A Satellite In Earth Orbit Has A Mass Of 93 Kg And Is At An Altitude Of 1.99x10^6 M. (assume That U =

A Satellite In Earth Orbit Has A Mass Of 93 Kg And Is At An Altitude Of 1.99x10^6 M. (assume That U =

Introduction to Satellite Orbits and Gravitational Potential Energy

Satellites are vital components of modern technology, enabling global communication, navigation, weather forecasting, and scientific research. Understanding the physics governing their motion involves examining gravitational forces, energy conservation, and orbital mechanics. When analyzing a satellite's behavior in orbit, especially in terms of energy, the concepts of gravitational potential energy, kinetic energy, and the total mechanical energy become essential.

In this article, we focus on a satellite with a mass of 93 kg orbiting Earth at an altitude of approximately 1.99 million meters. We will explore how to calculate its gravitational potential energy, kinetic energy, and total energy, assuming specific values for gravitational potential energy, and discuss the implications of these calculations for orbital stability and energy management.

Fundamental Concepts and Assumptions

Gravitational Potential Energy (U)

For a satellite orbiting Earth, the gravitational potential energy relative to Earth’s center is given by:

$$
U = - \frac{G M_e m}{r}
$$

where:

- G is the gravitational constant, $6.674 \times 10^{-11} \text{ N·m}^2/\text{kg}^2$,
- M_e is Earth's mass, approximately $5.972 \times 10^{24} \text{ kg}$,
- m is the satellite's mass, 93 kg,
- r is the distance from Earth's center to the satellite.

The negative sign indicates that the gravitational potential energy is considered zero at infinite separation and becomes more negative as the satellite approaches Earth.

Orbital Parameters

Given:

- Satellite mass $m = 93\, \text{kg}$,
- Altitude $h = 1.99 \times 10^{6}\, \text{m}$,
- Earth's radius $R_e \approx 6.371 \times 10^{6}\, \text{m}$,
- Total distance from Earth's center:

$$
r = R_e + h = 6.371 \times 10^{6}\, \text{m} + 1.99 \times 10^{6}\, \text{m} = 8.361 \times 10^{6}\, \text{m}
$$

Calculating Gravitational Potential Energy (U)

Using the formula:

$$
U = - \frac{G M_e m}{r}
$$

Plugging in the values:

$$
U = - \frac{(6.674 \times 10^{-11})(5.972 \times 10^{24})(93)}{8.361 \times 10^{6}}
$$

Calculating numerator:

$$
(6.674 \times 10^{-11})(5.972 \times 10^{24}) = 3.986 \times 10^{14}
$$

Then:

$$
3.986 \times 10^{14} \times 93 = 3.708 \times 10^{16}
$$

Dividing by r:

$$
U = - \frac{3.708 \times 10^{16}}{8.361 \times 10^{6}} \approx -4.436 \times 10^{9}\, \text{J}
$$

Result:

$$
\boxed{
U \approx -4.44 \times 10^{9}\, \text{J}
}
$$

This negative value indicates the satellite's potential energy is bound within Earth’s gravitational field.

Determining Orbital Velocity and Kinetic Energy

Orbital Velocity (v)

For a satellite in a stable circular orbit, the orbital velocity is derived from the balance of gravitational force and centripetal force:

$$
\frac{G M_e m}{r^2} = \frac{m v^2}{r}
$$

Solving for v:

$$
v = \sqrt{\frac{G M_e}{r}}
$$

Plugging in the known values:

$$
v = \sqrt{\frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{8.361
$$

\times 10^{6}}}
\]

Calculating numerator:

\[
6.674 \times 10^{-11} \times 5.972 \times 10^{24} = 3.986 \times 10^{14}
\]

Thus:

\[
v = \sqrt{\frac{3.986 \times 10^{14}}{8.361 \times 10^{6}}} = \sqrt{4.77
\times 10^{7}} \approx 6.9 \times 10^{3}\, \text{m/s}
\]

Result:

\[
\boxed{
v \approx 6900\, \text{m/s}
}
\]

Kinetic Energy (K)

Using:

\[
K = \frac{1}{2} m v^2
\]

Plugging in the values:

\[
K = 0.5 \times 93 \times (6900)^2
\]

Calculations:

\[
(6900)^2 = 4.761 \times 10^{7}
\]

Then:

\[
K = 0.5 \times 93 \times 4.761 \times 10^{7} \approx 0.5 \times 93 \times
4.761 \times 10^{7}

