A Solid Wire Of Radius 10 Cm Carries A Current Of 5.0 A Distributed Uniformly Over Its Cross-section.

Introduction

A solid wire of radius 10 cm carries a current of 5.0 A distributed uniformly over its cross-section. This seemingly simple statement opens the door to a rich exploration of electromagnetic principles, including current distribution, magnetic fields, and related phenomena. Understanding how current flows within a wire and the resulting magnetic effects is crucial in many applications, from electrical engineering to material science. In this article, we delve into the detailed physics of a uniformly distributed current within a thick wire, analyze the magnetic field it produces, and explore relevant concepts such as current density, magnetic field calculations via Ampère’s law, and potential implications in practical scenarios.

Understanding Uniform Current Distribution in a Solid Wire

Current and Current Density

- Current (I): The total charge passing through a cross-section per unit time, given as 5.0 A in this case.
- Current Density (J): The amount of current flowing per unit area, expressed as $J = \frac{I}{A}$, where A is the cross-sectional area.
- Uniform Distribution: The current is evenly spread across the entire cross-section, implying that J is constant throughout the wire’s cross-sectional area.

Calculating Cross-Sectional Area

Given the radius $r = 10\, \text{cm} = 0.10\, \text{m}$:

- Area A: $A = \pi r^2$

$$
A = \pi \times (0.10)^2 = \pi \times 0.01 = 0.031416\, \text{m}^2
$$

- Current Density J:

$$
J = \frac{I}{A} = \frac{5.0\, \text{A}}{0.031416\, \text{m}^2} \approx 159.15\, \text{A/m}^2
$$

This current density indicates how much current flows per unit area at every point within the wire's cross-section.

Magnetic Field Inside and Outside the Wire

The magnetic field generated by a current-carrying conductor depends on the position relative to the wire's axis. Using Ampère’s Law simplifies the calculation for symmetrical current distributions.

Applying Ampère’s Law

Ampère’s law states:

$$
\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}
$$

where:

- $\vec{B}$ is the magnetic field,
- $d\vec{\ell}$ is an element of the path,
- $\mu_0 = 4\pi \times 10^{-7}\, \text{H/m}$,
- I_{enc} is the current enclosed by the Amperian loop.

Magnetic Field at a Distance $r' \leq r$ from the Center

Since the current is uniformly distributed:

- Current density J: constant,
- Enclosed current I_{enc}:

$$
I_{\text{enc}} = J \times \text{Area within radius } r' = J \pi r'^2
$$

Substituting J:

$$
I_{\text{enc}} = 159.15\, \text{A/m}^2 \times \pi r'^2
$$

Applying Ampère’s Law:

$$
B(r') \times 2\pi r' = \mu_0 I_{\text{enc}} = \mu_0 J \pi r'^2
$$

Solving for $B(r')$:

$$
B(r') = \frac{\mu_0 J r'}{2}
$$

Expressed explicitly:

$$
B(r') = \frac{(4\pi \times 10^{-7}) \times 159.15 \times r'}{2}
$$

$$
B(r') \approx 1.0 \times 10^{-4} \times r' \quad (\text{in Tesla, with } r' \text{ in meters})
$$

- Inside the wire ($r' \leq 0.10\, \text{m}$):

$$
B(r') \approx 1.0 \times 10^{-4} \times r' \quad \text{Tesla}
$$

- At the surface ($r' = r = 0.10\, \text{m}$):

$$
B_{\text{surface}} = 1.0 \times 10^{-4} \times 0.10 = 1.0 \times 10^{-5}\, \text{Tesla}
$$

Magnetic Field Outside the Wire ($r' > r$)

- The entire current $I = 5.0\, \text{A}$ is enclosed by any loop outside the wire.

$$
B(r') = \frac{\mu_0 I}{2 \pi r'}
$$

- At a distance r' from the center:

$$
B(r') = \frac{4\pi \times 10^{-7} \times 5.0}{2 \pi r'} = \frac{2 \times 10^{-6} \times 5}{r'} = \frac{1 \times 10^{-5}}{r'}
$$

- Example at $r' = 0.20\, \text{m}$:

$$
B(0.20) = \frac{1 \times 10^{-5}}{0.20} = 5 \times 10^{-5}\, \text{Tesla}
$$

Implications and Practical Considerations

Magnetic Field Strength and Its Effects

- The magnetic field inside the wire increases linearly from zero at the center to a maximum at the surface.
- Outside, it decreases with the distance from the wire's center.
- The maximum magnetic field at the surface is quite small (~10 microTesla), indicating minimal magnetic effects for such a current and size.

Heat Dissipation and Resistance

- Resistivity and Resistance:

The resistance R of the wire depends on its material's resistivity ρ, length L, and cross-sectional area A.

- Power Dissipation:

The power dissipated as heat:

$$
P = I^2 R
$$

- For a thick wire (radius 10 cm), the cross-sectional area is large, reducing resistance and heat generation for a given material.

