

A Triangle Is 20 In Tall And 5 In Wide. If It Is Reduced To A Width Of 1 In Then How Tall Will It Be?

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Understanding how similar triangles work is essential when solving problems related to proportionality, especially in geometric contexts like this one. When a triangle is scaled down or up, its dimensions change proportionally, which means the ratios of corresponding sides remain constant. In this article, we will explore how to determine the height of a triangle after it has been scaled down in width, using the principles of similar triangles and proportional reasoning.

Introduction to Similar Triangles and Proportionality

What Are Similar Triangles?

Similar triangles are triangles that have the same shape but not necessarily the same size. They are characterized by:

- Equal corresponding angles
- Proportional corresponding sides

When two triangles are similar, their corresponding sides are in proportion, meaning the ratio of any two corresponding sides in one triangle is equal to the ratio of the corresponding sides in the other triangle.

The Concept of Proportionality

Proportionality refers to the equality of ratios. If two triangles are similar, then:

$$\frac{\text{Corresponding Side 1 in Triangle A}}{\text{Corresponding Side 1 in Triangle B}} = \frac{\text{Corresponding Side 2 in Triangle A}}{\text{Corresponding Side 2 in Triangle B}} = \dots$$

This principle allows us to find unknown dimensions when the scale factor between similar

figures is known.

Problem Breakdown: From the Given Dimensions

Let's analyze the problem carefully:

- Original triangle:
 - Height (H_1) = 20 inches
 - Width (W_1) = 5 inches
- Reduced triangle:
 - Width (W_2) = 1 inch
 - Height (H_2) = ?

Our goal is to determine the new height (H_2) when the width is scaled down to 1 inch.

Applying Proportionality to Find the Scaled Height

Step 1: Establish the Scale Factor

Since the triangles are similar (assuming the reduction is a uniform scale), the scale factor (k) for any side length is:

$$k = \frac{\text{New Width}}{\text{Original Width}} = \frac{W_2}{W_1}$$

Plugging in the values:

$$k = \frac{1}{5}$$

This means the new triangle is scaled down to one-fifth of the original width.

Step 2: Calculate the New Height

Because the triangle scales uniformly:

$$H_2 = H_1 \times k$$

Substituting the known values:

$$H_2 = 20 \times \frac{1}{5} = 4$$

Therefore, when the width is reduced to 1 inch, the height of the triangle becomes 4 inches.

Understanding the Concept of Scale Factors in Geometry

Why Do Dimensions Scale Uniformly?

In similar figures, all corresponding dimensions are scaled by the same factor. This property is fundamental in geometric problems involving resizing shapes while maintaining their proportions.

Implications of Proportionality

- The ratio of height to width remains constant:

$$\frac{H_1}{W_1} = \frac{H_2}{W_2}$$

- The scale factor applies equally to all dimensions, including heights, bases, sides, and other measurements.

Additional Examples and Applications of Similarity

Example 1: Scaling a Triangle

Suppose a triangle has a height of 15 inches and a base of 6 inches. If we want to reduce the base to 2 inches while maintaining similarity, what will be the new height?

- Original dimensions:
- Height: 15 inches
- Base: 6 inches
- New base: 2 inches

Calculate the scale factor:

$$k = \frac{2}{6} = \frac{1}{3}$$

New height:

$$H_{\text{new}} = 15 \times \frac{1}{3} = 5 \text{ inches}$$

Thus, the scaled-down triangle has a height of 5 inches.

Example 2: Enlarging a Triangle

If a triangle with a height of 10 inches and width of 4 inches is enlarged so that the width becomes 12 inches, what is the height?

- Original:
- Height: 10 inches
- Width: 4 inches
- New width: 12 inches

Calculate scale factor:

$$k = \frac{12}{4} = 3$$

New height:

$$H_{\text{new}} = 10 \times 3 = 30 \text{ inches}$$

This demonstrates how similar triangles can be scaled up or down proportionally.

Practical Applications of Similar Triangles and Proportions

Understanding the principles of similar triangles and proportionality is crucial in various fields and everyday situations, such as:

- Architecture and Engineering: Designing structures with scaled models
- Art and Design: Creating scaled drawings or sculptures
- Navigation and Astronomy: Calculating distances using similar triangles
- Photography: Understanding perspectives and scaling objects

Summary and Key Takeaways

- When a triangle is scaled proportionally, all dimensions change by the same scale factor.
- To find the new height after scaling the width, establish the scale factor and multiply the original height by this factor.
- In the given problem, reducing the width from 5 inches to 1 inch corresponds to a scale factor of $\frac{1}{5}$, leading to a new height of 4 inches.
- Recognizing the properties of similar triangles simplifies many geometric problems involving resizing.

Conclusion

By understanding the concept of similar triangles and proportionality, we can easily determine the dimensions of a scaled figure. In this specific case, reducing the width of a triangle from 5 inches to 1 inch results in the height decreasing from 20 inches to 4 inches. This reflects the fundamental principle that all linear dimensions of similar figures are scaled by the same factor—here, one-fifth.

Mastering these concepts enhances problem-solving skills in geometry, enabling you to analyze and solve various real-world problems involving scaling, measurement, and proportions. Whether you're a student preparing for exams or a professional applying geometry principles, understanding similar triangles is an invaluable skill in your mathematical toolkit.

Keywords: similar triangles, proportionality, scale factor, geometric scaling, triangle dimensions, problem-solving in geometry, proportional reasoning, scaled figures, triangle height calculation

Frequently Asked Questions

How do you find the height of a triangle when its width is scaled down proportionally?

You can use the concept of similar triangles; set up a proportion between the original and scaled dimensions to find the new height.

