

A Simple Random Sample Of 5 Observations From A Population Containing 400 Elements Was Taken, And The

A Simple Random Sample Of 5 Observations From A Population Containing 400 Elements Was Taken, And The scenario presents an essential concept in statistics: simple random sampling. This sampling method is widely used in research, survey design, and data analysis to ensure that each member of a population has an equal chance of being selected. Understanding the principles, calculations, and implications of such sampling is crucial for statisticians, data scientists, and researchers aiming for unbiased and representative data collection.

Understanding Simple Random Sampling

Definition and Principles

Simple random sampling (SRS) is a fundamental sampling technique where every possible subset of the population has an equal probability of being selected. This method ensures that each individual in the population has an equal chance of inclusion, leading to an unbiased representation of the population's characteristics.

Key principles of SRS include:

1. **Equal Probability:** Every element has the same chance of selection.
2. **Independence:** The selection of one element does not influence the selection of another.
3. **Randomness:** The process is conducted randomly, often using random number generators or lottery methods.

Advantages of Simple Random Sampling

- Reduces bias in the selection process.

- Provides a basis for statistical inference with well-understood properties.
- Simple to understand and implement for small to medium-sized populations.

Limitations and Challenges

- Impractical for very large populations due to logistical constraints.
- Requires a complete list of the population, which may not always be available.
- Potential for sampling error if the sample size is small relative to the population.

Calculating the Number of Possible Samples

Combinatorial Foundations

Given a population of 400 elements, selecting 5 observations involves understanding the total number of possible unique samples. This is a classic combinatorial problem, where the order of selection does not matter (since a sample is a set, not a sequence).

The total number of possible samples is given by the combination formula:

$$C(n, k) = n! / (k! (n - k)!)$$

where:

- n = total population size (400)
- k = sample size (5)

Calculating the Total Number of Samples

Applying the formula:

$$C(400, 5) = 400! / (5! (400 - 5)!)$$

This simplifies to:

$$C(400, 5) = 400! / (5! 395!)$$

Using factorial properties or computational tools, the value is:

1. Calculate numerator: $400 \times 399 \times 398 \times 397 \times 396$
2. Divide by $5! = 120$

Performing the calculation:

- Numerator: $400 \times 399 \times 398 \times 397 \times 396 = \text{approximately } 1.004 \times 10^{14}$
- Denominator: 120

Thus:

$$C(400, 5) \approx 8,366,762,560$$

This enormous number indicates the vast diversity of possible samples, emphasizing the importance of randomness to ensure representativeness.

Probability of Selecting a Specific Sample

Equal Likelihood of All Samples

In simple random sampling, each possible sample of size 5 has an equal chance of being selected. The probability of choosing any particular sample is:

$$P = 1 / C(400, 5)$$

which is approximately:

$$P \approx 1 / 8,366,762,560 \approx 1.2 \times 10^{-10}$$

This minuscule probability illustrates the uniqueness of each sample and underscores the randomness of the process.

Probability of Selecting a Specific Element

The probability that a particular element from the population is included in the sample is:

$$P(\text{element included}) = (\text{Number of samples including that element}) / (\text{Total number of samples})$$

Since each element appears in the same number of samples:

$$\text{Number of samples including a specific element} = C(399, 4)$$

which is:

$$C(399, 4) = 399! / (4! \cdot 395!) \approx 6,216,945$$

Therefore:

$$P(\text{element included}) = C(399, 4) / C(400, 5) \approx 6,216,945 / 8,366,762,560 \approx 0.000743$$

or roughly 0.0743%. This probability aligns with the expectation that each element has an equal chance of about 1/400, or 0.25, but slightly less because the sample size is small relative to the population.

Implications for Statistical Analysis

Estimating Population Parameters

One primary purpose of sampling is to estimate population parameters such as the mean, proportion, variance, etc. When using simple random sampling:

1. Sample mean ($\bar{x}$) is an unbiased estimator of the population mean (μ).
2. Sample proportion is an unbiased estimator of the population proportion.

3. The variance of the estimator decreases as the sample size increases.

Sampling Error and Margin of Error

Understanding the variability inherent in sampling allows researchers to quantify the potential deviation of the sample estimate from the true population value:

- Sampling error diminishes with larger sample sizes.
- Confidence intervals can be constructed to quantify uncertainty.

Role of the Finite Population Correction (FPC)

Since the population size (400) is finite and relatively small, the FPC factor adjusts variance estimates:

$$FPC = \sqrt{[(N - n) / (N - 1)]}$$

where $N = 400$ and $n = 5$.

This correction slightly reduces the estimated variance, reflecting the fact that sampling without replacement from a small population reduces variability.

Practical Applications and Considerations

Designing a Random Sample in Practice

To implement simple random sampling efficiently:

1. Compile a complete list of the population elements.
2. Use a random number generator or lottery method to select 5 unique identifiers.
3. Ensure the process is transparent and free from bias.

