A Rectangle Has A Length Of 40 Cm And Width Of 21 Cm. If A New Rectangle Is Formed By Adding The Same

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Understanding the properties of rectangles is fundamental in geometry, whether you're a student, teacher, or someone interested in practical applications like construction, design, or crafts. Rectangles are among the most common shapes in our daily lives, from the layout of rooms to the design of digital interfaces. This article explores a specific scenario involving a rectangle with given dimensions and investigates what happens when we modify its size by adding the same amount to its length and width. We'll delve into the concepts of area, perimeter, and how these changes impact the rectangle's properties, providing detailed explanations, calculations, and real-world examples.

Initial Rectangle: Dimensions and Properties

Let's begin by examining the original rectangle's properties.

Dimensions of the Original Rectangle

- Length: 40 cm
- Width: 21 cm

These dimensions are essential for calculating other properties such as area and perimeter.

Calculating the Area of the Original Rectangle

The area (A) of a rectangle is given by:
$$ A = \text{length} \times \text{width} $$
Substituting the given values:
$$ A = 40\, \text{cm} \times 21\, \text{cm} = 840\, \text{cm}^2 $$
This represents the surface space covered by the rectangle.

Calculating the Perimeter of the Original Rectangle

The perimeter (P) of a rectangle is calculated as:

$$ P = 2 \times (\text{length} + \text{width}) $$

Calculating:

$$ P = 2 \times (40\, \text{cm} + 21\, \text{cm}) = 2 \times 61\, \text{cm} = 122\, \text{cm} $$

The perimeter indicates the total boundary length of the rectangle.

Forming a New Rectangle by Adding the Same Length to Both Dimensions

The core concept here involves increasing both the length and width of the original rectangle by the same amount. This operation results in a new, larger rectangle.

Defining the Addition

Let:
- x = the amount added to both dimensions (in cm)

The new dimensions will be:
- New length: $40\, \text{cm} + x$
- New width: $21\, \text{cm} + x$

Properties of the New Rectangle

The key is to analyze how this addition affects the rectangle's area and perimeter, and to understand the relationships between the original and new shapes.

Calculations for the New Rectangle

To explore the effects thoroughly, we'll perform calculations for various values of x, such as 5 cm, 10 cm, and 15 cm. This will illustrate how incremental increases impact the rectangle's properties.

Example 1: Adding 5 cm

- New length: $40 + 5 = 45\, \text{cm}$
- New width: $21 + 5 = 26\, \text{cm}$

Area:
$$ A_{new} = 45\, \text{cm} \times 26\, \text{cm} = 1170\, \text{cm}^2 $$

Perimeter:
$$ P_{new} = 2 \times (45 + 26) = 2 \times 71 = 142\, \text{cm} $$

Increase in Area:
$$ 1170 - 840 = 330\, \text{cm}^2 $$

Increase in Perimeter:
$$ 142 - 122 = 20\, \text{cm} $$

Example 2: Adding 10 cm

- New length: $ 50\, \text{cm} $
- New width: $ 31\, \text{cm} $

Area:
$$ 50 \times 31 = 1550\, \text{cm}^2 $$

Perimeter:
$$ 2 \times (50 + 31) = 2 \times 81 = 162\, \text{cm} $$

Increase in Area:
$$ 1550 - 840 = 710\, \text{cm}^2 $$

Increase in Perimeter:
$$ 162 - 122 = 40\, \text{cm} $$

Example 3: Adding 15 cm

- New length: $ 55\, \text{cm} $
- New width: $ 36\, \text{cm} $

Area:
$$ 55 \times 36 = 1980\, \text{cm}^2 $$

Perimeter:

$$ 2 \times (55 + 36) = 2 \times 91 = 182\, \text{cm} $$

Increase in Area:
$$ 1980 - 840 = 1140\, \text{cm}^2 $$

Increase in Perimeter:
$$ 182 - 122 = 60\, \text{cm} $$

General Formulas and Relationships

Understanding the relationships between the original and new rectangles can be simplified using formulas.

Area of the New Rectangle

$$ A_{new} = (L + x) \times (W + x) = (40 + x)(21 + x) $$
Expanding:
$$ A_{new} = 840 + 40x + 21x + x^2 = 840 + 61x + x^2 $$
This quadratic formula shows that the area increases quadratically with x.

Perimeter of the New Rectangle

$$ P_{new} = 2[(L + x) + (W + x)] = 2(40 + x + 21 + x) = 2(61 + 2x) = 122 + 4x $$
This indicates the perimeter increases linearly with x.

Implications of the Formulas

- The area grows faster than the perimeter as x increases because of the quadratic relationship.
- For small changes in x, the perimeter increase is moderate, but the area increase can be significant.
- These formulas help in planning modifications to rectangle dimensions for practical purposes like fabric cutting, room expansion, or designing objects.

