A Triangle Is Defined By The Three Points =(3,10), =(6,9), And =(5,2).A=(3,10), B=(6,9), And C=(5,2).

A Triangle Is Defined By The Three Points =(3,10), =(6,9), And =(5,2).A=(3,10), B=(6,9), And C=(5,2).

Understanding the properties and characteristics of triangles is fundamental in geometry. When we specify the vertices of a triangle with their coordinate points, we can analyze its shape, size, and other geometric features precisely. In this article, we delve into the specifics of a triangle defined by the points A(3,10), B(6,9), and C(5,2). We will explore how to analyze such a triangle step-by-step, including calculating side lengths, midpoints, area, perimeter, and more. Whether you're a student studying geometry or a professional applying these concepts, this comprehensive guide will enhance your understanding of coordinate geometry related to triangles.

Introduction to Coordinate Geometry and Triangles

Coordinate geometry, also known as analytic geometry, allows us to analyze geometric figures using algebraic equations and coordinate points. This approach simplifies the process of calculating distances, slopes, areas, and other properties of figures like triangles on the Cartesian plane.

A triangle is a polygon with three sides and three vertices. When the vertices are given as coordinate points, the triangle's properties can be precisely calculated using formulas derived from the coordinate plane.

Given Points and Their Significance

Let's restate the points defining our triangle:

- A(3, 10)
- B(6, 9)
- C(5, 2)

- Point A (3,10): Located relatively high on the y-axis, with an x-coordinate of 3.
- Point B (6,9): Slightly to the right and slightly lower than point A.
- Point C (5,2): Significantly lower on the y-axis, located somewhat between points A and B horizontally.

These points form a unique triangle with specific properties that can be uncovered through calculations.

Calculating the Lengths of the Sides

The first step is to determine the lengths of the sides AB, BC, and AC using the distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Side AB

$$
AB = \sqrt{(6 - 3)^2 + (9 - 10)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10} \approx 3.16
$$

Side BC

$$
BC = \sqrt{(5 - 6)^2 + (2 - 9)^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1 + 49} = \sqrt{50} \approx 7.07
$$

Side AC

$$
AC = \sqrt{(5 - 3)^2 + (2 - 10)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68} \approx 8.25
$$

Summary of side lengths:

Side	Length	Approximate Value
AB	$\sqrt{10}$	3.16
BC	$\sqrt{50}$	7.07
AC	$\sqrt{68}$	8.25

Determining the Triangle's Type

Using the side lengths, we can classify the triangle:

- By side lengths:
- Since all sides are of different lengths, the triangle is scalene.

- By angles:
- To determine if the triangle is acute, right, or obtuse, we compare the

squares of the side lengths.

Calculate the squares:

- $AB^2 = 10$
- $BC^2 = 50$
- $AC^2 = 68$

Check the Pythagorean theorem:

- Sum of squares of shorter sides:

$$
AB^2 + BC^2 = 10 + 50 = 60
$$

- Is this equal to AC^2?

$$
68 \neq 60
$$

- Since $68 > 60$, the angle opposite the longest side (AC) is obtuse.

Conclusion: The triangle ABC is scalene and obtuse.

Midpoints and Segment Analysis

Knowing the midpoints of sides can help in further analysis, such as finding the centroid or understanding the triangle's symmetry.

Midpoint of AB

$$
M_{AB} = \left(\frac{3 + 6}{2}, \frac{10 + 9}{2} \right) = (4.5, 9.5)
$$

Midpoint of BC

$$
M_{BC} = \left(\frac{6 + 5}{2}, \frac{9 + 2}{2} \right) = (5.5, 5.5)
$$

Midpoint of AC

$$
M_{AC} = \left(\frac{3 + 5}{2}, \frac{10 + 2}{2} \right) = (4, 6)
$$

These midpoints are useful for constructing medians and centroid calculations.

Calculating the Area of Triangle ABC

The area of a triangle given coordinates can be calculated using the shoelace formula:

$$
\text{Area} = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |
$$

Plugging in the points:

$$
\begin{aligned}
\text{Area} &= \frac{1}{2} | 3(9 - 2) + 6(2 - 10) + 5(10 - 9) | \\
&= \frac{1}{2} | 3 \times 7 + 6 \times (-8) + 5 \times 1 | \\
&= \frac{1}{2} | 21 - 48 + 5 | \\
&= \frac{1}{2} | -22 | = 11
\end{aligned}
$$

Therefore, the area of triangle ABC is 11 square units.

Perimeter of Triangle ABC

Adding up the side lengths:

$$
\text{Perimeter} = AB + BC + AC \approx 3.16 + 7.07 + 8.25 = 18.48
$$

The perimeter provides insight into the size of the triangle.

