 \times 5 - 13 = 80 - 13 = 67 \quad \checkmark
$$

This confirms the correctness of the solution.

Implications and Further Exploration

The sequence $T_n = 16n - 13$ is a classic arithmetic progression with:

- First term (T_1):

$$
T_1 = 16 \times 1 - 13 = 16 - 13 = 3
$$

- Common difference:

$$
d = T_2 - T_1 = (16 \times 2 - 13) - (16 \times 1 - 13) = (32 - 13) - (16 - 13) = 19 - 3 = 16
$$

This matches the pattern of a linear sequence with a constant difference of 16, confirming the sequence's nature.

Additional Questions and Applications

What is the 10th term?

Using the formula:

$$
T_{10} = 16 \times 10 - 13 = 160 - 13 = 147
$$

Sum of the first n terms:

The sum of the first n terms in an arithmetic sequence is:

$$
S_n = \frac{n}{2} \times (T_1 + T_n)
$$

For $n=10$:

$$
S_{10} = \frac{10}{2} \times (3 + 147) = 5 \times 150 = 750
$$

What is the sequence's behavior?

The sequence increases linearly by 16 each step, illustrating steady growth. Such sequences are useful in modeling scenarios like uniform financial savings, consistent manufacturing outputs, or predictable population growth under ideal conditions.

Conclusion

Through a detailed analysis, we've demonstrated how to interpret a linear sequence with variable parameters, establish a general formula, and apply given conditions to find unknowns. The process underscores the importance of recognizing patterns, translating word problems into algebraic equations, and systematically solving for unknowns.

The sequence:

$$
T_n = 16n - 13
$$

not only satisfies the conditions provided but also exemplifies the elegance of linear sequences in mathematics. Understanding these sequences enhances problem-solving skills and provides a foundation for more complex mathematical concepts and real-world applications.

Summary of key steps:

- Recognize the pattern in the sequence's terms.
- Derive the general formula based on the pattern.
- Substitute known term values to form equations.
- Solve simultaneous equations to find unknowns.
- Verify the solution with the original conditions.
- Explore further terms and applications for comprehensive understanding.

By mastering these techniques, students and enthusiasts can confidently analyze and interpret linear sequences across various contexts, reinforcing the foundational principles of algebra and sequence analysis.

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