\]

\[
K \approx 46.5 \times 4.761 \times 10^{7} \approx 2.214 \times 10^{9}\,
\text{J}
\]

Result:

\[
\boxed{
K \approx 2.21 \times 10^{9}\, \text{J}
}
\]

Total Mechanical Energy (E)

The total energy in orbit is the sum of kinetic and potential energies:

\[
E = K + U
\]

Substituting the calculated values:

\[
E \approx 2.21 \times 10^{9}\, \text{J} - 4.44 \times 10^{9}\, \text{J} =
-2.23 \times 10^{9}\, \text{J}
\]

Result:

\[
\boxed{
E \approx -2.23 \times 10^{9}\, \text{J}
}
\]

The negative total energy confirms the satellite is gravitationally bound in its orbit.

Implications of the Energy Calculations

Understanding Orbital Stability

The negative total energy indicates a stable, bound orbit. To escape Earth's gravity, the satellite would need to gain at least $2.23 \times 10^{9}\, \text{J}$ of energy—specifically, enough to bring its total energy to zero or positive.

This energy requirement is critical for mission planning, especially for de-orbiting or escaping Earth's gravitational influence.

Energy Conservation and Satellite Maneuvers

Any change in the satellite's orbit involves energy exchanges:

- Increasing altitude: requires adding energy to the system, raising the total energy closer to zero.
- Decreasing altitude: involves removing energy, making the total energy more negative.
- Orbital transfers: often involve thrusters that modify kinetic energy, which in turn alters potential energy.

Efficiency and Fuel Considerations

The calculations highlight the importance of efficient fuel usage:

- Since kinetic energy is a significant part of the total energy budget, thrusters designed to modify orbital velocity must be carefully calibrated.
- The energy costs for orbital adjustments are substantial, emphasizing the importance of precise calculations during mission planning.

Additional Considerations and Assumptions

While the preceding calculations assume a circular orbit and neglect perturbations, real-world scenarios involve complexities such as:

- Orbital eccentricity: introduces variations in speed and potential energy.
- Atmospheric drag: especially at lower altitudes, causes gradual orbital decay.
- Earth's oblateness: leads to gravitational anomalies affecting orbit stability.

- Energy losses: due to radiation and other non-conservative forces.

Moreover, the initial assumption about U being a specific value (e.g., assuming a certain gravitational potential energy at a given altitude) emphasizes the importance of context in energy calculations. Precise values depend on the actual parameters and reference points used.

Conclusion

The analysis of a satellite with a mass of 93 kg orbiting at an altitude of approximately 1.99 million meters reveals the intricate balance of gravitational potential energy, kinetic energy, and total mechanical energy that governs orbital mechanics. The gravitational potential energy for this satellite is roughly -4.44×10^{9} Joules, while the kinetic energy is about 2.21×10^{9} Joules, resulting in a total energy of approximately -2.23×10^{9} Jou

Frequently Asked Questions

What is the gravitational potential energy (U) of a satellite with a mass of 93 kg at an altitude of 1.99×10^6 meters?

Using $U = -GMm/r$, where $G = 6.674\times10^{-11}\ N\cdot m^2/kg^2$, $M = 5.972\times10^{24}$ kg, $m = 93$ kg, and r = Earth's radius + altitude (6.371×10^6 m + 1.99×10^6 m = 8.361×10^6 m), $U = - (6.674\times10^{-11})(5.972\times10^{24})(93) / 8.361\times10^6 \approx -4.45\times10^9$ Joules.

How do you calculate the orbital velocity of the satellite at this altitude?

Orbital velocity $v = \sqrt{GM/r}$. Substituting G, M, and r as above, $v \approx \sqrt{6.674\times10^{-11} \times 5.972\times10^{24} / 8.361\times10^6} \approx 7.55$ km/s.

What is the significance of the value of U in the context of satellite motion?

The gravitational potential energy U indicates the energy stored due to the satellite's position in Earth's gravitational field; it's negative, reflecting a bound orbit.

If the satellite's mass increases, how does it affect the gravitational potential energy (U)?

U is directly proportional to the satellite's mass; increasing mass increases the magnitude of U (making it more negative).

What assumptions are made when calculating U for the satellite?

Assumptions include treating Earth as a point mass, ignoring atmospheric drag, and assuming a circular orbit at the given altitude.

How does the altitude of the satellite influence its gravitational potential energy?

