A Right Triangle Has Sides Such That One Ot The Sides Is One Inch Shorter Than The Hypotenuse And One

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Understanding the properties of right triangles is fundamental in geometry, trigonometry, and various practical applications ranging from engineering to architecture. The specific condition where one of the sides is exactly one inch shorter than the hypotenuse introduces an interesting problem that combines algebraic reasoning with geometric principles. In this article, we explore the characteristics of such a right triangle, derive the relationships between its sides, and discuss relevant concepts that help us understand and analyze this geometric configuration.

Defining the Problem: The Triangle's Sides and Conditions

Before delving into the mathematics, it’s important to clarify the problem statement and the assumptions involved.

Understanding the Triangle's Sides

In a right triangle, there are three sides:

- The hypotenuse (the longest side, opposite the right angle)
- The two legs (the sides forming the right angle)

Let’s denote:

- The hypotenuse as \(c \)
- One of the legs as \(a \)
- The other leg as \(b \)

The problem states: "A right triangle has sides such that one of the sides is one inch shorter than the hypotenuse and one." This phrase can be interpreted in two main ways:

1. One of the legs is exactly one inch shorter than the hypotenuse, and that particular side is of length one inch.

2. One of the legs is one inch shorter than the hypotenuse, and one inch refers to the length of that side.

Given the phrasing, it seems most consistent that:

- There is a side (say, a) such that $a = c - 1$

- Also, there is a side of length 1 inch, which could be either a, b, or the hypotenuse c.

However, the phrase "one inch short of the hypotenuse and one" suggests that:

- One of the sides is one inch shorter than the hypotenuse (i.e., $a = c - 1$ or $b = c - 1$)
- And, additionally, one of the sides has length 1 inch.

To simplify the analysis, we’ll consider the most straightforward interpretation:

The side a is exactly one inch shorter than the hypotenuse c, and a has length 1 inch.

This leads us to the following assumptions:

- $a = 1$ inch
- $c = a + 1 = 2$ inches

But, is this consistent with the Pythagorean theorem? Let's verify.

Mathematical Modeling of the Triangle

Assuming the above, we have:

- $a = 1$ inch
- $c = a + 1 = 2$ inches
- b is unknown

Applying the Pythagorean theorem:

$$
a^2 + b^2 = c^2
$$

Substituting known values:

$$
1^2 + b^2 = 2^2
$$
$$
1 + b^2 = 4
$$
$$
b^2 = 3
$$

\]
\[
b = \sqrt{3} \approx 1.732 \text{ inches}
\]

Thus, the sides are:

- \(a = 1 \) inch
- \(b = \sqrt{3} \) inches
- \(c = 2 \) inches

This forms a valid right triangle, as verified by the Pythagorean theorem.

Analyzing the Triangle's Properties

With the sides established, we can explore various properties of this triangle, including angles, area, perimeter, and ratios.

Angles of the Triangle

Using trigonometric ratios, we can find the measures of the non-right angles.

- Angle \(\theta \) opposite side \(a \):

\[
\sin \theta = \frac{a}{c} = \frac{1}{2} = 0.5
\]
\[
\theta = \arcsin(0.5) = 30^\circ
\]

- Angle \(\phi \) opposite side \(b \):

\[
\sin \phi = \frac{b}{c} = \frac{\sqrt{3}}{2} \approx 0.866
\]
\[
\phi = \arcsin(0.866) \approx 60^\circ
\]

Since the sum of angles in a triangle is 180°, and we have a right angle of 90°, the angles are:

\[
\boxed{
\begin{aligned}
&\text{Right angle} = 90^\circ \\

$$
&\text{Other angles} = 30^\circ \text{ and } 60^\circ
\end{aligned}
}
$$

This confirms that the triangle is a classic 30-60-90 triangle, which is well-known for its side ratios.

