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Understanding the relationship between different three-dimensional shapes such as spheres and cylinders is fundamental in geometry, engineering, and various applied sciences. When a sphere and a cylinder share the same radius and height, intriguing comparisons and calculations emerge, especially concerning their volumes. This article explores the mathematical principles behind these shapes, focusing on the given conditions: that both the sphere and the cylinder have the same radius and height, and that the volume of the cylinder is 50 cubic feet. Through detailed analysis, we will discover the dimensions, relationships, and implications of these parameters, providing a comprehensive understanding of the geometric properties involved.

Understanding the Basic Geometric Formulas

Volume of a Cylinder

The volume $V_{cylinder}$ of a cylinder is given by the formula:

- $V_{cylinder} = \pi r^2 h$

where r is the radius and h is the height of the cylinder.

Volume of a Sphere

The volume V_{sphere} of a sphere is calculated using:

- $V_{sphere} = \frac{4}{3} \pi r^3$

where r is the radius of the sphere.

Given Conditions and Their Implications

Same Radius and Height

Since the sphere and the cylinder have the same radius r and height h, their dimensions are directly comparable. This assumption simplifies calculations and allows us to analyze the relationships between their volumes more straightforwardly.

Volume of the Cylinder

It's given that the volume of the cylinder is 50 cubic feet:

- $V_{cylinder} = 50$ ft^3

Determining the Dimensions of the Cylinder

Calculating Radius and Height of the Cylinder

Using the volume formula for the cylinder:

- $\pi r^2 h = 50$

Suppose the height h is known or expressed in terms of the radius r. Alternatively, if the height h is also unknown, additional information is necessary to find specific values for r and h. However, since the problem states they share the same radius and height, we can treat r and h as equal variables:

- $r = h$

This assumption leads to the following relationship:

Expressing Volume in Terms of a Single Variable

Since $r = h$, substitute h with r in the volume formula:

- $\pi r^2 r = 50$

- $\pi r^3 = 50$

Rearranged, this gives:

- $r^3 = \frac{50}{\pi}$

Therefore, the radius r of the cylinder (and sphere, as they share the same radius) is:

- $r = \left(\frac{50}{\pi} \right)^{1/3}$

Numerical Approximation

Calculating the approximate value of r:

- $r \approx \left(\frac{50}{3.1416} \right)^{1/3}$
- $r \approx (15.915)^{1/3}$
- $r \approx 2.52$ feet

Since $h = r$, the height of the cylinder is also approximately 2.52 feet.

Calculating the Volume of the Sphere

Using the Shared Radius

The volume of the sphere with radius $r \approx 2.52$ feet is:

- $V_{sphere} = \frac{4}{3} \pi r^3$

Substituting the value of r^3 from earlier:

- $V_{sphere} = \frac{4}{3} \pi \times \frac{50}{\pi}$

Notice that π cancels out:

- $V_{sphere} = \frac{4}{3} \times 50$

- $V_{sphere} = \frac{200}{3} \approx 66.67$ cubic feet

Comparison of Volumes and Dimensions

Summary of Calculated Dimensions

- Radius of both shapes: approximately 2.52 feet
- Height of the cylinder: approximately 2.52 feet
- Volume of the sphere: approximately 66.67 cubic feet

Insights and Geometric Relationships

1. The sphere's volume exceeds that of the cylinder by about 16.67 cubic feet, despite sharing the same radius and height.
2. The sphere's volume is proportional to the cube of its radius, emphasizing how small changes in radius significantly impact volume.
3. The cylinder's volume depends linearly on height when radius is fixed, highlighting different scaling properties of these shapes.

Implications and Applications

Real-World Contexts

This geometric analysis has practical applications in fields such as manufacturing, storage, and design. For example:

- Designing containers with specific volume requirements using minimal material.
- Understanding capacity differences between spherical and cylindrical vessels of similar dimensions.
- Optimizing space utilization in engineering projects where volume and

shape matter.

Design Considerations

When choosing between a spherical or cylindrical container with the same radius and height, considerations include:

- Material efficiency: spherical shapes often provide maximum volume for minimal surface area.
- Structural stability: cylinders are easier to manufacture and stack.
- Space constraints: the shape that best fits spatial limitations depends on the specific application.

Further Mathematical Explorations

Varying One Parameter

Exploring how changing the radius or height affects their volumes can reveal more about shape efficiencies:

- Holding volume constant and varying radius or height.
- Analyzing surface area differences for the same volume.
- Optimizing shapes for specific criteria like minimal surface area for a given volume.

Extensions to Other Shapes

Similar analyses can be extended to other geometric shapes such as cones, pyramids, or ellipsoids, comparing their volumes and surface areas under shared parameters.

