A Spy Satellite Is In Circular Orbit Around Earth. It Makes One Revolution In 5.98 Hours.(a) How High

A Spy Satellite Is In Circular Orbit Around Earth. It Makes One Revolution In 5.98 Hours.(a) How High

Understanding the precise altitude of a spy satellite orbiting Earth involves a fascinating combination of physics, mathematics, and space science. Satellites are vital tools for reconnaissance, communication, weather monitoring, and scientific research. When it comes to spy satellites, their orbital parameters determine their coverage, resolution, and operational effectiveness. In this article, we will explore how to determine the altitude of a satellite given its orbital period—specifically, when a satellite completes one revolution around Earth in 5.98 hours. We will delve into the physics principles involved, perform the necessary calculations, and discuss the implications of the satellite's orbit.

Basics of Orbital Mechanics

Before calculating the altitude, it's essential to understand the fundamentals of orbital mechanics. Satellites orbit Earth due to the balance between gravitational force and the satellite's inertia moving forward.

Newton's Law of Universal Gravitation

According to Newton, the gravitational force F between Earth and a satellite is given by:

$$
F = \frac{G M_e m}{r^2}
$$

where:

- G is the gravitational constant ($6.674 \times 10^{-11} \, \mathrm{Nm^2/kg^2}$),

- M_e is Earth’s mass ($5.972 \times 10^{24} \, \mathrm{kg}$),
- m is the satellite's mass,
- r is the distance from Earth’s center to the satellite.

Orbital Velocity and Period

For a satellite in a circular orbit:

- The gravitational force provides the required centripetal force:

$$
\frac{G M_e m}{r^2} = \frac{m v^2}{r}
$$

- Simplifying:

$$
v = \sqrt{\frac{G M_e}{r}}
$$

- The orbital period T (the time to complete one revolution) relates to the orbital radius r:

$$
T = \frac{2\pi r}{v}
$$

By combining these equations, we can derive the relationship between orbital radius and period.

Calculating the Orbital Radius from Period

Given:

- Orbital period $T = 5.98$ hours

First, convert hours to seconds:

$$
T = 5.98 \times 3600 = 21,528 \, \text{seconds}
$$

Using the orbital period equation:

$$
T = 2\pi r / v
$$

and substituting $v = \sqrt{\frac{G M_e}{r}}$, we get:

$$
T = 2 \pi r / \sqrt{\frac{G M_e}{r}} = 2 \pi r^{3/2} / \sqrt{G M_e}
$$

Rearranged for r:

$$
r^{3/2} = \frac{T \sqrt{G M_e}}{2 \pi}
$$

$$
r^{3/2} = C \times T
$$

where:

$$
C = \frac{\sqrt{G M_e}}{2 \pi}
$$

Calculating C:

$$
\sqrt{G M_e} = \sqrt{6.674 \times 10^{-11} \times 5.972 \times 10^{24}} \approx \sqrt{3.986 \times 10^{14}} \approx 1.996 \times 10^7
$$

Then,

$$
C = \frac{1.996 \times 10^7}{2 \pi} \approx \frac{1.996 \times 10^7}{6.283} \approx 3.177 \times 10^6
$$

Now, solve for r:

$$
r^{3/2} = 3.177 \times 10^6 \times 21,528 \approx 68.36 \times 10^9
$$

To find r:

$$
r = \left(68.36 \times 10^9\right)^{2/3}
$$

Calculating:

$$
r = \left(68.36 \times 10^9\right)^{2/3}
$$

Express:

$$
68.36 \times 10^9 = 6.836 \times 10^{10}
$$

Then:

$$
r = \left(6.836 \times 10^{10}\right)^{2/3}
$$

Recall that:

$$
(a \times 10^{b})^{2/3} = a^{2/3} \times 10^{(2b/3)}
$$

Compute $a^{2/3}$:

$$
6.836^{2/3} \approx e^{(2/3) \ln 6.836} \approx e^{(2/3) \times 1.922} \approx e^{1.281} \approx 3.6
$$

And:

$$
10^{(2 \times 10)/3} = 10^{20/3} \approx 10^{6.666} \approx 4.6 \times 10^{6}
$$

Therefore:

$$
r \approx 3.6 \times 4.6 \times 10^{6} \approx 16.56 \times 10^{6} \text{ meters}
$$

or approximately:

$$
r \approx 1.656 \times 10^7 \, \text{meters}
$$

This is the orbital radius from Earth's center.

Determining the Altitude of the Satellite

Earth’s average radius:

$$
R_e = 6,371 \, \text{km} = 6.371 \times 10^6 \, \text{meters}
$$

The altitude h of the satellite above Earth’s surface:

$$
h = r - R_e
$$

$$
h = 1.656 \times 10^7 - 6.371 \times 10^6 \approx 1.019 \times 10^7 \, \text{meters}
$$

which is approximately:

$$
h \approx 10,190 \, \text{km}
$$

Thus, the spy satellite is orbiting at an altitude of roughly 10,190 km above Earth’s surface.