Sample Size Determination

While the scenario involves a sample size of 5, in practical settings, the sample size depends on:

- The desired precision of estimates.
- The variability within the population.
- The resources and time available.

For a population of 400, sample sizes are often larger to achieve reliable estimates, but small samples like 5 can still provide preliminary insights.

Ethical and Logistical Considerations

Ensuring fairness and minimizing bias involves:

- Maintaining randomness in selection.
- Documenting the process for reproducibility.
- Considering stratified sampling if subgroups are important.

Conclusion

A simple random sample of 5 observations from a population containing 400 elements exemplifies the core principles of unbiased data collection. The combinatorial calculations highlight the vast number of possible samples, reinforcing the importance of randomization to prevent bias. Proper understanding of the probabilities, sampling variability, and statistical implications ensures that researchers can make valid inferences about the population based on the sample. Whether used in surveys, experiments, or quality control, simple random sampling remains a cornerstone of statistical methodology, balancing simplicity with robustness for reliable data analysis.

Meta Description:

Learn about simple random sampling, including calculations for the number of possible samples, probabilities, implications for statistical inference, and best practices for designing unbiased surveys from a population of 400 elements.

Frequently Asked Questions

What is a simple random sample in the context of the given population?

A simple random sample is a subset of the population where each element has an equal chance of being selected, ensuring unbiased representation of the entire population.

How many possible simple random samples of size 5 can be drawn from a population of 400 elements?

The number of possible samples is given by the combination formula $C(400, 5)$, which equals 2,585,720,300 different samples.

What is the probability that a specific element from the population is included in the sample?

The probability that a particular element is included is $5/400$, or $1/80$, because each element has an equal chance of being selected in a sample of 5.

If the sample mean is calculated from the 5 observations, how can it be used to estimate the population mean?

The sample mean serves as an unbiased estimator of the population mean, and averaging over many such samples can approximate the true population mean.

What are the advantages of using a simple random sample in statistical analysis?

It minimizes bias, simplifies analysis, and ensures each element has an equal chance of selection, making the results more representative of the population.

How does the sample size of 5 impact the precision of estimating the population parameters?

A small sample size like 5 provides limited information and may lead to higher variability and less precise

estimates of the population parameters.

What are potential limitations or challenges when selecting a simple random sample of this size from a population of 400?

Challenges include ensuring true randomness, managing sampling variability, and the possibility that the small sample may not adequately represent the population's diversity.

Additional Resources

Random Sampling

When conducting statistical analysis, research, or data collection, understanding the concept of random sampling is fundamental. In particular, examining a simple random sample (SRS) of five observations from a population of 400 elements provides insight into the principles of probability, sampling techniques, and the implications for data inference. This article offers an in-depth exploration of this scenario, breaking down the process, calculations, and significance of such sampling methods, akin to an expert review or detailed product feature.

Understanding Simple Random Sampling (SRS)

Definition and Key Characteristics

Simple Random Sampling (SRS) is a fundamental sampling method in which every subset of the population has an equal chance of being selected. This technique ensures the sample's representativeness, minimizes bias, and allows for straightforward statistical analysis.

Key features include:

- **Equal Probability:** Each element in the population has an identical chance of being chosen.
- **Independence:** The selection of one element does not influence the selection of another.
- **Randomness:** The process relies on a random mechanism, often computerized or using random number generators.

Why Use SRS?

- To produce representative samples that reflect the entire population.
- To facilitate statistical inference with well-understood properties.

- To simplify calculations of probabilities and variances.

The Scenario: Sampling 5 from a Population of 400

Imagine a scenario where a researcher wants to select a small, random subset of data points or elements from a large population of 400. The goal is to analyze or infer properties about the entire population based on this sample.

Key parameters:

- Population Size (N): 400 elements
- Sample Size (n): 5 observations

This specific setup raises several questions about the probabilities involved, the possible combinations, and the implications of choosing such a sample.

Sampling Process and Its Mechanics

Methodology

The sampling process involves:

1. Assigning a unique identifier to each of the 400 elements.
2. Using a random mechanism (such as a random number generator) to select 5 unique numbers corresponding to these identifiers.
3. Collecting the data points associated with the selected identifiers.

Ensuring Fairness

To guarantee the sample's randomness, the process must:

- Use a reliable random number generator that produces uniform distributions.
- Avoid bias by ensuring each element has the same selection probability.
- Prevent duplication if sampling without replacement.

Sampling Without Replacement

Typically, in such scenarios, sampling is performed without replacement, meaning:

- Once an element is selected, it cannot be chosen again.
- The sample comprises 5 distinct elements.

Calculating Probabilities and Combinatorial Aspects

A thorough understanding of the probabilities involved in selecting specific elements or subsets is essential. This involves combinatorial calculations, primarily binomial coefficients, to quantify possible sample arrangements.