Material Considerations

- Conductors like copper or aluminum are typical choices for such wires.
- The large cross-section means high mechanical strength and ability to carry significant current without overheating, provided the material properties are suitable.

Summary of Key Points

- The current density in a uniformly conducting wire of radius 10 cm carrying 5.0 A is approximately

159.15 A/m².
- The magnetic field inside the wire varies linearly with radius, from zero at the center to about 10 microTesla at the surface.
- Outside the wire, the magnetic field decreases inversely with distance from the center.
- The small magnetic field for this current and size suggests minimal electromagnetic interference.
- Practical considerations include resistance, heat dissipation, and material choice for efficient and safe operation.

Conclusion

Analyzing a solid wire with a uniform current distribution provides fundamental insights into electromagnetic phenomena. The calculations for current density, magnetic field distribution, and external effects serve as foundational knowledge for designing electrical systems and understanding magnetic interactions. While the current and dimensions in this scenario produce modest magnetic fields, understanding these principles is crucial for scaling up to larger currents and sizes, where effects become significantly more pronounced. Such analysis underscores the importance of physics in electrical engineering and the necessity of considering electromagnetic effects in practical applications.

Frequently Asked Questions

What is the magnetic field at the center of a solid wire of radius 10 cm carrying a 5.0 A current?

The magnetic field at the center of the wire is zero because the magnetic field inside a current-carrying conductor is zero at its center due to symmetry.

How does the magnetic field vary with distance from the center of the wire?

Inside the wire ($r < 10$ cm), the magnetic field increases linearly with distance from the center, proportional to r. Outside the wire ($r > 10$ cm), it decreases with $1/r$, following Ampère's law.

What is the magnetic field at the surface of the wire (r = 10 cm)?

Using Ampère's law, the magnetic field at the surface is $B = (\mu_0 I) / (2\pi r) = (4\pi \times 10^{-7}$ T·m/A 5.0 A) / (2π 0.10 m) $\approx 1.0 \times 10^{-6}$ T.

How is the current distributed within the cross-section of the wire?

The current is uniformly distributed across the wire's cross-sectional area, meaning the current density is constant throughout the cross-section.

What is the significance of the wire’s radius in calculating magnetic fields?

The radius determines the region where the magnetic field is calculated: inside the wire ($r < R$) or outside ($r > R$), affecting how the magnetic field varies with distance.

Can the magnetic field inside the wire be considered uniform? Why or why not?

No, the magnetic field inside the wire is not uniform; it varies linearly with the distance from the center, reaching zero at the center and maximum at the surface.

Additional Resources

A Solid Wire of Radius 10 cm Carries a Current of 5.0 A Distributed Uniformly Over Its Cross-Section

In the realm of electrical engineering and physics, understanding how electric currents distribute through conductive materials is fundamental. When a solid wire carries current, the manner in which this current is spread across its cross-section influences various phenomena, including magnetic field generation, heat dissipation, and electrical resistance. This article delves deeply into the case of a solid wire of radius 10 cm carrying a current of 5.0 A, with the current distributed uniformly across its cross-sectional area. Through comprehensive analysis, we will explore the physical principles, mathematical formulations, and practical implications of this scenario.

Understanding the Fundamental Concepts

What is a Solid Wire in Electrical Conductors?

A solid wire refers to a conductor made from a single piece of material, typically metal such as copper or aluminum, with a uniform cross-sectional area along its length. Unlike stranded wires, which consist of multiple smaller conductors twisted together, a solid wire offers simplicity in manufacturing and is often used for low to medium current applications.

Key characteristics of a solid wire include:

- Uniform material properties throughout its length.
- Cross-sectional area that remains constant.
- Current distribution that can be either uniform or non-uniform depending on the conditions.

Current Distribution in Conductors: Uniform vs. Non-Uniform

The distribution of current within a conductor is vital for understanding its electrical behavior. When current is distributed uniformly, each infinitesimal element of the cross-section carries an equal amount of current per unit area. Conversely, non-uniform distributions occur when factors like magnetic fields, skin effect, or material impurities cause the current to concentrate in certain regions.

In our case, the assumption of uniform distribution simplifies analysis and is valid for direct current (DC) and low-frequency AC conditions where skin effects are negligible.

Physical and Mathematical Description of the Scenario

Given Data and Assumptions

- Radius of the wire (r_0): 10 cm = 0.10 meters
- Current (I): 5.0 A
- Distribution: Uniform across cross-section
- Material properties: Assumed to be a typical metal with known resistivity, though specific resistivity is not provided here.

Cross-Sectional Area Calculation

The cross-sectional area (A) of the wire is a critical parameter:

$$ A = \pi r_0^2 $$

Substituting the given radius:

$$ A = \pi \times (0.10)^2 = \pi \times 0.01 \approx 0.0314\, \text{m}^2 $$

This large cross-sectional area reflects a relatively thick wire, which influences the current density and magnetic field characteristics.