Implications of the Satellite’s Orbit

Knowing the altitude of a satellite provides insights into its operational capabilities and mission profile.

Orbit Type and Coverage

- The calculated altitude indicates a high Earth orbit, well beyond Low Earth Orbit (LEO), which typically ranges from 160 km to 2,000 km.
- Such an altitude suggests a Medium Earth Orbit (MEO) or Geostationary Orbit (GEO), though it’s not exactly GEO because GEO satellites orbit at approximately 35,786 km altitude.
- Given the period of 5.98 hours, this orbit is faster than GEO but significantly higher than LEO, providing wide coverage and persistent observation.

Advantages for Spy Satellites

- Higher altitudes enable broad surveillance coverage, capturing large geographic areas in a single pass.
- It reduces the number of satellites needed for global coverage.
- Better resistance to atmospheric drag, enabling longer operational lifespans.

Conclusion and Final Remarks

Calculating the altitude of a spy satellite based on its orbital period involves applying principles of physics and orbital mechanics. With an orbital period of approximately 5.98 hours, the satellite is orbiting at an altitude of roughly 10,190 km above Earth’s surface. This high-altitude orbit allows for extensive surveillance capabilities, making it invaluable for intelligence gathering and national security.

Understanding satellite orbits is not only crucial for space scientists and engineers but also for policymakers and strategists involved in satellite deployment. As technology advances, the precision with which we can determine and manipulate satellite orbits continues to improve, enhancing our ability to monitor and understand our planet from space.

Remember: The calculations presented here assume a perfect circular orbit and neglect factors like Earth's oblateness, gravitational perturbations, and atmospheric drag, which can slightly influence real-world satellite orbits. Nonetheless, this provides a solid approximation of the satellite’s altitude based on its orbital period.

Frequently Asked Questions

What is the orbital period of the spy satellite in hours?

The satellite completes one revolution in 5.98 hours.

How can we determine the altitude of the satellite based on its orbital period?

Using Kepler's third law and the known Earth's radius, we can calculate the altitude from the orbital period by applying the formula for circular orbits.

What is the significance of the satellite's orbital period being 5.98 hours?

The 5.98-hour period indicates the satellite's speed and altitude, helping to understand its orbit and coverage area around Earth.

How does the orbital radius relate to the orbital period for a satellite around Earth?

The orbital radius increases with the orbital period; a longer period means a higher altitude, according to Kepler's third law.

What assumptions are made when calculating the satellite's altitude in this scenario?

Assumptions include a circular orbit, Earth's mass being concentrated at its center, and neglecting atmospheric drag and other perturbations.

What is the approximate altitude of the satellite if it completes a revolution in 5.98 hours?

By applying Kepler's third law, the altitude is approximately 300 km above Earth's surface, assuming Earth's radius is about 6371 km.

Why is understanding the orbital parameters important for spy satellites?

Knowing the orbital parameters helps optimize coverage, ensure stealth, and improve the effectiveness of surveillance missions.

Additional Resources

Spy Satellite in Circular Orbit: Calculating Its Altitude

Understanding the motion of spy satellites around the Earth involves applying fundamental principles of physics and orbital mechanics. When a satellite is placed in a circular orbit, it maintains a constant altitude and velocity, completing each revolution in a specific period. This analysis focuses on a spy satellite that completes one orbit every 5.98 hours and aims to determine its altitude – that is, how high above the Earth's surface it is orbiting.

Introduction to Orbital Mechanics and Spy Satellites

Earth's satellites, especially spy or reconnaissance satellites, play a crucial role in national security and intelligence gathering. These satellites are designed to orbit the Earth at specific altitudes and speeds, providing continuous coverage of targeted regions.

Key aspects of satellite orbits:
- Circular orbits: The satellite maintains a constant radius from Earth's center, meaning its altitude remains fixed.
- Orbital period (T): The time it takes for the satellite to complete one full revolution.
- Orbital velocity (v): The constant speed at which the satellite moves along its circular path.
- Altitude (h): The height of the satellite above Earth's surface, which is derived from the orbital radius.

Understanding the Given Data

The problem states:
- The satellite makes one revolution in 5.98 hours.
- The orbit is circular.
- The Earth's radius (R_e) is approximately 6371 km.
- The Earth's mass (M) is approximately 5.972 × 10^{24} kg.

- The universal gravitational constant (G) is 6.674×10^{-11} N·m²/kg².

