

A Simple Random Sample Of 40 Items Resulted In A Sample Mean Of 25. The Population Standard Deviation

A Simple Random Sample Of 40 Items Resulted In A Sample Mean Of 25. The Population Standard Deviation

Introduction

In the realm of statistics, understanding how to analyze and interpret data collected through sampling methods is essential for making informed decisions and drawing meaningful conclusions about larger populations. One common scenario involves taking a simple random sample from a population and calculating the sample mean. When the sample size is sufficiently large and the population standard deviation is known, statisticians can perform various hypothesis tests and construct confidence intervals to estimate population parameters accurately.

Imagine a situation where you take a simple random sample of 40 items from a large population. Upon analyzing this sample, you find that the mean of these items is 25. Now, the key question arises: what does this sample mean tell us about the overall population? Is it significantly different from a hypothesized population mean? To answer such questions, we often need to understand concepts such as the sampling distribution, standard error, and the role of the population standard deviation.

This article delves into the statistical principles behind analyzing such data, including how to perform hypothesis testing, construct confidence intervals, and interpret results when the population standard deviation is known. We will explore the relevant formulas, assumptions, and step-by-step procedures, all while emphasizing the importance of sample size and randomness in ensuring valid inferences.

Understanding the Context and Key Concepts

What is a Simple Random Sample?

A simple random sample (SRS) is a subset of individuals or items selected from a larger population, where each item has an equal chance of being chosen, and every possible sample of the specified size has the same probability of selection. This method minimizes bias and ensures that the sample is representative of the population.

Sample Mean and Population Mean

- Sample Mean ($\bar{x}$): The average of the sample data points. In our case, $\bar{x} = 25$.
- Population Mean (μ): The true average of the entire population. Often unknown, but we attempt to estimate it using sample data.

Population Standard Deviation (σ)

The population standard deviation measures the dispersion of data points in the population. When σ is known, it simplifies many statistical procedures, particularly hypothesis testing and confidence interval estimation.

Sample Size (n)

- Sample size: 40 items.
- Larger samples tend to produce more reliable estimates of the population parameters, thanks to the Central Limit Theorem.

Significance of Sample Size and Standard Deviation

- With a sample of size 40, the sample mean distribution can be approximated as normal under certain conditions, especially when the population distribution is not heavily skewed.
- Knowing σ allows us to use z-tests and z-confidence intervals, which are more straightforward than t-tests that are used when σ is unknown.

The Importance of Sampling Distribution in Statistical Inference

What is the Sampling Distribution of the Sample Mean?

The sampling distribution describes the distribution of the sample mean ($\bar{x}$) over many samples drawn from the population. According to the Central Limit Theorem, regardless of the population distribution, the sampling distribution of the mean approaches a normal distribution as the sample size increases, typically $n \geq 30$.

Properties of the Sampling Distribution

- Mean: The mean of the sampling distribution is equal to the population mean (μ).
- Standard Error: The standard deviation of the sampling distribution, called the standard error (SE), is given by:

$$SE = \frac{\sigma}{\sqrt{n}}$$

where:

- σ is the population standard deviation,
- n is the sample size.

Implications for Our Scenario

Given:

- $n = 40$,
- $\bar{x} = 25$,
- σ is known (though its value is not specified here).

Knowing the standard error allows us to assess how far the sample mean is from the

hypothesized population mean and whether this difference is statistically significant.

Performing Hypothesis Testing with Known Population Standard Deviation

Objective

To determine whether the observed sample mean of 25 provides enough evidence to conclude that the population mean differs from a specified value, say μ_0 .

Step 1: State the Hypotheses

- Null hypothesis (H_0): $\mu = \mu_0$
- Alternative hypothesis (H_a): $\mu \neq \mu_0$ (for a two-tailed test)

Note: The specific value of μ_0 depends on the context or research question.

Step 2: Choose the Significance Level (α)

Common choices are 0.05 or 0.01, representing 5% or 1% risk of Type I error.

Step 3: Calculate the Test Statistic

Since the population standard deviation σ is known, we use the z-test:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Step 4: Determine the Critical Value(s)

Using standard normal distribution tables, find the z-value corresponding to the chosen α . For a two-tailed test at $\alpha = 0.05$:

- Critical z-values: ± 1.96

Step 5: Make a Decision

- If $|z|$ is greater than the critical value, reject H_0 .
- If $|z|$ is less than or equal to the critical value, fail to reject H_0 .

Example:

Suppose $\mu_0 = 24$:

$$z = \frac{25 - 24}{\sigma / \sqrt{40}}$$

If σ is known (say, $\sigma=5$), then:

$$z = \frac{1}{5 / \sqrt{40}} = \frac{1}{5 / 6.3246} \approx \frac{1}{0.789} \approx 1.27$$

Since $1.27 < 1.96$, we fail to reject (H_0) , indicating that the sample mean does not significantly differ from 24 at the 5% significance level.

Constructing Confidence Intervals for the Population Mean

Purpose

To estimate the range within which the true population mean (μ) lies with a certain level of confidence.