Centroid of Triangle ABC

The centroid is the point where the three medians intersect and can be calculated as the average of the vertices' coordinates:

$$
G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)
$$

$$
G = \left(\frac{3 + 6 + 5}{3}, \frac{10 + 9 + 2}{3} \right) = \left(\frac{14}{3}, \frac{21}{3} \right) = \left(4.\overline{6}, 7 \right)
$$

The centroid's coordinates are approximately (4.67, 7). It is the triangle's balancing point and divides medians in a 2:1 ratio.

Additional Geometric Properties

Orthocenter, Circumcenter, and Incenter

While calculating these centers precisely involves more advanced formulas and is beyond the scope of this article, understanding their approximate locations provides deeper insights into the triangle's geometry.

- Circumcenter: The point equidistant from all vertices, found at the intersection of the perpendicular bisectors of sides.
- Incenter: The intersection of angle bisectors, equidistant from all sides.
- Orthocenter: Intersection point of the altitudes.

These centers are critical in various geometric constructions and proofs.

Practical Applications of Triangle Analysis in Coordinate Geometry

Understanding how to analyze triangles with given coordinate points has numerous real-world applications:

- Engineering and Design: Precise measurements and layouts.
- Computer Graphics: Rendering objects based on coordinate points.
- Navigation and GIS: Calculating areas and distances between locations.
- Robotics: Path planning within specified boundaries.

Summary of Key Calculations

Property	Calculation	Result
Side AB	$\sqrt{(6-3)^2 + (9-10)^2}$	$\sqrt{10} \approx 3.16$
Side BC	$\sqrt{(5-6)^2 + (2-9)^2}$	$\sqrt{50} \approx 7.07$
Side AC	\(\sqrt{(5-3)^2 + (2-10	

Frequently Asked Questions

How do you find the area of the triangle with vertices A(3,10), B(6,9), and C(5,2)?

Use the shoelace formula: Area = 0.5 |x1(y2-y3) + x2(y3-y1) + x3(y1-y2)|. Substituting the points gives Area = 0.5 |3(9-2) + 6(2-10) + 5(10-9)| = 0.5 |37 + 6(-8) + 51| = 0.5 |21 - 48 + 5| = 0.5 |-22| = 11. So, the area is 11 square units.

Are the points A(3,10), B(6,9), and C(5,2) collinear?

To check collinearity, calculate the slope between pairs of points. Slope AB = $(9-10)/(6-3) = -1/3$, Slope AC = $(2-10)/(5-3) = -8/2 = -4$. Since slopes are different, the points are not collinear and form a triangle.

What is the length of side AB in the triangle with points A(3,10) and B(6,9)?

Use the distance formula: distance = $\sqrt{[(x2-x1)^2 + (y2-y1)^2]}$. So, AB = $\sqrt{[(6-3)^2 + (9-10)^2]} = \sqrt{[3^2 + (-1)^2]} = \sqrt{[9 + 1]} = \sqrt{10} \approx 3.16$ units.

How can you determine the type of triangle formed by points A(3,10), B(6,9), and C(5,2)?

Calculate the lengths of all sides using the distance formula. Then compare the lengths; if all sides are equal, it's equilateral; if two are equal, isosceles; if none are equal, scalene. Based on calculations, the sides are approximately AB ≈ 3.16, BC ≈ 7.07, and AC ≈ 8.06, so the triangle is scalene.

What is the centroid of the triangle with vertices A(3,10), B(6,9), and C(5,2)?

The centroid is the average of the x-coordinates and y-coordinates: $\bar{x} = (3+6+5)/3 = 14/3 \approx 4.67$, $\bar{y} = (10+9+2)/3 = 21/3 = 7$. Therefore, the centroid is approximately at (4.67, 7).

How can you find the equation of the line passing through points A(3,10) and C(5,2)?

First, find the slope: $m = (2-10)/(5-3) = -8/2 = -4$. Then, use point-slope form with point A: $y-10 = -4(x-3)$. Simplify to get $y = -4x + 12 + 10 = -4x + 22$. So, the line equation is $y = -4x + 22$.

Additional Resources

A Triangle Is Defined By The Three Points =(3,10), =(6,9), And =(5,2).A=(3,10), B=(6,9), And C=(5,2).

Understanding the fundamental properties of triangles formed by specific points in the coordinate plane is a cornerstone of geometry. Whether you're a student, educator, or enthusiast delving into the intricacies of coordinate geometry, analyzing the triangle with vertices at A(3, 10), B(6, 9), and C(5, 2) offers valuable insights into geometric relationships, calculations, and properties. This article explores the key aspects of this triangle, including how to determine its side lengths, classify its type, find its area, and analyze its other geometric features—all through a detailed, yet approachable, technical lens.

Setting the Stage: The Coordinates and Their Significance

The Coordinates of the Vertices

The triangle in question is defined by three points:

- Point A: (3, 10)
- Point B: (6, 9)
- Point C: (5, 2)

These points are plotted on the Cartesian coordinate plane, where each point's position is specified by its x (horizontal) and y (vertical) coordinates. The coordinates provide a precise location, allowing us to perform various calculations to understand the properties of the triangle.

