# A Point On The Terminal Side Of Angle 0 Is Given. Find The Exact Value Of The Indicated Trigonometric

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Understanding how to find the exact value of trigonometric functions given a point on the terminal side of an angle is a fundamental skill in trigonometry. Whether you're solving for sine, cosine, tangent, or their reciprocal functions, mastering this process is essential for success in mathematics. This article provides a comprehensive guide on how to approach such problems, offering clarity and step-by-step instructions to help you confidently determine the exact values of trigonometric functions based on a given point.

## Understanding the Basic Concepts

### What Is an Angle and Its Terminal Side?

In the coordinate plane, an angle is formed by rotating a ray (the initial side) from the positive x-axis to another ray (the terminal side). The terminal side is the position of the ray after rotation, and it can lie in any quadrant or on axes.

### Points on the Terminal Side

A point on the terminal side of an angle is represented by its coordinates (x, y). These coordinates are crucial because they allow us to determine the exact trigonometric values associated with the angle.

### The Role of the Radius (r)

The distance from the origin (0,0) to the point (x, y)—denoted as r—is the hypotenuse of a right triangle formed in the coordinate plane. It is calculated using the Pythagorean theorem:

- $r = \sqrt{(x^2 + y^2)}$

This value is essential because it normalizes the coordinate point and helps in computing the exact values of sine, cosine, and tangent.

# Step-by-Step Method to Find Exact Trigonometric Values

## Step 1: Identify the Coordinates (x, y)

Given the point on the terminal side of the angle, note down its x and y coordinates.

## Step 2: Calculate the Radius r

Compute the hypotenuse using the formula:

- $r = \sqrt{(x^2 + y^2)}$

This step ensures normalization of the point concerning the unit circle.

## Step 3: Find the Basic Trigonometric Functions

Using the coordinates and radius, you can find the exact values of:

- **Sine (sin θ):** $\sin \theta = y / r$
- **Cosine (cos θ):** $\cos \theta = x / r$
- **Tangent (tan θ):** $\tan \theta = y / x$ (provided $x \neq 0$)

Note that these are exact ratios, and they may simplify depending on the values.

## Step 4: Determine Reciprocal Functions

Once sine, cosine, and tangent are known, the reciprocal functions are straightforward:

- **Cosecant (csc θ):** $\csc \theta = r / y$ (if $y \neq 0$)
- **Secant (sec θ):** $\sec \theta = r / x$ (if $x \neq 0$)
- **Cotangent (cot θ):** $\cot \theta = x / y$ (if $y \neq 0$)

Understanding these steps allows you to find the exact value of any indicated trigonometric function for the given point.

# Examples Illustrating the Process

## Example 1: Point in Quadrant I

Suppose the point given is (3, 4).

- Calculate r:

$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

- Find sin θ:

$\sin \theta = y / r = 4 / 5 = 0.8$

- Find cos θ:

$\cos \theta = x / r = 3 / 5 = 0.6$

- Find tan θ:

$\tan \theta = y / x = 4 / 3 \approx 1.333$

Similarly, reciprocal functions:

- $\csc \theta = 5 / 4 = 1.25$
- $\sec \theta = 5 / 3 \approx 1.666$
- $\cot \theta = 3 / 4 = 0.75$

## Example 2: Point in Quadrant III

Suppose the point is (-6, -8).

- Calculate r:

$r = \sqrt{(-6)^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

- sin θ:

$\sin \theta = y / r = -8 / 10 = -0.8$

- cos θ:

$\cos \theta = x / r = -6 / 10 = -0.6$

- tan θ:

$\tan \theta = y / x = -8 / -6 = 4 / 3 \approx 1.333$

Reciprocal functions:

- $\csc \theta = 10 / (-8) = -1.25$
- $\sec \theta = 10 / (-6) \approx -1.666$
- $\cot \theta = -6 / -8 = 3 / 4 = 0.75$

These examples demonstrate how to find exact trigonometric values using coordinate points.

# Special Cases and Considerations

## Points on the Axes

- If the point lies on the x-axis (y=0), then:
- $\sin \theta = 0$
- $\cos \theta = \pm 1$, depending on the sign of x
- $\tan \theta = 0$
- If the point lies on the y-axis (x=0), then:
- $\cos \theta = 0$
- $\sin \theta = \pm 1$, depending on y
- $\tan \theta$ is undefined

## Angles on the Axes and the Unit Circle

Angles such as 0°, 90°, 180°, and 270° have points on axes, making their trigonometric values straightforward.