Conclusion

The exploration of a sphere and a cylinder sharing the same radius and height, with the cylinder's volume specified as 50 cubic feet, reveals fascinating insights into geometric relationships. By deriving the dimensions and calculating the respective volumes, we see that the sphere, with the same radius, has a larger volume, approximately 66.67 cubic feet, compared to the cylinder's 50 cubic feet. These calculations not only deepen our understanding of shape properties but also have practical implications in various scientific and engineering contexts. Recognizing how dimensions influence volume and other attributes enables better design, optimization, and application of these fundamental shapes in real-world scenarios.

Frequently Asked Questions

What is the radius and height of the sphere if it has the same volume as the cylinder?

Since both the sphere and the cylinder have the same radius and height, and the cylinder’s volume is 50 cubic feet, the radius and height are the same for both. To find the radius, use the cylinder volume formula $V = \pi r^2 h$. Given $V = 50$ and $h = h$, you'd need the height to determine the radius; if h is known, then $r = \text{sqrt}(V / (\pi h))$.

How do you calculate the volume of the sphere given the same radius as the cylinder?

The volume of a sphere is calculated using $V = (4/3)\pi r^3$. If the sphere has the same radius as the cylinder, and the radius is known, plug it into the formula to find the sphere’s volume.

If the volume of the cylinder is 50 cubic feet, what is its height if the radius is 2 feet?

Using the formula $V = \pi r^2 h$, substitute $V = 50$ and $r = 2$: $50 = \pi(2)^2 h \rightarrow 50 = \pi 4h \rightarrow h = 50 / (4\pi) \approx 50 / 12.566 \approx 3.98$ feet.

Can the sphere and the cylinder have the same volume if they have the same radius and height?

Yes, if both have the same radius and height, their volumes are related but not necessarily equal unless the dimensions are chosen accordingly. In this case, since the cylinder’s volume is 50 cubic feet and the sphere shares the same radius, the sphere’s volume can be calculated independently.

What is the formula for the volume of a cylinder, and how is it used here?

The volume of a cylinder is $V = \pi r^2 h$. Given the volume is 50 cubic feet and the radius and height are equal, you can use this formula to find the missing dimension or verify the dimensions if known.

How does the volume of a sphere compare to that of a cylinder with the same radius and height?

The volume of a sphere with radius r is $(4/3)\pi r^3$, while the volume of a cylinder with the same radius and height h is $\pi r^2 h$. If h equals 2r, for example, the sphere’s volume is (2/3) times that of the cylinder, highlighting the difference in their volume formulas.

Additional Resources

A Sphere And A Cylinder Have The Same Radius And Height. The Volume Of The Cylinder Is 50 Feet Cubed.

Understanding the geometric relationship between different three-dimensional shapes is fundamental in mathematics, engineering, architecture, and various scientific disciplines. Among these shapes, the sphere and the cylinder frequently appear due to their unique properties and applications. When comparing a sphere and a cylinder with the same radius and height, intriguing questions arise about their respective volumes, surface areas, and other geometrical characteristics. This analysis explores these aspects in depth, focusing on the given scenario where the cylinder's volume is exactly 50 cubic feet.

Understanding the Geometric Shapes: Sphere and Cylinder

Before delving into calculations and comparisons, it’s essential to understand the basic properties of a sphere and a cylinder.

The Sphere

- A sphere is a perfectly symmetrical three-dimensional object, where every point on its surface is equidistant from its center.
- The key defining parameter of a sphere is its radius, denoted as \(r \).
- The surface area of a sphere is given by:
\[
A_{sphere} = 4 \pi r^2
\]
- The volume of a sphere is given by:
\[

V_{sphere} = \frac{4}{3} \pi r^3
\]
- Spheres are often associated with natural and physical phenomena such as bubbles, planets, and atoms.

The Cylinder

- A cylinder is a three-dimensional shape with two parallel circular bases connected by a curved lateral surface.
- It is characterized by its radius \(r \) and height \(h \).
- The surface area of a cylinder is:
\[
A_{cylinder} = 2 \pi r^2 + 2 \pi r h
\]
- The volume of a cylinder is:
\[
V_{cylinder} = \pi r^2 h
\]
- Cylinders are common in everyday objects such as cans, pipes, and storage tanks.

Given Conditions and Their Implications

The problem states:
> A sphere and a cylinder have the same radius and height, and the volume of the cylinder is 50 cubic feet.

This information leads to specific constraints:

1. Equal Radius and Height:
\[
r_{sphere} = r_{cylinder} = r
\]
\[
h_{cylinder} = h
\]

2. Volume of Cylinder:
\[
V_{cylinder} = 50 \text{ ft}^3
\]
Using the volume formula for the cylinder:
\[
\pi r^2 h = 50
\]

3. Sphere's Volume:

$$
V_{sphere} = \frac{4}{3} \pi r^3
$$

Since the radius is shared, the sphere's volume depends solely on r.