Total Number of Possible Samples

The total number of different simple random samples of size 5 from 400 elements is given by the combination formula:

$$\boxed{\binom{400}{5} = \frac{400!}{5! \times (400 - 5)!}}$$

Calculating this value:

- Factorials: 400! is the factorial of 400.
- Denominator: 5! (which is 120) times 395! (factorial of 395).

Approximate Calculation

$$\binom{400}{5} \approx \frac{400 \times 399 \times 398 \times 397 \times 396}{5 \times 4 \times 3 \times 2 \times 1}$$

Breaking down the numerator:

- $(400 \times 399 = 159,600)$
- $(159,600 \times 398 \approx 63,460,800)$
- $(63,460,800 \times 397 \approx 25,195,737,600)$

$$- (25,195,737,600 \times 396 \approx 9,977,846,329,600)$$

Divide by 120:

$$\left[\frac{9,977,846,329,600}{120} \approx 83,148,719,413 \right]$$

Conclusion:

There are approximately 83.15 billion different possible samples of size 5 from a population of 400.

Probability of Selecting a Specific Sample

Since each sample is equally likely:

$$\left[P(\text{any specific sample}) = \frac{1}{\binom{400}{5}} \approx \frac{1}{83,148,719,413} \right]$$

This illustrates the rarity of any particular sample, emphasizing the importance of multiple samples for reliable inference.

Probability of Selecting Specific Elements

Suppose we are interested in the probability of selecting a specific element, say element 123, in our sample:

- The probability that element 123 is included in the sample:

$$\left[P(\text{element 123 is in the sample}) = \frac{\text{Number of samples including element 123}}{\text{Total number of samples}} \right]$$

Number of samples that include element 123:

- Fix element 123 as part of the sample.
- Select the remaining 4 elements from the remaining 399 elements:

$\left[$

$$\binom{399}{4} = \frac{399!}{4! \times 395!}$$

Calculating:

$$\binom{399}{4} \approx \frac{399 \times 398 \times 397 \times 396}{24}$$

- $(399 \times 398 = 158,802)$
- $(158,802 \times 397 \approx 63,032,394)$
- $(63,032,394 \times 396 \approx 24,977,675,224)$

Divide by 24:

$$\frac{24,977,675,224}{24} \approx 1,041,569,801$$

Thus,

$$P(\text{element 123}) = \frac{\binom{399}{4}}{\binom{400}{5}} \approx \frac{1,041,569,801}{83,148,719,413} \approx 0.0125$$

or about 1.25%.

This confirms that each individual element has an equal probability ($\sim 1.25\%$) of being included in the sample, consistent with the principles of SRS.

Implications for Data Analysis and Inference

Sampling a small subset from a large population has several important implications for statistical inference, data reliability, and decision-making.

Representativeness and Variability

- Representativeness: Because every element has an equal chance, the sample is expected to reflect the

population's characteristics, assuming random sampling is performed correctly.

- Sampling Variability: Small samples, like $n=5$, inherently have high variability. The sample mean, variance, or proportion may fluctuate significantly from the true population parameters.

Estimation and Confidence

- Point Estimators: Measures from the sample can serve as estimates for population parameters (e.g., sample mean as an estimate of the population mean).
- Confidence Intervals: With small samples, confidence intervals tend to be wider, indicating less precision.
- Hypothesis Testing: Small samples reduce statistical power, making it harder to detect significant effects or differences.

Practical Considerations

- When sampling just 5 elements from 400, the likelihood of capturing rare traits or outliers decreases.
- To improve reliability, larger samples are generally preferred, but resource constraints might limit the sample size.

Advanced Topics and Extensions

1. Sampling With or Without Replacement

- Without Replacement: As in our scenario, each element can only be selected once, leading to fewer possible samples.
- With Replacement: Elements can be selected multiple times, increasing the total number of possible samples to (400^5) .

2. Stratified and Cluster Sampling

While simple random sampling is straightforward, in practice, researchers may use stratification or clustering to improve efficiency, especially when the population has distinct subgroups.

3. Variance and Standard Error Calculations

Understanding the variance associated with the sample estimate is crucial. For example, the variance of the sample mean depends on the population variance and sample size.

Conclusion: The Power and Limitations of Small Random Samples

Sampling five observations from a population of 400 elements exemplifies the fundamental principles of simple random sampling. It ensures each element has an equal chance of inclusion, leading to unbiased and representative data collection. However, the small sample size introduces significant variability and limits the precision of estimations.

While the probability calculations reveal the immense number of potential samples and the rarity of any specific one, they also underscore the importance of multiple samples and larger sample sizes for robust statistical inference. The process exemplifies a key balance in data collection: the need for randomness and fairness against practical constraints like resources and time.

In conclusion, simple random sampling remains a cornerstone of statistical methodology, and understanding its intricacies—such as selecting 5 from 400 elements—provides a solid foundation for conducting reliable and valid research across diverse fields. Whether in academic research, market analysis, or quality control, mastering these concepts empowers practitioners to make informed decisions and derive meaningful insights from their data.

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