## Conclusion: Mastering the Calculation of Exact Trigonometric Values

Finding the exact value of a trigonometric function from a point on the terminal side of an angle involves understanding the relationship between the coordinates, the radius, and the trigonometric ratios. By identifying the point, calculating the hypotenuse, and applying the fundamental ratios, you can accurately determine sine, cosine, tangent, and their reciprocals.

This skill is not only vital for solving various mathematical problems but also foundational for understanding more advanced topics in calculus, physics, and engineering. Practice with different points and quadrants enhances your proficiency, enabling you to approach trigonometric problems with confidence and precision.

Remember, the key steps are: identify the point (x, y), compute r, and then use ratios to find the exact values. With consistent practice, these calculations will become intuitive, solidifying your grasp of trigonometry's core principles.

## Frequently Asked Questions

### How do you determine the exact value of a trigonometric function when a point lies on the terminal side of an angle in standard position?

You identify the coordinates of the point on the terminal side, then use the definitions of the trigonometric functions (e.g., sine = y/r, cosine = x/r, tangent = y/x) to find the exact value.

### What is the significance of the point's coordinates on the terminal side of an angle in finding trigonometric values?

The coordinates (x, y) directly represent the values needed to compute the trigonometric functions, with r being the distance from the origin to the point, which normalizes these values.

### If a point on the terminal side of angle 0 has coordinates (3, 4), what is the exact value of sin 0?

First, find $r = \sqrt{(3^2 + 4^2)} = 5$. Then, sin 0 = y/r = 4/5.

### How can the Pythagorean theorem assist in finding the exact values of trigonometric functions at a given point?

It helps determine the hypotenuse r when the coordinates are known, enabling calculation of sine, cosine, and tangent directly from x, y, and r.

## What is the process to find the exact value of cotangent when given a point on the terminal side of an angle?

Calculate tangent as y/x, then take the reciprocal for cotangent, which is x/y.

## Can the signs of the coordinates affect the sign of the trigonometric functions? How?

Yes. The signs of x and y determine the quadrant, which in turn affects whether sine, cosine, tangent, and their reciprocals are positive or negative.

## How do you evaluate the exact value of secant or cosecant given a point on the terminal side?

Calculate $r = \sqrt{x^2 + y^2}$. Then, secant = r/x and cosecant = r/y, provided x and y are non-zero.

## What is the importance of knowing the exact values of trigonometric functions at specific points on the terminal side?

They are essential for solving trigonometric equations, analyzing angles, and applying in real-world problems involving periodic phenomena.

## How does understanding the unit circle help in finding the exact values of trigonometric functions at points on the terminal side?

The unit circle provides standard reference points with known coordinates, simplifying the process of determining exact trigonometric values for various angles.

# Additional Resources

Understanding the Problem: A Point On The Terminal Side Of Angle θ Is Given – Finding the Exact Value of the Indicated Trigonometric Function

When delving into the world of trigonometry, one of the foundational tasks involves determining the values of trigonometric functions based on a given point in the coordinate plane. A common problem scenario is when a point lies on the terminal side of an angle θ, and we are asked to find the exact value of a specific trigonometric function associated with that angle. This comprehensive guide aims to explore this problem in depth, providing clarity on the concepts, methods, and calculations involved.

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# Introduction to Trigonometric Functions and Angles in the Coordinate Plane

Before tackling the problem, it is essential to revisit some core principles of trigonometry and coordinate geometry.

What Is an Angle in Standard Position?

- An angle θ in standard position is formed by rotating the initial side (usually along the positive x-axis) counterclockwise in the coordinate plane.
- The terminal side of the angle is the ray that terminates after rotation, passing through a specific point.

Points on the Terminal Side of an Angle

- Any point (x, y) lying on the terminal side of an angle θ (except the origin) can be used to determine the values of trigonometric functions.
- The distance from the origin (0,0) to the point is called the radius or hypotenuse, often denoted as r.

The Significance of the Point (x, y)

- The coordinates (x, y) directly relate to the functions sine, cosine, tangent, and their reciprocals.
- These relationships are defined as:
- sin θ = y / r
- cos θ = x / r
- tan θ = y / x
- csc θ = r / y
- sec θ = r / x
- cot θ = x / y

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# Understanding the Given Data: Point on the Terminal Side

Suppose you're given a specific point on the terminal side of an angle θ. Typically, the problem provides:

- The coordinates of the point, say (x, y).
- Sometimes, the radius r, or it may need to be calculated.
- The specific trigonometric function to evaluate (e.g., sin θ, cos θ, tan θ, etc.).