Formula for a Confidence Interval When (σ) is Known

$$\bar{x} \pm z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

where:

- $(\bar{x})$ = sample mean,
- $(z_{\alpha/2})$ = critical z-value for the desired confidence level,
- (σ) = population standard deviation,
- (n) = sample size.

Example: 95% Confidence Interval

Using the previous example $(\bar{x} = 25, \sigma=5, n=40)$:

$$\text{Margin of Error} = 1.96 \times \frac{5}{\sqrt{40}} \approx 1.96 \times 0.789 \approx 1.55$$

So, the confidence interval:

$$25 \pm 1.55 \rightarrow (23.45, 26.55)$$

This means we are 95% confident that the true population mean (μ) lies between 23.45 and 26.55.

Factors Influencing the Analysis

Sample Size and Its Impact

- Larger samples reduce the standard error, leading to narrower confidence intervals and increased test power.
- The Central Limit Theorem ensures the sampling distribution approaches normality for sufficiently large n , even if the population distribution is skewed.

Known vs. Unknown Population Standard Deviation

- When σ is known, z-tests and z-intervals are appropriate.
- When σ is unknown, t-tests and t-intervals are used, which account for additional uncertainty.

Assumptions in Statistical Testing

- Random sampling ensures representativeness.
- Independence of observations.
- Normality of the population distribution, or a sufficiently large sample size to invoke the Central Limit Theorem.

Practical Applications and Real-World Examples

Quality Control in Manufacturing

Suppose a factory produces items with a known standard deviation of weight. Sampling 40 items yields a mean weight of 25 grams. Testing whether the population mean weight differs from a target weight involves similar methods.

Educational Testing

Analyzing scores from a random sample of students to infer the average performance of all students, especially when the population standard deviation is known from previous data.

Medical Research

Estimating average blood pressure or cholesterol levels in a population based on a sample, using known population standard deviations for precise inference.

Summary and Key Takeaways

- A simple random sample of 40 items with a mean of 25 provides valuable information about the population, especially when the population standard deviation is known.
- The standard error ($SE = \sigma / \sqrt{n}$) measures how much the sample mean is expected to vary from the true population mean.
- Hypothesis tests using the z-statistic allow researchers to determine whether observed differences are statistically significant.
- Confidence intervals offer a range within which the true population mean likely resides,

Frequently Asked Questions

What is a simple random sample?

A simple random sample is a subset of a population where each item has an equal chance of being selected, ensuring unbiased representation.

Given a sample size of 40 and a sample mean of 25, how can we estimate the population mean?

If the population standard deviation is known, the sample mean (25) is an unbiased estimate of the population mean.

How do you construct a confidence interval for the population mean with known population standard deviation?

Use the formula: $\text{sample mean} \pm Z(\text{population standard deviation} / \sqrt{n})$, where Z is the Z -score for the desired confidence level.

What is the significance of knowing the population standard deviation in this context?

Knowing the population standard deviation allows for more precise confidence intervals using the Z -distribution instead of the t -distribution.

How do you determine the margin of error in this scenario?

Calculate it as $Z(\text{population standard deviation} / \sqrt{n})$, where Z depends on the chosen confidence level.

If the population standard deviation was unknown, what alternative method could be used?

Use the sample standard deviation and apply the t -distribution to construct the confidence interval.

What is the effect of increasing the sample size on the confidence interval?

Increasing the sample size decreases the standard error, resulting in a narrower confidence interval and more precise estimates.

How can this data be used to make decisions about the population?

By constructing confidence intervals, one can estimate the population mean with a specified level of confidence, aiding in informed decision-making.

Additional Resources

A Simple Random Sample Of 40 Items Resulted In A Sample Mean Of 25. The Population Standard Deviation is a statement that encapsulates a fundamental concept in statistical inference. It indicates that from a larger population, a randomly selected subset of 40 items was analyzed, and the average value of this subset was found to be 25. This phrase serves as a gateway into understanding the principles of sampling, estimation, and hypothesis testing, which are essential tools in statistics for making inferences about populations based on sample data.

Understanding the Context of the Sample

What Is a Simple Random Sample?

A simple random sample (SRS) involves selecting items from a population such that each item has an equal chance of being chosen, and every possible subset of the specified size has an equal probability of selection. This sampling method minimizes bias and ensures that the sample is representative of the population, making statistical inferences more reliable.

Features of SRS:

- Equal probability for all items.
- No systematic bias.
- Ease of implementation in many scenarios.

Pros:

- Simplifies analysis due to randomness.
- Reduces bias.
- Enables straightforward application of probability theory.

Cons:

- May be impractical for large or dispersed populations.
- Requires a complete list of the population.

The Significance of the Sample Size (n=40)

Choosing a sample size of 40 reflects a balance between resource constraints and the

need for statistical accuracy. While larger samples tend to produce more precise estimates, they also require more effort and resources.

Implications:

- The Central Limit Theorem (CLT) applies reasonably well at $n=40$, allowing the sample mean to be approximated by a normal distribution.
- The sample size influences the confidence intervals and hypothesis testing outcomes.

