

A Rectangle Has A Perimeter Of 44 Inches And An Area Of 72 Square Inches. What Are The Lengths Of The

A Rectangle Has A Perimeter Of 44 Inches And An Area Of 72 Square Inches.

What Are The Lengths Of The This question is a classic problem in geometry that involves understanding the relationships between the length, width, perimeter, and area of a rectangle. Solving such problems not only enhances mathematical problem-solving skills but also deepens comprehension of fundamental geometric concepts. In this comprehensive guide, we will explore how to determine the length and width of a rectangle given its perimeter and area, step-by-step, with detailed explanations and tips for solving similar problems efficiently.

Understanding the Basics: Perimeter and Area of a Rectangle

Before diving into the problem, it's essential to review the basic formulas related to rectangles:

Perimeter of a Rectangle

- The perimeter (P) of a rectangle is the total distance around it.

- Formula:

$$P = 2 \times (\text{length} + \text{width})$$

or

$$P = 2L + 2W$$

Area of a Rectangle

- The area (A) is the amount of surface enclosed within the rectangle.

- Formula:

$$A = \text{length} \times \text{width}$$

or

$$A = L \times W$$

Knowing these formulas allows us to translate the problem's given information into mathematical equations, setting the stage for solving for the unknowns.

Given Data and What We Need to Find

The problem provides:

- Perimeter, $P = 44$ inches
- Area, $A = 72$ square inches

Our goal:

- Find the length (L) and width (W) of the rectangle.

Setting Up the Equations

Based on the formulas, we can express the problem as a system of equations:

Equation 1: Perimeter

- $2L + 2W = 44$

Simplify:

$$L + W = 22$$

Equation 2: Area

- $L \times W = 72$

This system involves two equations with two variables, which can be solved using substitution or elimination methods.

Step-by-Step Solution Process

Let's walk through the process of solving for L and W.

Step 1: Express one variable in terms of the other

From Equation 1:

$$L + W = 22$$

Express W in terms of L:

$$W = 22 - L$$

Step 2: Substitute into the area equation

Replace W in Equation 2:

$$L \times (22 - L) = 72$$

Step 3: Simplify and form a quadratic equation

Expand:

$$L \times 22 - L^2 = 72$$

$$22L - L^2 = 72$$

Bring all to one side:

$$-L^2 + 22L - 72 = 0$$

Multiply through by -1 to make the quadratic standard:

$$L^2 - 22L + 72 = 0$$

Step 4: Solve the quadratic equation

Quadratic formula:

$$L = [22 \pm \sqrt{(22^2 - 4 \times 1 \times 72)}] / (2 \times 1)$$

Calculate discriminant:

$$22^2 = 484$$

$$4 \times 1 \times 72 = 288$$

$$\text{Discriminant: } \sqrt{(484 - 288)} = \sqrt{196} = 14$$

Calculate roots:

$$L = [22 \pm 14] / 2$$

Two solutions:

$$- L = (22 + 14) / 2 = 36 / 2 = 18$$

$$- L = (22 - 14) / 2 = 8 / 2 = 4$$

Step 5: Find corresponding W values

Recall $W = 22 - L$:

$$- \text{ If } L = 18, \text{ then } W = 22 - 18 = 4$$

$$- \text{ If } L = 4, \text{ then } W = 22 - 4 = 18$$

Interpreting the Results: Lengths and Widths

The solutions indicate that the rectangle's dimensions are:

- Length = 18 inches and Width = 4 inches, or
- Length = 4 inches and Width = 18 inches

Since the problem asks for "the lengths," both solutions are valid; typically, the longer side is considered the length.

Final dimensions:

- Length: 18 inches
- Width: 4 inches

or

- Length: 4 inches
- Width: 18 inches

Verifying the Solution

It's always good practice to verify the solution:

1. Check perimeter:

$$2L + 2W = 2 \times 18 + 2 \times 4 = 36 + 8 = 44 \text{ inches } \checkmark$$

2. Check area:

$$L \times W = 18 \times 4 = 72 \text{ square inches } \checkmark$$

Both conditions are satisfied, confirming the correctness of the solution.