Applications in Real-Life Scenarios

Understanding how adding to rectangle dimensions affects area and perimeter is valuable in various contexts:

1. Construction and Architecture

- Expanding a room or building involves increasing dimensions, directly affecting space (area) and boundary length (perimeter).
- Accurate calculations ensure optimal use of materials and space planning.

2. Manufacturing and Fabric Cutting

- When cutting fabrics or materials into rectangular pieces, knowing the impact of adding margins or seams on total area and edge length is essential.

3. Design and Aesthetics

- Modifying shapes for visual appeal or functionality, such as enlarging a picture frame or a billboard, requires understanding these geometric principles.

4. Education and Learning

- Demonstrating the relationships between shape dimensions, area, and perimeter enhances geometric understanding for students.

Conclusion

Modifying the dimensions of a rectangle by adding the same amount to both length and width results in predictable changes to its area and perimeter. Starting with an initial rectangle measuring 40 cm by 21 cm, we see that increasing each dimension by a certain value x causes the area to grow quadratically and the perimeter linearly. This understanding is crucial in practical applications, from construction to design, where precise measurements influence the final outcome.

By applying the formulas:
- Area of the new rectangle: $(40 + x)(21 + x) = 840 + 61x + x^2$
- Perimeter of the new rectangle: $122 + 4x$

you can plan and execute modifications effectively. Whether you're resizing fabric, planning a room extension, or exploring geometric concepts, these principles serve as foundational tools for accurate calculations and informed decision-making.

Understanding these relationships enhances mathematical literacy and supports real-world problem-solving in various fields, making the study of rectangles both practical and engaging.

Frequently Asked Questions

What is the area of the original rectangle with length 40 cm and width 21 cm?

The area is 40 cm × 21 cm = 840 square centimeters.

If a new rectangle is formed by adding the same length to the original rectangle, what could be its dimensions?

Without specific details, if the length is increased by a certain amount, the new rectangle's dimensions would be (40 + x) cm by 21 cm, where x is the added length.

How does adding the same length to both sides of the rectangle affect its area?

Adding the same length to both sides increases the area, and the new area becomes (40 + x) × (21 + x) square centimeters.

What is the perimeter of the original rectangle?

The perimeter is 2 × (40 cm + 21 cm) = 2 × 61 cm = 122 centimeters.

If the new rectangle has a length increased by 10 cm, what are its dimensions?

The new rectangle's dimensions would be 50 cm (40 + 10) and 21 cm.

How does increasing the length of the rectangle affect its area and perimeter?

Increasing the length increases both the area and perimeter proportionally, depending on how much is added.

If the width is increased by the same amount as the length, what is the new area?

If both length and width are increased by x cm, the new area is (40 + x) × (21 + x).

Can the area of the rectangle be doubled by adding the same length to the sides? How?

Yes, by choosing an appropriate value of x such that (40 + x)(21 + x) equals twice the original area (1680 sq cm), we can double the area.

What is the significance of adding the same amount to both dimensions of a rectangle?

Adding the same amount to both dimensions results in a larger rectangle with increased area and perimeter, maintaining the shape's proportionality.

Additional Resources

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Introduction

Mathematics often appears as a straightforward discipline, yet beneath its surface lies a realm of intriguing problems and insightful concepts. One such problem involves the transformation of geometric figures through the extension of their dimensions. Specifically, when considering rectangles, a common question arises: what happens when we uniformly increase their length and width by the same amount? This exploration delves into the properties of rectangles, the implications of such modifications, and the broader mathematical insights that emerge from this seemingly simple problem.

Understanding the Initial Rectangle

The problem begins with a rectangle measuring 40 cm in length and 21 cm in width. These dimensions serve as the baseline for further analysis.

Basic Properties

- Original Area: The area of the initial rectangle is calculated by multiplying length and width:

\[
\text{Area}_1 = 40 \times 21 = 840\, \text{cm}^2

\]

- Original Perimeter: The perimeter is the sum of all sides:

$$
\text{Perimeter}_1 = 2 \times (40 + 21) = 2 \times 61 = 122\, \text{cm}
$$

Understanding these foundational properties provides a reference point for subsequent modifications.

The Transformation: Adding the Same Length and Width

Suppose we introduce a positive value, x centimeters, which is added uniformly to both the length and the width. The new dimensions become:

- New Length: $40 + x$ cm
- New Width: $21 + x$ cm

This creates a new rectangle whose properties depend on the chosen value of x.

Mathematical Representation

- New Area:

$$
\text{Area}_2 = (40 + x)(21 + x) = 840 + 40x + 21x + x^2 = 840 + 61x + x^2
$$

- New Perimeter:

$$
\text{Perimeter}_2 = 2 \times [(40 + x) + (21 + x)] = 2 \times (61 + 2x) = 122 + 4x
$$

This algebraic formulation allows us to analyze how the rectangle's area and perimeter change as x varies.