Higher altitude (larger r) results in a less negative U (less bound), indicating the satellite is farther from Earth's gravitational well.

Is the gravitational potential energy positive or negative for the satellite in orbit?

It is negative, as gravitational potential energy in bound systems is always negative, indicating a bound orbit.

Can the value of U be used to determine the total mechanical energy of the satellite?

Yes, the total mechanical energy $E = K + U$, where K is the kinetic energy; for a stable orbit, E is typically negative and equal to $U/2$.

How does the gravitational potential energy relate to the satellite's kinetic energy in orbit?

In a circular orbit, the kinetic energy is half the magnitude of the gravitational potential energy ($K = -U/2$), reflecting energy conservation in the satellite's orbit.

What is the impact of the satellite's altitude on its orbital period?

Higher altitude (larger r) leads to a longer orbital period, calculated via Kepler's third law, $T = 2\pi\sqrt{r^3/GM}$.

Additional Resources

A Satellite in Earth Orbit Has a Mass of 93 Kg and Is at an Altitude of 1.99×10^6 m — this scenario offers an excellent opportunity to explore the fundamental principles of orbital mechanics, gravitational potential energy, and the physics governing satellites in space. Understanding such a problem requires a deep dive into Newtonian gravity, the relationship between mass, distance, and gravitational forces, and the practical implications for satellite operation and stability. In this comprehensive guide, we'll break down the problem step-by-step, analyze the relevant physics, and provide insights into how these principles apply to real-world satellite missions.

Understanding the Scenario

Imagine a satellite with a mass of 93 kg orbiting Earth at an altitude of 1.99×10^6 meters. This altitude is significantly above Earth's surface, and analyzing the satellite's potential energy, gravitational force, and orbital velocity requires understanding the key concepts of gravitational physics.

Key Data Points:
- Satellite mass, $m = 93\, \text{kg}$
- Altitude above Earth's surface, $h = 1.99 \times 10^6\, \text{m}$
- Earth's radius, $R_e \approx 6.371 \times 10^6\, \text{m}$
- Earth's mass, $M_e \approx 5.972 \times 10^{24}\, \text{kg}$
- Universal gravitational constant, $G \approx 6.674 \times 10^{-11}\, \text{N}\,\text{m}^2/\text{kg}^2$

The Physics Behind Satellite Orbits

Gravitational Potential Energy (U)

The gravitational potential energy of an object in the gravitational field of a planet like Earth is given by:

$$
U = -\frac{G M_e m}{r}
$$

where:
- $r = R_e + h$ is the distance from Earth's center to the satellite.

The negative sign indicates that the satellite is bound within Earth's gravitational field.

Gravitational Force (F)

Newton's law of universal gravitation states:

$$
F = \frac{G M_e m}{r^2}
$$

This force provides the necessary centripetal force for the satellite's circular orbit.

Orbital Velocity (v)

For a satellite in a stable circular orbit:

$$
v = \sqrt{\frac{G M_e}{r}}
$$

This velocity ensures the satellite remains in orbit without falling back to Earth or escaping into space.

Step-by-Step Calculations

1. Determine the Distance from Earth's Center

Since the altitude is given relative to Earth's surface:

$$
r = R_e + h = 6.371 \times 10^6\, \text{m} + 1.99 \times 10^6\, \text{m} = 8.361 \times 10^6\, \text{m}
$$

This is the orbital radius from Earth's center.

2. Compute the Gravitational Potential Energy (U)

Using the formula:

$$
U = -\frac{G M_e m}{r}
$$

Plugging in the values:

$$
U = -\frac{(6.674 \times 10^{-11})(5.972 \times 10^{24})(93)}{8.361 \times 10^{6}}
$$

Calculate numerator:

\[

(6.674 \times 10^{-11}) \times (5.972 \times 10^{24}) \times 93 \approx 6.674 \times 5.972 \times 93 \times 10^{(-11 + 24)}
\]

Calculations:

- \(6.674 \times 5.972 \approx 39.878\)
- \(39.878 \times 93 \approx 3709.1\)

So,

\[
U \approx -\frac{3709.1 \times 10^{13}}{8.361 \times 10^{6}} = -\frac{3.7091 \times 10^{16}}{8.361 \times 10^{6}}
\]

Divide:

\[
U \approx -4.437 \times 10^{9}\, \text{J}
\]

Result: The gravitational potential energy of the satellite at this altitude is approximately -4.44×10^9 Joules.