Key Takeaways for Solving Similar Problems

When approaching problems involving rectangles with given perimeters and areas, keep these key points in mind:

Key Points:

- Translate the word problem into algebraic equations based on formulas.
- Use substitution to reduce the system to a quadratic equation.
- Solve the quadratic carefully, considering both roots.
- Verify the solutions to ensure they meet the original conditions.
- Remember that dimensions can be interchangeable; focus on the context to assign the correct labels.

Additional Tips for Geometry Problem Solving

- Understand the problem thoroughly before jumping into calculations.
- Write down all given information and the formulas involved.
- Simplify equations step-by-step to avoid mistakes.
- Check your solutions in the original equations.
- When dealing with dimensions, always consider the physical context to interpret the results correctly.

Conclusion: Mastering Rectangle Dimensions from Perimeter and Area

Determining the lengths of a rectangle given its perimeter and area is a fundamental problem in geometry that combines algebra and spatial reasoning. By setting up accurate equations, solving quadratic equations, and verifying solutions, you can confidently find the dimensions of any rectangle based on its perimeter and area. Practice similar problems to strengthen your understanding, and remember that a methodical approach leads to success in solving complex geometric puzzles.

Summary of the Solution:

- Perimeter: 44 inches
- Area: 72 square inches
- Dimensions: 18 inches by 4 inches (or vice versa)

This problem exemplifies how algebra can be applied to geometric problems, providing valuable skills for academic success and real-world applications such as construction, design, and engineering.

Frequently Asked Questions

What are the possible length and width measurements of a rectangle with a perimeter of 44 inches and an area of 72 square inches?

The possible dimensions are length and width pairs where the length and width satisfy the equations $2(l + w) = 44$ and $l \times w = 72$. Solving these yields the pairs approximately (12, 6) inches and (9, 8) inches.

How do you find the side lengths of a rectangle given its perimeter and area?

Set up equations for perimeter ($2(l + w) = 44$) and area ($l \times w = 72$). Solve the perimeter equation for one variable (e.g., $l = 22 - w$) and substitute into the area equation to find the possible side lengths.

Can a rectangle with a perimeter of 44 inches and an area of 72 square inches have multiple solutions?

Yes, because solving the equations results in quadratic equations that can have two positive solutions, corresponding to different pairs of length and width, such as (12, 6) and (9, 8).

What steps are involved in determining the side lengths of the rectangle described?

First, write the perimeter equation $2(l + w) = 44$, then express l in terms of w as $l = 22 - w$. Substitute into the area equation $l \times w = 72$ to get a quadratic in w , solve it to find the possible widths, then calculate the corresponding lengths.

Are the dimensions of the rectangle with area 72 sq inches and perimeter 44 inches unique?

No, there are multiple possible pairs of length and width that satisfy both conditions; specifically, (12, 6) and (9, 8) inches are two solutions.

How can I verify the dimensions of the rectangle once I find potential length and width values?

Plug the length and width back into the perimeter and area formulas to confirm that they satisfy both conditions: perimeter = 44 inches and area = 72 square inches.

Is it possible for the length to be greater than the width in this rectangle, and what are the implications?

Yes, either dimension can be greater. For example, in the pair (12, 6), length is greater than width. This flexibility means multiple dimension pairs are possible, and the specific orientation depends on how you assign length and width.

Additional Resources

A Rectangle Has A Perimeter Of 44 Inches And An Area Of 72 Square Inches. What Are The Lengths Of The Sides?

Understanding the dimensions of a rectangle when given its perimeter and area is a classic problem in geometry that combines algebra and spatial reasoning. When a rectangle's perimeter of 44 inches and area of 72 square inches are provided, the challenge lies in finding the precise lengths of its sides. This type of problem not only tests your algebraic skills but also enhances your understanding of the relationships between the sides of a rectangle. In this comprehensive guide, we will explore how to approach this problem systematically, breaking down each step to uncover the length of each side with clarity and confidence.

Introduction to the Problem

Suppose you are given a rectangle where:

- The perimeter is 44 inches.
- The area is 72 square inches.

Your goal is to determine the exact lengths of the rectangle's sides—let's denote them as length (L) and width (W).