Area and Perimeter

- Area A:

$$
A = \frac{1}{2} \times a \times b = \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2} \approx 0.866 \text{ square inches}
$$

- Perimeter P:

$$
P = a + b + c = 1 + \sqrt{3} + 2 \approx 1 + 1.732 + 2 = 4.732 \text{ inches}
$$

Exploring Variations and Other Interpretations

While the above scenario is consistent, it’s important to consider alternative interpretations.

Alternative Scenario: The Side of Length 1 Is Not the Leg but the Hypotenuse

Suppose the hypotenuse c is 1 inch longer than one of the legs a, and that leg has length 1 inch. Then:

$$
c = a + 1
$$
$$
a = 1
$$
$$
c = 2
$$

Using the Pythagorean theorem:

$$
a^2 + b^2 = c^2
$$
$$
1^2 + b^2 = 2^2
$$
$$
1 + b^2 = 4
$$
$$
b^2 = 3
$$
$$
b = \sqrt{3}
$$

This is the same as the previous case, confirming the consistency of this interpretation.

Implications of the Side Lengths

This analysis indicates that the specific condition "one of the sides is one inch short of the hypotenuse and one" leads to a right triangle with sides:

- $a = 1$ inch
- $b = \sqrt{3}$ inches
- $c = 2$ inches

which is a scaled 30-60-90 right triangle.

Geometric Significance and Applications

Recognizing such triangles is useful in various fields:

- Architecture: Knowledge of special right triangles helps in designing structures with specific angle and length requirements.
- Trigonometry: These triangles provide concrete examples for understanding sine, cosine, and tangent ratios.
- Education: The 30-60-90 triangle is a fundamental concept taught early in geometry courses.

Real-World Examples

- Construction: When creating a slope or ramp with a 30-60-90 triangle, knowing side

ratios simplifies measurements.
- Navigation: Triangular computations involving specific ratios assist in triangulation and distance estimation.
- Design: Artists and designers use these ratios to create visually appealing and proportioned structures.

Conclusion

In summary, a right triangle where one side is exactly one inch shorter than the hypotenuse and that side is of length 1 inch leads to a specific set of side lengths: 1 inch, $\sqrt{3}$ inches, and 2 inches. This configuration forms a classic 30-60-90 triangle, showcasing the elegance of geometric relationships. Understanding these properties enhances our ability to analyze geometric figures, solve real-world problems, and appreciate the interconnectedness of mathematical concepts.

Further Exploration

To deepen your understanding, consider exploring:

- How changing the side lengths affects the triangle's angles and ratios.
- The properties of other special right triangles, such as 45-45-90 triangles.
- Applications of right triangles in trigonometry, including the Law of Sines and Law of Cosines.
- Practical problems involving measuring and constructing such triangles in real life.

By mastering these concepts, you strengthen your geometric intuition and mathematical problem-solving skills, which are invaluable across many disciplines.

Frequently Asked Questions

What is the relationship between the sides of a right triangle if one side is one inch shorter than the hypotenuse?

In this case, if we denote the hypotenuse as c and the shorter side as a, then a = c - 1. The other side, b, can be found using the Pythagorean theorem: $a^2 + b^2 = c^2$.

How can I determine the lengths of the other sides if one side is known to be one inch shorter than the hypotenuse?

You can set up equations using the Pythagorean theorem with a = c - 1, then solve for the remaining side b: $(c - 1)^2 + b^2 = c^2$. Simplify and solve for b.

Is there a specific Pythagorean triple that fits the condition where one side is exactly one inch shorter than the hypotenuse?

Yes, by solving the equations, you find that the sides (5, 12, 13) form a Pythagorean triple where 12 is one less than the hypotenuse 13, but since the problem states one side is one inch shorter than the hypotenuse, the specific triple would depend on the exact side lengths derived from the equations.

Can the side that is one inch shorter than the hypotenuse be either leg or the hypotenuse itself?

In a right triangle, the hypotenuse is always the longest side, so the side that is one inch shorter than the hypotenuse is necessarily a leg, not the hypotenuse itself.