Key question: Given the cylinder's volume and the shared radius and height, what is the sphere's volume? How do their volumes compare? And what insights can we derive about their sizes?

Calculating the Radius and Height of the Cylinder

Given:

$$
\pi r^2 h = 50
$$

We notice that without additional information, the problem can have multiple solutions. To proceed, we need to consider the possible relationships between r and h.

Assumption 1: The height h is equal to the diameter of the sphere

This is a common scenario in practical applications, where the height of a cylinder matches the diameter of the sphere it contains or relates to.

- Diameter of the sphere:

$$
D_{sphere} = 2r
$$

- Assume:

$$
h = 2r
$$

Substituting into the volume equation:

$$
\pi r^2 (2r) = 50
$$

$$
2 \pi r^3 = 50
$$

$$
r^3 = \frac{50}{2\pi} = \frac{25}{\pi}
$$

\[

$$
r = \sqrt[3]{\frac{25}{\pi}}
$$

Calculating approximately:

$$
r \approx \sqrt[3]{7.9577} \approx 1.99 \text{ ft}
$$

Thus, the radius is approximately 1.99 feet, and the height:

$$
h = 2r \approx 3.98 \text{ ft}
$$

Sphere's Volume:

Using the radius:

$$
V_{sphere} = \frac{4}{3} \pi r^3
$$

$$
V_{sphere} = \frac{4}{3} \pi \times 7.9577 \approx \frac{4}{3} \pi \times 7.958
$$

Calculations:

$$
V_{sphere} \approx \frac{4}{3} \times 3.1416 \times 7.958 \approx 4.1888 \times 7.958 \approx 33.4 \text{ ft}^3
$$

Summary:

Parameter	Value
Radius r	approximately 1.99 ft
Height h	approximately 3.98 ft
Sphere Volume	approximately 33.4 ft^3

Alternative Assumption: Equal Radius and Height for Both Shapes

Suppose instead that the height is equal to the radius, meaning:

$$
h = r
$$

then from the cylinder volume:

$$
\pi r^2 h = 50
$$
$$
\pi r^2 \times r = 50
$$
$$
\pi r^3 = 50
$$
$$
r^3 = \frac{50}{\pi}
$$
$$
r = \sqrt[3]{\frac{50}{\pi}} \approx \sqrt[3]{15.915} \approx 2.52 \text{ ft}
$$
then:
$$
h = r \approx 2.52 \text{ ft}
$$

Sphere volume:
$$
V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times 3.1416 \times 15.915 \approx 4.1888 \times 15.915 \approx 66.9 \text{ ft}^3
$$

In this case, the sphere's volume exceeds the cylinder's volume, as expected, because the sphere with larger radius encloses more volume.

Impact of Shared Dimensions on Volume and Surface Area

Comparison of volumes:

- When the height equals twice the radius, the sphere’s volume is approximately 33.4 ft^3.
- When the height equals the radius, the sphere’s volume increases to approximately 66.9 ft^3.

This demonstrates that, for the same radius, increasing the height of the cylinder increases its volume, but the sphere's volume depends solely on its radius. The relationship between the two shapes’ volumes depends critically on the dimensions specified.

Surface area considerations:

- Sphere:
$$
A_{sphere} = 4 \pi r^2
$$
- Cylinder:
$$
A_{cylinder} = 2 \pi r^2 + 2 \pi r h
$$

Using the earlier approximate radius ($r \approx 1.99 \text{ ft}$) and height ($h \approx 3.98 \text{ ft}$):

- Sphere surface area:
$$
A_{sphere} \approx 4 \times 3.1416 \times (1.99)^2 \approx 4 \times 3.1416 \times 3.96 \approx 4 \times 12.45 \approx 49.8 \text{ ft}^2
$$
- Cylinder surface area:
$$
A_{cylinder} \approx 2 \times 3.1416 \times (1.99)^2 + 2 \times 3.1416 \times 1.99 \times 3.98
$$
$$
\approx 2 \times 3.1416 \times 3.96 + 2 \times 3.1416 \times 1.99 \times 3.98
$$
$$
\approx 2 \times 12.45 + 2 \times 3.1416 \times 1.99 \times 3.98
$$
$$
\approx 24.9 + 2 \times 3.1416 \times 7.92
$$
$$
\approx 24.9 + 2 \times 24.9 \approx 24.9 + 49.8 \approx 74.7 \text{ ft}^2
$$

Observation:

The sphere has a smaller surface area compared to the cylinder with the same radius and height, highlighting the efficiency of spheres in enclosing volume with minimal surface area.

Practical Applications and Real-World Significance