This problem involves two key geometric formulas:

- Perimeter of a rectangle:

$$(P = 2L + 2W)$$

- Area of a rectangle:

$$(A = L \times W)$$

By leveraging these formulas, we can set up equations to solve for the unknowns.

Step 1: Establishing the Equations

Given:

- Perimeter $(P = 44)$ inches
- Area $(A = 72)$ square inches

Using the formulas:

1. Perimeter equation:

$$(2L + 2W = 44)$$

Simplify:

$$(L + W = 22)$$

2. Area equation:

$$(L \times W = 72)$$

Now, our goal is to find L and W that satisfy both equations.

Step 2: Expressing One Variable in Terms of the Other

From the perimeter equation:

$$L + W = 22$$

Express (L) in terms of (W) :

$$L = 22 - W$$

This substitution simplifies the problem to a single variable.

Step 3: Formulate a Quadratic Equation

Substitute $(L = 22 - W)$ into the area equation:

$$(22 - W) \times W = 72$$

Expanding:

$$22W - W^2 = 72$$

Bring all terms to one side to set the quadratic equation to zero:

$$-W^2 + 22W - 72 = 0$$

Multiply through by (-1) to simplify:

$$W^2 - 22W + 72 = 0$$

This quadratic equation is easier to solve:

$$W^2 - 22W + 72 = 0$$

Step 4: Solving the Quadratic Equation

Use the quadratic formula:

$$W = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:

- $a = 1$
- $b = -22$
- $c = 72$

Calculate the discriminant:

$$D = b^2 - 4ac = (-22)^2 - 4 \times 1 \times 72 = 484 - 288 = 196$$

Since $D = 196$, which is a perfect square (14^2), the solutions are straightforward:

$$W = \frac{22 \pm \sqrt{196}}{2} = \frac{22 \pm 14}{2}$$

Calculate each:

1. $W = \frac{22 + 14}{2} = \frac{36}{2} = 18$
2. $W = \frac{22 - 14}{2} = \frac{8}{2} = 4$

Now, use these to find corresponding L values:

- If $W = 18$, then $L = 22 - 18 = 4$
- If $W = 4$, then $L = 22 - 4 = 18$

Interestingly, both solutions are valid and essentially represent the same rectangle, just with the sides swapped.

Conclusion: The Dimensions of the Rectangle

The rectangle's sides are:

- Length = 18 inches
- Width = 4 inches

or vice versa, since rectangles are not orientation-dependent.

Verification of the Solution

It's good practice to verify the solutions:

- Perimeter check:

$$2 \times 18 + 2 \times 4 = 36 + 8 = 44 \text{ inches}$$

- Area check:

$$(18 \times 4 = 72) \text{ square inches}$$

Both checks confirm the correctness of the solution.

Additional Insights: Why Did the Equation Form as It Did?

This problem elegantly demonstrates how algebraic methods intersect with geometric principles:

- The sum $(L + W = 22)$ comes directly from the perimeter formula.
- The quadratic arises from the area relationship, which involves the product $(L \times W)$.
- Solving the quadratic provides the possible pairs of side lengths.

Real-World Applications and Considerations

Understanding such problems has practical implications:

- Design and Architecture: Calculating dimensions based on constraints like perimeter and area.
- Manufacturing: Ensuring materials fit specified area and perimeter requirements.
- Education: Building foundational algebraic skills through geometric applications.

Always remember to verify solutions by plugging the values back into the original formulas to confirm their validity.

Summary: Key Takeaways

- Set up equations based on given perimeter and area.
- Express one variable in terms of the other.
- Derive and solve the quadratic equation.
- Interpret solutions in the context of the problem.
- Verify solutions to ensure correctness.

Final Thoughts

By carefully translating the problem into algebraic equations and methodically solving, you can confidently determine the dimensions of a rectangle given its perimeter and area. This approach not only solves the

specific problem but also enhances your overall problem-solving skills, enriching your understanding of the interconnectedness between algebra and geometry. Whether for academic pursuits, practical applications, or simply sharpening your analytical thinking, mastering such problems is a valuable step forward in mathematical proficiency.

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