Key Steps to Solve the Problem

1. Identify the Coordinates: Extract x and y from the given point.

2. Calculate the Radius (r): Use the distance formula:

$$
r = \sqrt{x^2 + y^2}
$$

3. Determine the Trigonometric Function: Use the definitions relating x, y, and r.
4. Simplify to Exact Value: Express the result in simplest radical form, if necessary, to find the exact value.

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# Deep Dive into Calculations and Methodology

Let's explore each step with detailed explanations and examples.

Step 1: Extracting Coordinates

- The problem statement provides a point, such as (x, y) = (3, 4).
- The point's location relative to axes determines the quadrant:

| Quadrant | Conditions | Example Coordinates |
|---|---|---|
| I | x > 0, y > 0 | (3, 4) |
| II | x < 0, y > 0 | (-3, 4) |
| III | x < 0, y < 0 | (-3, -4) |
| IV | x > 0, y < 0 | (3, -4) |

- The sign of x and y influences the sign of the trigonometric functions.

Step 2: Calculating the Radius (r)

- The radius, or hypotenuse, is the distance from origin to the point:

$$
r = \sqrt{x^2 + y^2}
$$

- For example, for point (3, 4):

$$
r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

- The value of r is crucial because many trigonometric functions depend on this length.

Step 3: Applying Trigonometric Definitions

- Once x, y, and r are known, the functions can be expressed precisely:

- Sine: $\sin \theta = y / r$
- Cosine: $\cos \theta = x / r$
- Tangent: $\tan \theta = y / x$
- Cosecant: $\csc \theta = r / y$
- Secant: $\sec \theta = r / x$
- Cotangent: $\cot \theta = x / y$

Step 4: Exact Value and Simplification

- Simplify fractions and radical expressions to obtain the exact value.
- For example, with (x, y) = (3, 4):

$$
\sin \theta = \frac{4}{5}
$$
$$
\cos \theta = \frac{3}{5}
$$
$$
\tan \theta = \frac{4}{3}
$$

- These are already in simplest form.

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# Special Cases and Considerations

While the above steps are straightforward, certain cases require additional attention:

When x or y is Zero

- If x = 0, then $\tan \theta$ is undefined, but $\sin \theta$ and $\csc \theta$ are either 1 or -1, depending on y.
- If y = 0, then $\sin \theta = 0$, and $\cos \theta$ is either 1 or -1 based on x.

When r Is Zero

- The point is at the origin (0, 0), and the angle is undefined because the radius is zero.

Quadrant Sign Considerations

- The signs of x and y determine the positive or negative values of the functions, consistent with the quadrant:

| Quadrant | sin | cos | tan | csc | sec | cot |
|---|---|---|---|---|---|---|
| I | + | + | + | + | + | + |
| II | + | - | - | + | - | - |

| | | | | | | |
|---|---|---|---|---|---|---|
| III | - | - | + | - | - | + |
| IV | - | + | - | - | + | - |

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## Worked Examples and Practice Problems

Let's solidify understanding with specific problems.

Example 1: Point in Quadrant I

Given: A point (6, 8) on the terminal side of an angle θ.

Find: Exact values of all six trigonometric functions.

Solution:

1. Calculate r:

$$
r = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10
$$

2. Compute functions:

- $\sin \theta = \frac{8}{10} = \frac{4}{5}$
- $\cos \theta = \frac{6}{10} = \frac{3}{5}$
- $\tan \theta = \frac{8}{6} = \frac{4}{3}$
- $\csc \theta = \frac{10}{8} = \frac{5}{4}$
- $\sec \theta = \frac{10}{6} = \frac{5}{3}$
- $\cot \theta = \frac{6}{8} = \frac{3}{4}$

Example 2: Point in Quadrant II

Given: Point (-3, 4).

Find: $\sin \theta$

Solution:

- r:

$$
r = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

- $\sin \theta = \frac{4}{5}$

- Sign considerations:

- x = -3 (negative), y=4 (positive), so the point is in Quadrant II, where sine is positive.

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## Special Trigonometric Values and Rationalization

Some problems involve angles where the point leads to well-known special triangle ratios, such as 30°, 45°, 60°, etc.

- For instance, points on the unit circle at these angles yield:

| Angle | Coordinates (x, y) | Exact Trigonometric Ratios |
|---------|--------------------|---------------------------|
| 30° / \(\pi/6\) | \((\sqrt{3}/2, 1/2)\) | \(\sin 30° = 1/2

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