Interpreting the Sample Mean and Population Standard Deviation

Sample Mean (25)

The sample mean (denoted as $\bar{x}$) indicates the average of the sampled data points. In this case, it is 25, suggesting that, on average, the items in the sample have a value of 25.

Implications:

- Acts as an estimate for the population mean (μ).
- Its accuracy depends on the variability of the data and sample size.

Population Standard Deviation (σ)

The population standard deviation measures the spread or dispersion of the entire population data. Although not explicitly provided, knowing or estimating σ is crucial for many statistical procedures, especially when constructing confidence intervals or conducting hypothesis tests.

Features:

- Quantifies variability.
- Assumed known or estimated from the sample.

Pros of knowing σ :

- Simplifies calculations in z-tests.
- Enables more precise confidence intervals.

Cons:

- Often unknown in practice.
- Estimating σ introduces additional uncertainty.

Statistical Inference Using the Sample Data

Estimating the Population Mean

Given the sample mean and size, statisticians often seek to estimate the true population mean (μ). Using the sample mean as a point estimate is straightforward, but constructing confidence intervals provides a range likely to contain μ .

Confidence Interval Construction:

- When σ is known, the confidence interval for μ is typically based on the z-distribution.
- The formula: $\bar{x} \pm z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$, where $z_{\alpha/2}$ is the critical value from the standard normal distribution.

Advantages:

- Provides a range of plausible values for μ .
- Incorporates sampling variability.

Limitations:

- Relies on accurate knowledge of σ .
- Assumes the sample is representative and drawn randomly.

Hypothesis Testing about the Population Mean

Suppose a researcher wants to test whether the population mean differs from a hypothesized value μ_0 . Using the sample mean, population standard deviation, and sample size, a z-test can be performed.

Steps:

1. State the null hypothesis ($H_0: \mu = \mu_0$).
2. Calculate the test statistic: $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$.
3. Compare the calculated z to the critical z-value for the chosen significance level.

Pros:

- Straightforward when σ is known.
- Suitable for large samples ($n \geq 30$) due to CLT.

Cons:

- Sensitive to violations of normality if the sample is small and σ is unknown.
- Cannot be used if σ is estimated from the sample unless adjusted with t-distribution.

Challenges and Considerations in Practical Application

Assumption of Known Population Standard Deviation

In many real-world scenarios, σ is unknown, which complicates the analysis. When σ is unknown and the sample size is small, the t-distribution replaces the normal distribution for inference.

Implications:

- Use of t-confidence intervals.
- T-tests instead of z-tests.

Sampling Variability and Bias

While a simple random sample reduces bias, it does not eliminate it entirely. Other sources of bias, such as non-response or measurement errors, can influence results.

Strategies to Mitigate Bias:

- Proper sampling techniques.
- Adequate sample size.
- Validation of data collection processes.

Limitations of the Sample Size

While 40 is often considered sufficient for approximate normality of the sample mean, smaller samples might not satisfy CLT assumptions. Larger samples generally improve estimate accuracy but are more costly.

Conclusion and Practical Recommendations

Choosing a simple random sample of 40 items that results in a sample mean of 25 provides a solid foundation for statistical analysis. The key lies in understanding the assumptions involved, especially regarding the population standard deviation. When σ is known, the inference process is more straightforward, utilizing z-distribution-based methods. However, in practical applications where σ is unknown, statisticians often estimate it from the sample, adjusting their methods accordingly.

Best practices include:

- Ensuring the sample truly is random to minimize bias.
- Verifying that the sample size is appropriate for the assumptions of normality.
- Recognizing the limitations of the data and the potential for sampling errors.

- Using confidence intervals to express the uncertainty around the population mean.
- Conducting hypothesis tests to evaluate specific claims about the population.

Summary of features and considerations:

- Pros:
 - Simple random sampling reduces bias.
 - Large enough sample size allows for CLT application.
 - Provides a clear estimate of the population mean.
- Cons:
 - Assumes knowledge of σ , which may not be available.
 - Potential sampling bias if randomness is compromised.
 - Sample may not perfectly represent the population due to inherent variability.

In conclusion, the scenario of sampling 40 items with a mean of 25 and known or estimated population standard deviation underscores the importance of proper sampling techniques and statistical methods. Whether used for estimation or hypothesis testing, understanding these components ensures that inferences about the broader population are both valid and meaningful.

A Simple Random Sample Of 40 Items Resulted In A Sample Mean Of 25 The Population Standard Deviation

A Simple Random Sample Of 40 Items Resulted In A Sample Mean Of 25 The Population Standard Deviation

Related Articles

- [A Manager Is Using The Organizations Erp Software To Generate A Report That Links Its Human Resources](#)
- [An Investor Whose Highest Priority Is Getting The Largest Gains Possible, Even If it Requires A Lot Of](#)
- [An Estuary Is An Area At The Mouth Of A River Where The River Joins A Sea. Estuaries Contain Brackish](#)

[Back to Home](#)