

A Normal Distribution Has A Mean Of 15 And A Standard Deviation Of 2. Find The Value That Corresponds

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Understanding the properties of normal distribution is fundamental in statistics, especially when analyzing data that tends to cluster around a central value. When we are presented with a normal distribution characterized by its mean and standard deviation, the key question often is: What specific value corresponds to a certain percentile or probability? In this article, we will explore how to find the value that corresponds to a given probability or percentile in a normal distribution with a mean of 15 and a standard deviation of 2. We will delve into the concepts of z-scores, standard normal distribution tables, and the use of statistical software or calculators to determine these values.

Introduction to Normal Distribution

The normal distribution, often referred to as the bell curve, is a probability distribution that is symmetric about the mean. It is widely used in natural and social sciences to represent real-valued random variables whose distributions are not known precisely but tend to cluster around an average.

Characteristics of a Normal Distribution

- Symmetry: The distribution is perfectly symmetric around the mean.
- Mean, Median, Mode: All three are equal and located at the center.
- Bell-shaped curve: The highest point is at the mean, with tails extending infinitely in both directions.
- 68-95-99.7 Rule: Approximately 68% of data falls within one standard deviation, 95% within two, and 99.7% within three.

The Given Parameters

For our specific case:

- Mean (μ): 15
- Standard Deviation (σ): 2

Understanding these parameters allows us to compute the value corresponding to a particular percentile or probability.

The Concept of Z-Score

To find the value that corresponds to a particular probability in a normal

distribution, we use the concept of a z-score.

What is a Z-Score?

A z-score indicates how many standard deviations an element is from the mean. It is calculated using:

$$z = \frac{X - \mu}{\sigma}$$

Where:

- X: The value in the distribution
- μ : The mean
- σ : The standard deviation

This transformation allows us to compare scores from different normal distributions by standardizing them to a standard normal distribution with mean 0 and standard deviation 1.

Using Z-Score to Find Corresponding Values

Given a probability or percentile, we can:

1. Find the z-score that corresponds to that probability using standard normal distribution tables or statistical software.
2. Rearranged the z-score formula to solve for X:

$$X = z \times \sigma + \mu$$

This process is central to many statistical analyses, especially in hypothesis testing and confidence interval estimation.

Finding Values Corresponding to Specific Probabilities

Suppose we want to find the value X in our distribution that corresponds to a particular probability p (for example, the 90th percentile, which is the value below which 90% of the data falls).

Step-by-Step Procedure

1. Determine the probability (p): The cumulative probability up to the value X.
2. Find the z-score (z): Use a z-table or calculator to find the z-score that corresponds to p.
3. Calculate X: Use the formula $X = z \times \sigma + \mu$.

Example: Find the 90th Percentile

Let's walk through an example where:

- p = 0.90 (90th percentile)

Step 1: Find the z-score for $p = 0.90$

Using a z-table or calculator, the z-score corresponding to 0.90 is approximately 1.28.

Step 2: Calculate the value X

$$X = 1.28 \times 2 + 15 = 2.56 + 15 = 17.56$$

Result: The 90th percentile value in the distribution is approximately 17.56.

This means that 90% of the data falls below approximately 17.56 in this distribution.

Using Statistical Software and Calculators

While z-tables are useful, modern tools make it easier and more accurate to find these values:

- Excel: Use the function `=NORM.INV(probability, mean, standard_deviation)`
- R: Use `qnorm(p, mean, sd)`
- Python: Use `scipy.stats.norm.ppf(p, loc=mean, scale=standard_deviation)`
- Online Calculators: Many websites offer free normal distribution calculators.