Deep Dive: Impact of Adding the Same Length to Both Dimensions

The process of increasing both dimensions by the same amount is a fundamental geometric operation, often used in practical applications like resizing images, designing layouts, or optimizing materials. It's essential to understand the implications of such modifications.

Effect on Area

The area increases quadratically with x, as seen in the equation:

$$
\text{Area}_2 = 840 + 61x + x^2
$$

This quadratic relationship indicates that as x grows, the area expands rapidly, especially for larger values of x.

Examples:

x (cm)	New Length (cm)	New Width (cm)	New Area (cm^2)	Change in Area (cm^2)
0	40	21	840	0
5	45	26	$45 \times 26 = 1170$	+330
10	50	31	$50 \times 31 = 1550$	+710
15	55	36	$55 \times 36 = 1980$	+1140

This table illustrates how incremental increases in x significantly boost the area.

Effect on Perimeter

The perimeter increases linearly with x:

$$
\text{Perimeter}_2 = 122 + 4x
$$

For the same x values:

x (cm)	New Perimeter (cm)	Change in Perimeter (cm)
0	122	0
5	$122 + 20 = 142$	+20
10	$122 + 40 = 162$	+40
15	$122 + 60 = 182$	+60

The linear increase indicates a predictable growth in boundary length, which is significant for applications where border materials or fencing costs are considered.

Geometric and Practical Implications

Understanding how uniform dimension increases influence area and perimeter has both theoretical and practical relevance.

Geometric Significance

- Scaling: The process resembles geometric scaling, where an object is enlarged proportionally.
- Area Growth: Quadratic scaling emphasizes how area is sensitive to linear dimension changes.
- Perimeter Growth: Linear scaling of perimeter underscores that boundary length increases at a constant rate with respect to x.

Practical Applications

- Construction and Design: When enlarging rooms or surfaces, knowing how dimensions affect material and space is crucial.
- Manufacturing: Modifying component sizes often involves similar considerations for cost and efficiency.
- Mathematical Modelling: These principles underpin models in physics, engineering, and economics where proportional changes occur.

Determining the Value of x for Specific Goals

Suppose a designer wants to double the area of the original rectangle. How much should they add to each dimension?

Solving for x when the area doubles:

$$
\text{Area}_2 = 2 \times \text{Area}_1 = 2 \times 840 = 1680
$$

Using the area formula:

$$
840 + 61x + x^2 = 1680
$$

Rearranged as:

$$
x^2 + 61x + 840 - 1680 = 0
$$

$$
x^2 + 61x - 840 = 0
$$

Applying the quadratic formula:

$$
x = \frac{-61 \pm \sqrt{61^2 - 4 \times 1 \times (-840)}}{2}
$$

Calculate discriminant:

$$
\Delta = 61^2 - 4 \times 1 \times (-840) = 3721 + 3360 = 7081
$$

Square root of discriminant:

$$
\sqrt{7081} \approx 84.2
$$

Solutions:

$$
x = \frac{-61 \pm 84.2}{2}
$$

Two possible solutions:

- $x \approx \frac{-61 + 84.2}{2} = \frac{23.2}{2} = 11.6$
- $x \approx \frac{-61 - 84.2}{2} = \frac{-145.2}{2} = -72.6$

Since a negative increase is not feasible in this context, the meaningful solution is:

$$
x \approx 11.6\, \text{cm}
$$

This indicates that increasing both dimensions by approximately 11.6 cm will double the area.

Implication for Design and Engineering

This approach demonstrates how precise calculations inform decision-making in real-world modifications, ensuring cost-effective and goal-oriented adjustments.

Broader Mathematical Insights and Extensions

The problem of adding the same length to both sides of a rectangle opens avenues for various mathematical explorations:

1. Generalization to Other Shapes

- Extending the concept to squares, triangles, or circles involves similar principles, emphasizing proportional scaling.
- For circles, for example, increasing the radius by a constant leads to quadratic growth in area.

2. Optimization Problems

- Finding the value of x that maximizes or minimizes certain properties (like area or perimeter) under constraints.
- For instance, determining the minimal perimeter for a fixed area, or vice versa.

3. Relation to Similar Figures

- The new rectangle is similar to the original if the ratios of corresponding sides are equal.
- Since both sides increase by the same amount, the rectangle remains similar, and the scale factor is:

$$
\text{Scale factor} = \frac{40 + x}{40} = \frac{21 + x}{21}
$$

- Equating the ratios:

$$
\frac{40 + x}{40} = \frac{21 + x}{21}
$$

- Cross-multiplied:

$$
(40 + x) \times 21 = (21 + x) \times 40
$$

- Simplify:

$$
840 + 21x = 840 + 40x
$$

- Solve:

$$
840 + 21x = 840 + 40x \Rightarrow 21x = 40x \Rightarrow 0 = 19x \Rightarrow x = 0
$$

- The only solution is $x = 0$, which confirms the figures are similar only when no change occurs.

4. Implications in Real-world Applications

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