Converting Orbital Period to SI Units

Since the period is given in hours, converting it to seconds simplifies calculations:

$$
T = 5.98 \text{ hours} \times 3600 \text{ seconds/hour} = 21528 \text{ seconds}
$$

Fundamental Principles of Orbital Motion

The motion of the satellite obeys Newton's law of gravitation and circular motion dynamics:

$$
\text{Centripetal force} = \text{Gravitational force}
$$

$$
\frac{m v^2}{r} = \frac{G M m}{r^2}
$$

Where:

- m is the mass of the satellite (which cancels out),
- v is the orbital velocity,

- r is the orbital radius (Earth's radius + altitude).

Rearranging for v:

$$
v = \sqrt{\frac{G M}{r}}
$$

The orbital period T relates to r and v:

$$
T = \frac{2 \pi r}{v}
$$

Substituting the expression for v:

$$
T = 2 \pi r \sqrt{\frac{r}{G M}} = 2 \pi \sqrt{\frac{r^3}{G M}}
$$

This is the key relationship used to find the orbital radius r:

$$
r^3 = \frac{G M T^2}{4 \pi^2}
$$

Calculating the Orbital Radius

Plugging in known values:

$$
r^3 = \frac{(6.674 \times 10^{-11}) \times (5.972 \times 10^{24}) \times (21528)^2}{4 \pi^2}
$$

Step-by-step:

1. Calculate $G M$:

$$
G M = 6.674 \times 10^{-11} \times 5.972 \times 10^{24} \approx 3.986 \times 10^{14} \text{ m}^3/\text{s}^2
$$

2. Square the period:

$$
T^2 = (21528)^2 \approx 4.636 \times 10^8 \text{ s}^2
$$

3. Compute numerator:

$$
G M T^2 = 3.986 \times 10^{14} \times 4.636 \times 10^8 \approx 1.849 \times 10^{23}
$$

4. Compute denominator:

$$
4 \pi^2 \approx 4 \times (3.1416)^2 \approx 39.478
$$

5. Calculate r^3:

$$
r^3 = \frac{1.849 \times 10^{23}}{39.478} \approx 4.684 \times 10^{21} \text{ m}^3
$$

6. Find r:

$$
r = \sqrt[3]{4.684 \times 10^{21}} \approx 1.67 \times 10^7 \text{ m}
$$

Determining the Satellite's Altitude (h)

Since r is the distance from Earth's center to the satellite:

$$
r = R_e + h
$$

where:

- R_e = Earth's radius = 6371 km = 6.371×10^6 m,
- h = satellite's altitude in meters.

Calculating h:

$$
h = r - R_e = (1.67 \times 10^7) - (6.371 \times 10^6) \approx 1.03 \times 10^7 \text{ m}
$$

Convert meters to kilometers:

$$
h \approx 10,300 \text{ km}
$$

Implications of the Satellite's Altitude

This altitude indicates that the spy satellite is orbiting approximately 10,300 km above Earth's surface. To put this into perspective:

- Low Earth Orbit (LEO): 160 km – 2,000 km
- Medium Earth Orbit (MEO): 2,000 km – 35,786 km
- Geostationary Orbit (GEO): approximately 35,786 km

The calculated altitude places the satellite well within the MEO range, which is typical for certain reconnaissance or communication satellites designed for broad coverage and high vantage points.

Understanding the Significance of the Orbit

Why is this altitude notable?

- Coverage and Resolution: Higher orbits enable a broader field of view, useful for surveillance of large regions.
- Orbital Period: At 5.98 hours, the satellite completes about 4 revolutions per day, offering frequent revisit times for high-priority targets.
- Orbital Stability: Circular orbits at this altitude are stable, allowing for consistent imaging and data collection.

Additional Considerations and Real-World Factors

While the above calculations are idealized, real-world scenarios involve complex factors:

- Atmospheric Drag: At higher altitudes, atmospheric drag is negligible, enhancing orbital stability.
- Orbital Decay: Not significant at this altitude, but still a factor over long durations.
- Inclination: The orbit's inclination (angle relative to Earth's equator) affects coverage patterns but doesn't influence altitude calculations directly.
- Orbital Perturbations: Gravitational influences from the Moon, Sun, and Earth's oblateness can slightly alter the orbit over time.

Summary and Final Thoughts

In summary, by applying fundamental physics principles, we determine that a spy satellite completing an orbit every 5.98 hours is approximately 10,300 km above Earth's surface. This high-altitude orbit balances broad coverage with the ability to revisit target areas frequently, making it suitable for reconnaissance purposes.

Key takeaways:

- The orbital radius r is about 16,700 km from Earth's center.
- The altitude h is approximately 10,300 km.
- The satellite's orbital period and altitude are interconnected through Kepler's third law, illustrating the elegant relationship between time and distance in celestial mechanics.

Understanding these calculations offers insight into how satellite missions are planned and optimized, emphasizing the fundamental role of physics in space technology.

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