Example with Excel

To find the 90th percentile:

```
```\nexcel\n=NORM.INV(0.90, 15, 2)\n```
```

Result: 17.56 (matching our manual calculation).

### Applications of Finding Corresponding Values

Understanding how to find these values is critical in various fields:

- Quality Control: Determining the acceptable range of product measurements.
- Finance: Calculating Value at Risk (VaR).
- Psychology and Education: Establishing percentile ranks for test scores.
- Medical Research: Determining thresholds for clinical measurements.

### Common Percentiles and Their Corresponding Values

Below are some typical percentiles and their approximate corresponding values in our distribution:

Percentile	Z-Score	Calculation	Corresponding Value (X)
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5th	-1.64	$\backslash ( X = -1.64 \times 2 + 15 \backslash )$	11.72
25th	-0.67	$\backslash ( X = -0.67 \times 2 + 15 \backslash )$	13.66
50th (Median)	0	$\backslash ( X = 0 \times 2 + 15 \backslash )$	15.00
75th	0.67	$\backslash ( X = 0.67 \times 2 + 15 \backslash )$	16.34
95th	1.64	$\backslash ( X = 1.64 \times 2 + 15 \backslash )$	18.28

These calculations demonstrate how percentile positions translate into raw data values.

## Interpreting the Results

The ability to find the corresponding value for a given probability helps in decision-making processes where understanding the distribution of data is crucial. For example:

- Setting thresholds: If a company wants to identify the top 10% of performers, they would look at the 90th percentile.
- Risk assessment: In finance, knowing the value at a certain percentile helps assess potential losses.
- Educational assessments: Percentile ranks help interpret student test scores relative to the population.

## Summary and Conclusion

In this comprehensive guide, we explored how to determine the value that corresponds to a specific probability in a normal distribution with a mean of 15 and a standard deviation of 2. The key steps involved:

- Understanding the properties of the normal distribution.
- Calculating or finding the z-score associated with the desired probability.
- Applying the z-score formula to find the raw data value.

By mastering these concepts, statisticians, researchers, data analysts, and students can accurately interpret data distributions and make informed decisions based on probabilistic thresholds.

## Final Tips:

- Always verify the probability or percentile you want to find.
- Use reliable tools like statistical software or online calculators for precision.
- Remember that the same principles apply regardless of the mean and standard deviation; only the parameters change.

Understanding how to find the corresponding values in a normal distribution is a vital skill in statistics that bridges theoretical concepts with practical applications across various domains. Whether you're analyzing test scores, financial risks, or quality control data, these techniques empower you to interpret and utilize data more effectively.

## Frequently Asked Questions

### **What is the probability that a value from the normal distribution is less than 17?**

Using the z-score formula:  $(17 - 15) / 2 = 1.0$ . The probability that a value is less than 17 is approximately 84.13%.

### **What value corresponds to the 95th percentile in this distribution?**

The z-score for the 95th percentile is approximately 1.645. Therefore, the value is  $15 + (1.645)(2) = 15 + 3.29 = 18.29$ .

### **If a value is 19, what is its z-score?**

$Z = (19 - 15) / 2 = 2.0$ . So, the z-score for 19 is 2.0.

### **What is the probability that a value is between 13 and 17?**

Calculate z-scores:  $(13 - 15)/2 = -1.0$ ,  $(17 - 15)/2 = 1.0$ . The probability between  $z = -1$  and  $z = 1$  is approximately 68.27%.

### **Find the value that corresponds to the 50th percentile (median).**

In a normal distribution, the median is equal to the mean, so the value is 15.

### **What is the probability of obtaining a value greater than 19?**

$Z = (19 - 15)/2 = 2.0$ . The probability of a z-score greater than 2.0 is approximately 2.28%.

## Additional Resources

**A Normal Distribution Has A Mean Of 15 And A Standard Deviation Of 2. Find The Value That Corresponds**

Understanding the nuances of the normal distribution is fundamental in statistics, especially when it comes to interpreting data and making predictions. When given a normal distribution characterized by a mean ( $\mu$ ) and a standard deviation ( $\sigma$ ), one of the most common questions posed is: What is

the value corresponding to a specific probability or percentile? In this article, we will explore how to find the specific value (often called the raw score or x-value) that aligns with a particular probability within a normal distribution that has a mean of 15 and a standard deviation of 2.

This comprehensive review will cover the theoretical foundation of the normal distribution, how to interpret probabilities and percentiles, the use of the standard normal distribution (z-scores), and practical steps to find the corresponding value. We will also analyze real-world applications and common pitfalls in this process.

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## Understanding the Normal Distribution: Basics and Significance

### What Is a Normal Distribution?

The normal distribution, often referred to as the bell curve, is a probability distribution that is symmetric about its mean. It describes how the values of a continuous variable are distributed, with most data points clustering around the central value (mean), and fewer occurring as you move further away in either direction.

Mathematically, the probability density function (PDF) for a normal distribution is given by:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{\left\{ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right\}}$$

where:

- $\mu$  is the mean (center of the distribution),
- $\sigma$  is the standard deviation (spread or dispersion),
- $x$  is the variable of interest.

The shape of the distribution is entirely determined by these two parameters.

### Why Is the Normal Distribution Important?

The normal distribution is fundamental in statistics because:

- Many natural phenomena tend to follow a normal distribution (e.g., heights, test scores, measurement errors).

- It forms the basis for many statistical inference techniques, including hypothesis testing and confidence intervals.
- It simplifies calculations related to probabilities, especially when combined with the standard normal distribution.

In our scenario, the distribution has:

- Mean ( $\mu$ ) = 15
- Standard deviation ( $\sigma$ ) = 2

This means most data points will cluster around 15, with about 68% within one standard deviation (13–17), 95% within two standard deviations (11–19), and 99.7% within three standard deviations (9–21).

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## Interpreting Probabilities and Percentiles in a Normal Distribution

### What Does a Probability or Percentile Represent?

In the context of the normal distribution, probabilities relate to the area under the curve:

- The probability that a random variable falls within a certain range.
- The percentile rank indicates the percentage of data below a specific value.

For example, the 50th percentile (median) in a symmetric distribution like the normal distribution is the mean, which is 15 in our case.

Suppose we are asked: "Find the value corresponding to the 95th percentile." This means we need the value below which 95% of the data falls.

### Using the Standard Normal Distribution

Calculating probabilities directly from the original distribution can be complex. To simplify, statisticians convert the original data into a standardized form called the z-score, which transforms the distribution into a standard normal distribution with:

- Mean ( $\mu = 0$ )
- Standard deviation ( $\sigma = 1$ )

The z-score formula is:

$$z = \frac{x - \mu}{\sigma}$$

Once converted, we use standard normal distribution tables or computational tools to find the probability corresponding to a z-score, or vice versa.

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## Calculating the Corresponding Value: Step-by-Step Approach

Suppose you are asked to find the value  $x$  corresponding to a specific percentile (say, the 95th percentile) in our distribution with  $\mu=15$ ,  $\sigma=2$ .

Step 1: Identify the Required Probability

- For the 95th percentile, the probability  $P = 0.95$ .

Step 2: Find the Corresponding z-Score

Using a standard normal distribution table or calculator, look up the z-score that corresponds to a cumulative probability of 0.95.

- Commonly,  $z_{0.95} \approx 1.645$ .

Step 3: Convert z-Score Back to the Original Scale

Using the z-score formula rearranged:

$$x = \mu + z \times \sigma$$

Substitute the values:

$$x = 15 + 1.645 \times 2$$

$$x = 15 + 3.29$$

$$x \approx 18.29$$

Result: The value corresponding to the 95th percentile is approximately 18.29.

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# Applying the Method to Various Scenarios

## 2.1 Finding a Value for a Given Probability

Suppose you are asked: "Find the value of  $x$  such that 25% of the data falls below it."

Solution Steps:

- $P = 0.25$
- Find  $z_{0.25}$  from z-tables, approximately -0.674.
- Compute:

$$x = 15 + (-0.674) \times 2 = 15 - 1.348 = 13.652$$

Interpretation: About 25% of data falls below approximately 13.65.

## 2.2 Finding a Value for a Specific Interval

If you want to find the value between the 40th and 60th percentiles:

- 40th percentile:  $z \approx -0.253$
- 60th percentile:  $z \approx 0.253$

Calculate the corresponding x-values:

- 40th percentile:

$$x_{40} = 15 + (-0.253) \times 2 = 15 - 0.506 = 14.494$$

- 60th percentile:

$$x_{60} = 15 + 0.253 \times 2 = 15 + 0.506 = 15.506$$

These values show the interquartile range around the median.

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# Practical Applications of These Calculations

Understanding how to find these corresponding values has broad implications across multiple fields:

## 2.1 Education and Testing

Standardized tests often report scores in percentiles. Knowing how to convert these percentiles back into raw scores helps educators interpret student

performances on different scales.

## 2.2 Quality Control and Manufacturing

Manufacturers use normal distribution models to determine acceptable ranges of product measurements. For example, if a product's weight is normally distributed with known parameters, determining the cutoff for the top 5% of weights can guide quality standards.

## 2.3 Finance and Risk Assessment

Financial analysts assess asset returns assuming they follow a normal distribution. Calculating the value at risk (VaR) involves identifying the loss threshold that corresponds to a certain confidence level.

## 2.4 Health and Medical Research

Clinical measurements, such as blood pressure or cholesterol levels, are often modeled with normal distributions. Determining thresholds that correspond to specific percentiles helps in diagnosing and setting health standards.

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# Common Pitfalls and Limitations in Using Normal Distribution

While the normal distribution is a powerful tool, several pitfalls can occur:

## 2.1 Over-reliance on Normality Assumption

Many real-world data sets are not perfectly normal. Applying normal distribution calculations to skewed data can lead to inaccurate results.

## 2.2 Misinterpretation of Percentiles

Assuming that the 95th percentile corresponds to a specific raw value without considering the actual data distribution can be misleading, especially when the data is not normal.

## 2.3 Ignoring Outliers and Variability

Outliers or heteroscedasticity (non-constant variance) can distort the distribution, affecting the accuracy of the calculated values.

## 2.4 Using Approximate z-Values

Sometimes, the z-values are approximated; small errors can lead to

significant deviations, especially in critical decision-making contexts.

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## **Advanced Considerations and Tools**

### **2.1 Using Software and Calculators**

Modern statistical software packages (e.g., R, Python's SciPy, SPSS, Excel) simplify the process:

- Functions like ``scipy.stats.norm.ppf()`` in Python give the inverse cumulative distribution function (percent point function) directly.
- Excel's ``NORM.INV()`` function allows for quick calculations.

### **2.2 Confidence Intervals and Hypothesis Testing**

Understanding the relationship between raw scores and z-scores also aids in constructing confidence intervals and conducting hypothesis tests, which are foundational in scientific research.

### **2.3 The Role of Sample Size**

While the above calculations assume the distribution parameters are known, in practice, they are estimated from sample data. Larger samples yield more reliable estimates, reducing uncertainty in the calculated values.

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## **Conclusion: The Power of Normal Distribution in Data Analysis**