Current Density and Its Significance

Definition of Current Density (J)

Current density (J) signifies the amount of current flowing per unit area:

$$ J = \frac{I}{A} $$

Assuming uniform distribution, the current density across the entire cross-section is:

$$ J = \frac{5.0\, \text{A}}{0.0314\, \text{m}^2} \approx 159.2\, \text{A/m}^2 $$

This value indicates how densely the current is packed within the conductor.

Implications of Current Density

- Thermal considerations: Higher current densities generate more heat due to resistive losses.
- Material limits: Conductors have maximum permissible current densities to prevent damage or melting.
- Design implications: The large radius and resulting low current density suggest minimal heating and high safety margins.

Magnetic Field Generated by the Current in the Wire

Using Ampère’s Law for a Long, Straight Conductor

Ampère’s circuital law relates the magnetic field (B) at a distance (r) from a long, straight conductor carrying a current I:

$$ \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} $$

Where:

- μ_0 = permeability of free space = $4\pi \times 10^{-7} \, \text{T}\cdot\text{m/A}$
- I_{enc} = current enclosed within radius r

For points outside the wire ($r \geq r_0$), the magnetic field is:

$$ B(r) = \frac{\mu_0 I}{2\pi r} $$

For points inside the wire ($r \leq r_0$), the enclosed current is proportional to the area:

$$ I_{enc}(r) = I \times \frac{\pi r^2}{\pi r_0^2} = I \times \frac{r^2}{r_0^2} $$

Thus, the magnetic field at radius r inside the wire:

$$ B(r) = \frac{\mu_0 I_{enc}(r)}{2\pi r} = \frac{\mu_0 I r^2}{2\pi r_0^2 r} = \frac{\mu_0 I r}{2\pi r_0^2} $$

Magnetic Field Distribution

- Outside the wire ($r \geq r_0$):

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

- Inside the wire ($r \leq r_0$):

$$B(r) = \frac{\mu_0 I r}{2\pi r_0^2}$$

This indicates that:

- The magnetic field inside the wire increases linearly with r.
- The magnetic field outside decreases inversely with r.

Given the large radius (10 cm), the magnetic field at the surface ($r = r_0$) is:

$$B(r_0) = \frac{\mu_0 I}{2 \pi r_0}$$

Substituting values:

$$B(0.10\, \text{m}) = \frac{4\pi \times 10^{-7} \times 5.0}{2 \pi \times 0.10} = \frac{2 \times 10^{-6} \times 5.0}{0.20} = \frac{1 \times 10^{-5}}{0.20} = 5 \times 10^{-5}\, \text{T}$$

or 50 μT, a relatively weak magnetic field, consistent with typical currents in such conductors.

Electrical Resistance and Power Dissipation

Resistance of the Wire

The resistance (R) of the wire depends on the resistivity (ρ) of the material, length (L), and cross-sectional area:

$$R = \frac{\rho L}{A}$$

While the length (L) and resistivity (ρ) are not specified, the calculation underscores the importance of these parameters in heat dissipation and efficiency.

Power Dissipated as Heat

Power (P) dissipated due to resistance:

$$P = I^2 R$$

High currents and resistances lead to significant heat generation, which necessitates proper thermal management in practical applications.

Practical Implications and Applications

Design Considerations for Conductors

The analysis of current distribution and magnetic fields informs several practical aspects:

- Material selection: Conductors with low resistivity are preferred to minimize heat.
- Cross-sectional dimensions: Larger radii reduce current density, lowering heating risks.
- Thermal management: Adequate cooling mechanisms are essential for high-current systems.
- Magnetic shielding: Weak magnetic fields generated can be mitigated or harnessed depending on application.

Real-World Applications

- Power transmission lines: Thick copper or aluminum wires are used to transport electricity efficiently over long distances.
- Electromagnetic devices: Understanding magnetic field distribution aids in designing transformers, inductors, and motors.
- Electrical safety: Knowledge of magnetic and thermal effects guides safety standards and insulation requirements.

Advanced Topics and Considerations

Skin Effect and Alternating Currents

While the current distribution is uniform for DC, at higher frequencies, skin effect causes currents to concentrate near the surface, altering the distribution and magnetic field profile. For our static case, this effect is negligible.

Material Impurities and Non-Uniformities

Real conductors may have imperfections, affecting current distribution, resistance, and heat dissipation. Manufacturing processes aim to minimize these effects for optimal performance.

Impact of External Magnetic Fields

External magnetic fields can influence current paths, induce eddy currents, and affect the conductor's behavior, especially in complex electromagnetic environments.

Conclusion

The case of a solid wire of radius 10 cm carrying a current of 5.0 A illustrates fundamental principles of electromagnetism and electrical conduction. The uniform distribution of current simplifies analysis, revealing straightforward relationships between current, magnetic field,

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