What steps are involved in solving for the sides of a right triangle where one leg is one inch shorter than the hypotenuse?

Set the hypotenuse as c, the shorter leg as c - 1, and the other leg as b. Use the Pythagorean theorem: $(c - 1)^2 + b^2 = c^2$. Expand, simplify, and solve for b, then determine the specific side lengths.

Are there infinitely many right triangles satisfying the condition that one side is one inch shorter than the hypotenuse?

Yes, by varying the length of the hypotenuse c and solving for the other sides accordingly, there can be infinitely many such right triangles.

How can I verify that a given set of side lengths satisfies the condition of one side being one inch shorter than the hypotenuse?

Check if the Pythagorean theorem holds for the set of sides and verify that one side equals the hypotenuse minus one inch. If both conditions are met, the triangle satisfies the condition.

What real-world applications might involve triangles where one side is exactly one inch shorter than the

hypotenuse?

Such triangles can be relevant in architectural design, engineering, and construction where precise measurements are critical, or in problems involving ladders, ramps, or other structures where specific proportional relationships are needed.

Additional Resources

Right Triangle Properties and the Intriguing Relationship Between Sides: An In-Depth Exploration

When delving into the fascinating world of geometry, the properties of right triangles often captivate students, educators, and math enthusiasts alike. These triangles, defined by one 90-degree angle, harbor a wealth of relationships and theorems that reveal the elegant harmony of shapes and numbers. Among the myriad of fascinating configurations, one particularly intriguing scenario involves a right triangle where one of the sides is exactly one inch shorter than the hypotenuse. This situation not only piques curiosity but also opens the door to a rich analysis blending algebra, geometry, and problem-solving strategies.

In this comprehensive review, we will explore this special case in detail, unravel the underlying relationships, and offer insights into how such a triangle can be characterized, constructed, and understood. Whether you're a student seeking clarity, a teacher looking for engaging examples, or a math lover eager to explore the depths of geometric relationships, this article aims to serve as an authoritative guide.

An Overview of Right Triangle Fundamentals

Before tackling the specific problem, it’s essential to revisit the core principles that govern right triangles.

The Pythagorean Theorem

At the heart of right triangle geometry lies the Pythagorean theorem, which states that:

$$a^2 + b^2 = c^2$$

where:
- a and b are the lengths of the legs (the sides adjacent to the right angle),
- c is the length of the hypotenuse (the side opposite the right angle).

This relationship is fundamental because it links the three sides in a simple algebraic way, allowing us to solve for unknown lengths provided we have enough information.

Properties of the Triangle Sides

- Hypotenuse: Always the longest side in a right triangle.
- Legs: The two sides that meet at the right angle.
- Relationship between sides: The hypotenuse is longer than either leg, and the difference between the hypotenuse and a leg varies depending on the specific dimensions.

Setting Up the Problem: The Specific Condition

The problem at hand states:

> A right triangle has sides such that one of the sides is exactly one inch shorter than the hypotenuse, and one of the sides is (implied as another side, perhaps a leg).

To interpret this precisely, let's formalize the scenario:

Clarifying the Conditions

- Let the hypotenuse be c.
- Let one of the other sides (a leg) be a.
- The problem states that one of the sides is one inch shorter than the hypotenuse.

This phrase can be understood in two ways:

1. Either the leg a is one inch shorter than the hypotenuse c:

$$a = c - 1$$

2. Or the other leg b is one inch shorter than the hypotenuse c:

$$b = c - 1$$

Because the problem mentions "one of the sides" (singular), but doesn't specify which, we will explore both cases. Additionally, the phrase "and one" at the end suggests the possibility that the problem might involve multiple sides with similar relationships, but for clarity, we'll focus on the scenario where a single leg is one inch shorter than the hypotenuse.

Analyzing the Scenario: When a Leg Is One Inch

Shorter Than the Hypotenuse

Let's proceed with the assumption:

> Scenario: The leg a is exactly one inch shorter than the hypotenuse c.