---

3. Calculate the Gravitational Force (F)

Using:

\[
F = \frac{G M_e m}{r^2}
\]

Plugging in values:

\[
F = \frac{(6.674 \times 10^{-11})(5.972 \times 10^{24})(93)}{(8.361 \times 10^{6})^2}
\]

Calculate numerator, as above:

\[
6.674 \times 5.972 \times 93 \approx 3709.1
\]

Calculate denominator:

\[

$$
(8.361 \times 10^{6})^2 = 8.361^2 \times 10^{12} \approx 69.92 \times 10^{12} = 6.992 \times 10^{13}
$$

Now, force:

$$
F \approx \frac{3709.1 \times 10^{13}}{6.992 \times 10^{13}} = \frac{3709.1}{6.992} \approx 530.4\, \text{N}
$$

Result: The gravitational force acting on the satellite at this altitude is approximately 530.4 N.

---

4. Find the Orbital Velocity (v)

Using:

$$
v = \sqrt{\frac{G M_e}{r}}
$$

Plug in the values:

$$
v = \sqrt{\frac{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}{8.361 \times 10^{6}}}
$$

Calculate numerator:

$$
6.674 \times 5.972 \approx 39.878
$$

Thus,

$$
v = \sqrt{\frac{39.878 \times 10^{13}}{8.361 \times 10^{6}}}
$$

Divide:

$$
\frac{39.878 \times 10^{13}}{8.361 \times 10^{6}} \approx 4.77 \times 10^{6}
$$

Take square root:

$$
v \approx \sqrt{4.77 \times 10^{6}} \approx 2184\, \text{m/s}
$$

Result: The satellite must travel at approximately 2184 meters per second to maintain a stable circular orbit at this altitude.

Additional Insights

Total Mechanical Energy (E)

For a satellite in a circular orbit, total mechanical energy is:

$$
E = K + U
$$

where K is the kinetic energy:

$$
K = \frac{1}{2} m v^2
$$

Calculate K:

$$
K = 0.5 \times 93 \times (2184)^2 \approx 0.5 \times 93 \times 4.77 \times 10^6 \approx 0.5 \times 93 \times 4.77 \times 10^{6}
$$

$$
K \approx 46.5 \times 4.77 \times 10^{6} \approx 222 \times 10^{6} = 2.22 \times 10^{8}\, \text{J}
$$

Recall $U \approx -4.44 \times 10^9\, \text{J}$.

Total energy:

$$
E = 2.22 \times 10^{8} - 4.44 \times 10^{9} \approx -4.22 \times 10^{9}\, \text{J}
$$

This negative value confirms the bound state of the satellite.

Practical Applications and Implications

Satellite Design Considerations

- Orbital Velocity: To stay in orbit at this altitude, the satellite must be launched with a velocity of approximately 2184 m/s.
- Energy Requirements: The energy needed to reach and maintain this orbit includes overcoming Earth's gravity, atmospheric drag (if at lower altitudes), and the spacecraft's own propulsion costs.
- Stability: The gravitational force and energy calculations help mission planners design stable orbits, anticipate fuel needs, and plan station-keeping maneuvers.

Orbital Altitude and Period

Using Kepler's Third Law, the orbital period T can be found:

$$
T = 2\pi \sqrt{\frac{r^3}{G M_e}}
$$

Calculating:

$$
T = 2\pi \sqrt{\frac{(8.361 \times 10^{6})^3}{6.674 \times 10^{-11} \times 5.972 \times 10^{24}}}
$$

This period indicates how long the satellite takes to complete one orbit, which is critical for communication, imaging, or scientific missions.

Summary

In this detailed exploration, we've analyzed a satellite with a mass of 93 kg at an altitude of 1.99×10^6 meters:

- Its gravitational potential energy is approximately

A Satellite In Earth Orbit Has A Mass Of 93 Kg And Is At An Altitude Of 1 99x10 6 M Assume That U

A Satellite In Earth Orbit Has A Mass Of 93 Kg And Is At An Altitude Of 1 99x10 6 M Assume That U

Related Articles

- A Race Car Starts From Rest And Travels East Along A Straight And Level Track. For The First 5.0 S Of

- A Student Is Asked To Create Models Of Both Prokaryotic And Eukaryotic Cells. They Will Use Materials
- A Person Invested $ 3 , 600 $3,600 In An Account Growing At A Rate Allowing The Money To Double Every

Back to Home