What is the height of the triangle if its width is reduced from 5 inches to 1 inch?

The height will be 4 inches, since the scale factor is $\frac{1}{5}$ (from 5 inches to 1 inch), so multiply the original height of 20 inches by $\frac{1}{5}$.

What is the scale factor when the width decreases from 5 inches to 1 inch?

The scale factor is $\frac{1}{5}$ because 1 divided by 5 equals $\frac{1}{5}$.

How do similar triangles help in solving this problem?

They allow us to set up a proportion between corresponding sides, enabling us to find the unknown height after scaling.

Is the relationship between the width and height proportional in this problem?

Yes, because the triangle is scaled uniformly, so the height and width change proportionally.

What is the final answer to the problem of the triangle's height when scaled to 1 inch wide?

The height will be 4 inches after scaling the width from 5 inches to 1 inch.

Additional Resources

Understanding the Problem: A Triangle's Scaling and Proportionality

When dealing with geometric figures, especially triangles, understanding the relationship

between their dimensions is crucial. The problem at hand involves a triangle that is originally 20 inches tall and 5 inches wide, and it is scaled down to a width of 1 inch. The primary question is: If the triangle is reduced to a width of 1 inch, how tall will it be? To answer this, we need to explore concepts of similar triangles, proportionality, and scaling.

Fundamentals of Similar Triangles and Proportionality

What Are Similar Triangles?

- Definition: Two triangles are similar if their corresponding angles are equal, and their corresponding sides are in proportion.
- Key Property: Corresponding sides of similar triangles are proportional, meaning the ratios of their lengths are equal.
- Implication: When a triangle is scaled uniformly (all sides scaled by the same factor), it remains similar to its original shape.

Understanding Proportionality

- When a figure is scaled uniformly:
- Each dimension (height, width, sides) is multiplied by the same scale factor.
- The ratios of corresponding sides remain constant.
- Example:
- Original triangle: height = 20 inches, width = 5 inches.
- Scaled triangle: width = 1 inch (given), height = ? (to be determined).

Applying Proportionality to the Given Triangle

Initial Dimensions and Scale Factor

- Original dimensions:
- Height (H_1) = 20 inches
- Width (W_1) = 5 inches
- Scaled dimensions:
- Width (W_2) = 1 inch
- Height (H_2) = ? (unknown)

Determining the Scale Factor

- To find the scale factor (k), compare the new width to the original:

$$k = \frac{\text{New Width}}{\text{Original Width}} = \frac{W_2}{W_1} = \frac{1}{5}$$

- Since the scaling is uniform, this same factor applies to the height:

$$H_2 = H_1 \times k$$

Calculating the New Height

- Substitute the known values:

$$H_2 = 20 \times \frac{1}{5} = 20 \times 0.2 = 4 \text{ inches}$$

- Result: When the width of the triangle is reduced from 5 inches to 1 inch, the height will be scaled down from 20 inches to 4 inches.

Visualizing the Scaling Process

Graphical Representation

- Visualize the original triangle:
 - Base (width) = 5 inches
 - Height = 20 inches
- After scaling:
 - Base = 1 inch
 - Height = 4 inches
- The shape remains similar; only the size changes proportionally.

Why Does This Work?

- Because the triangle's shape remains the same, the ratios of corresponding sides are constant.
- The ratio of the new width to the original width is applied directly to the height, maintaining similarity.

Additional Considerations and Applications

Scaling in Different Contexts

- Engineering & Architecture: When designing scaled models, proportions must be maintained for accuracy.
- Art & Design: Artists often use proportional scaling to create accurate representations.
- Mathematics Education: Understanding similar triangles helps in solving real-world problems involving scale and proportion.

Limitations and Assumptions in This Problem

- Uniform Scaling: Assumes the triangle is scaled uniformly in all directions.
- No Deformation: The shape remains a perfect triangle; no distortions occur.
- Constant Ratios: The proportionality principle holds true only under similar transformations.

Extensions and Related Problems

What If the Triangle Is Not Uniformly Scaled?

- If only the width is scaled, while the height remains unchanged, the triangle would no longer be similar to the original.
- The problem then involves more complex transformations, possibly non-proportional, requiring different calculations.

Scaling Other Dimensions

- Suppose the triangle's sides are scaled by different factors:
- How does this affect the angles and the shape?
- The shape would no longer be similar unless all sides are scaled uniformly.

Applying to Other Shapes

- Similar principles apply to rectangles, circles, and polygons.
- The key is the concept of similarity and proportionality.

Summary and Final Thoughts

- The problem illustrates a fundamental geometric principle: similar triangles scale proportionally.
- When a triangle's width is decreased from 5 inches to 1 inch, its height decreases proportionally from 20 inches to 4 inches.
- The scale factor is:

$$\text{Scale factor} = \frac{\text{New width}}{\text{Original width}} = \frac{1}{5}$$

- The resulting height:

$$\boxed{4 \text{ inches}}$$

- Understanding these core concepts is essential in various fields, including mathematics, engineering, art, and design.

Final Remarks

Grasping the relationship between dimensions in similar figures enables us to solve a wide array of real-world problems efficiently. Whether designing scaled models, analyzing architectural plans, or engaging in mathematical problem-solving, the principle of proportionality remains a fundamental tool. The problem of scaling a triangle from 20 inches tall and 5 inches wide to a width of 1 inch exemplifies this beautifully, reinforcing the importance of similarity and ratios in geometry.

In conclusion, the height of the scaled-down triangle, when its width is reduced to 1 inch, will be 4 inches. This straightforward yet powerful application of similar triangles underscores the elegance and utility of geometric principles in understanding and solving practical problems.

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