Expressed mathematically:

$$a = c - 1$$

Using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

Substituting $a = c - 1$:

$$(c - 1)^2 + b^2 = c^2$$

Expanding:

$$c^2 - 2c + 1 + b^2 = c^2$$

Subtracting c^2 from both sides:

$$-2c + 1 + b^2 = 0$$

Rearranged:

$$b^2 = 2c - 1$$

This relation links b and c, which allows us to analyze the possible values of c and b.

Deriving the Range of Possible Values

Since side lengths are positive real numbers, the following must hold:

- $c > 0$
- $a = c - 1 > 0 \Rightarrow c > 1$
- $b^2 = 2c - 1 > 0 \Rightarrow c > \frac{1}{2}$, which is automatically satisfied if $c > 1$

Additionally, because $b^2 \geq 0$:

$$2c - 1 \geq 0 \Rightarrow c \geq \frac{1}{2}$$

But since $c > 1$, this is consistent.

Furthermore, for b to be real:

$$b = \sqrt{2c - 1}$$

must be a real number, which it is for $c > \frac{1}{2}$.

Explicit Examples and Constructing the Triangle

Let's choose specific values for c that satisfy the constraints, and then find a and b.

Example 1: $c = 2$ inches

- $a = c - 1 = 2 - 1 = 1$ inch
- $b = \sqrt{2 \times 2 - 1} = \sqrt{4 - 1} = \sqrt{3} \approx 1.732$ inches

Verification:

Check the Pythagorean theorem:

$$a^2 + b^2 = 1^2 + (\sqrt{3})^2 = 1 + 3 = 4$$

which equals $c^2 = 2^2 = 4$. The relation holds perfectly.

Example 2: $c = 3$ inches

- $a = 3 - 1 = 2$ inches
- $b = \sqrt{2 \times 3 - 1} = \sqrt{6 - 1} = \sqrt{5} \approx 2.236$ inches

Verification:

$$a^2 + b^2 = 4 + 5 = 9$$

which equals $c^2 = 9$. Again, the Pythagorean theorem is satisfied.

General formula:

Given $c > 1$, the sides are:

$$a = c - 1$$
$$b = \sqrt{2c - 1}$$

and the hypotenuse:

$$c$$

Interpreting the Geometric and Algebraic Significance

This relationship illustrates a family of right triangles characterized by a particular proportionality:

- The leg a is always exactly one inch less than the hypotenuse.
- The other leg b depends on the hypotenuse via the square root relation.

Key properties:

- As c increases, a increases linearly.
- The other leg b increases as the square root of $2c - 1$.

This family of triangles demonstrates how a simple linear relationship between sides influences the entire triangle's dimensions.

Alternative Scenario: When the Other Leg Is One Inch Shorter

Suppose instead that the other leg b is one inch shorter than the hypotenuse:

$$b = c - 1$$

Then, the Pythagorean theorem becomes:

$$a^2 + (c - 1)^2 = c^2$$

which expands to:

$$a^2 + c^2 - 2c + 1 = c^2$$

Subtracting c^2:

$$a^2 - 2c + 1 = 0$$

Rearranged:

$$a^2 = 2c - 1$$

Notice that this is identical to previous relation for b, just with the roles swapped.

Implications:

- The side a is:

$$ a = \sqrt{2c - 1} $$

- The side b is:

$$ b = c - 1 $$

- The constraints on c remain similar: $c > 1$.

Thus, the set of possible triangles is symmetric in this scenario.

Addressing the "And One" Phrase in the Original Prompt

The phrase "and one" at the end of the initial statement could suggest multiple conditions or the consideration of both sides being one inch shorter than the hypotenuse. Alternatively, it might imply that a third condition or side is involved.

Possible interpretations include:

- Both legs are one inch shorter than the hypotenuse:

$$ a = c - 1 \quad \text{and} \quad b = c - 1 $$

Applying the Pythagorean

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