Why Coordinates Matter

In coordinate geometry, the use of points enables us to:

- Calculate side lengths using the distance formula.
- Determine angles via slope and trigonometric relationships.
- Classify the triangle (scalene, isosceles, equilateral, right-angled, etc.)
- Compute the area using coordinate-based formulas.
- Analyze symmetry and other geometric properties.

This approach provides a systematic method to analyze geometric figures mathematically, moving beyond visual estimations.

Calculating Side Lengths: The Foundation of Triangle Analysis

The Distance Formula: A Quick Recap

The distance between any two points (x_1, y_1) and (x_2, y_2) in the plane is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

This formula is derived from the Pythagorean theorem, representing the length of the hypotenuse of a right triangle formed by the differences in x and y coordinates.

Applying the Distance Formula to Our Triangle

Let's compute the three sides:

- Side AB:

$$
AB = \sqrt{(6 - 3)^2 + (9 - 10)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10} \approx 3.16
$$

- Side BC:

$$
BC = \sqrt{(5 - 6)^2 + (2 - 9)^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1 + 49} = \sqrt{50} \approx 7.07
$$

- Side CA:

$$
CA = \sqrt{(5 - 3)^2 + (2 - 10)^2} = \sqrt{2^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68} \approx 8.25
$$

Summary of Side Lengths

Side	Length (approximate)
AB	3.16
BC	7.07
CA	8.25

The side lengths reveal that none of the sides are equal, suggesting the triangle is scalene. But further classification requires analyzing angles and other properties.

Classifying the Triangle: Types Based on Sides and Angles

Side-Based Classification

Since the side lengths are all distinct, the triangle is classified as scalene—no two sides are equal.

Angle-Based Classification

To determine if the triangle is right-angled, acute, or obtuse, we can use the Law of Cosines or analyze the dot products of vectors. The Law of Cosines relates side lengths to the angles:

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

where a, b, c are the side lengths, and C is the angle opposite side c.

Let's check whether any angle is exactly 90°, indicating a right triangle.

Determining Whether the Triangle is Right-Angled

Using the Dot Product of Vectors

The dot product method is efficient:

- Two vectors are perpendicular if their dot product is zero.
- For triangle ABC:
- $\vec{AB} = (x_B - x_A, y_B - y_A) = (6 - 3, 9 - 10) = (3, -1)$
- $\vec{AC} = (5 - 3, 2 - 10) = (2, -8)$
- $\vec{BC} = (5 - 6, 2 - 9) = (-1, -7)$

Check each pair:

- At A:

$$
\vec{AB} \cdot \vec{AC} = (3)(2) + (-1)(-8) = 6 + 8 = 14 \neq 0
$$

- At B:

$$
\vec{BA} = (3 - 6, 10 - 9) = (-3, 1)
$$
$$
\vec{BC} = (5 - 6, 2 - 9) = (-1, -7)
$$
$$
\vec{BA} \cdot \vec{BC} = (-3)(-1) + (1)(-7) = 3 - 7 = -4 \neq 0
$$

- At C:

$$
\vec{CA} = (3 - 5, 10 - 2) = (-2, 8)
$$
\[

$$
\vec{CB} = (6 - 5, 9 - 2) = (1, 7)
$$

$$
\vec{CA} \cdot \vec{CB} = (-2)(1) + (8)(7) = -2 + 56 = 54 \neq 0
$$

Since none of the dot products are zero, the triangle is not right-angled.

Using the Law of Cosines for Confirmation

Calculate the cosines of angles:

- Angle at A:

$$
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
$$

where:

- $a = BC \approx 7.07$
- $b = CA \approx 8.25$
- $c = AB \approx 3.16$

Plug in the values:

$$
\cos A = \frac{(8.25)^2 + (7.07)^2 - (3.16)^2}{2 \times 8.25 \times 7.07}
$$

Calculate numerator:

$$
68.06 + 50 + 10 = 128.06
$$

Calculate denominator:

$$
2 \times 8.25 \times 7.07 \approx 2 \times 58.4 \approx 116.8
$$

Then:

$$
\cos A \approx \frac{128.06}{116.8} \approx 1.095
$$

Since cosine cannot be greater than 1, this suggests a miscalculation—probably due to rounding. Let's be more precise with the side lengths:

- $AB = \sqrt{10} \approx 3.162$
- $BC = \sqrt{50} \approx 7.071$
- $CA = \sqrt{68} \approx 8.246$

Calculate numerator:

$$
(8.246)^2 + (7.071)^2 - (3.162)^2 = 68 + 50 + 10 = 128
$$

Calculate denominator:

$$
2 \times 8.246 \times 7.071 \approx 2 \times 58.33 \approx 116.66
$$

Thus,

$$
\cos A \approx \frac{128}{116.66} \approx 1.096
$$

Again, exceeding 1, indicating that the angle at A is obtuse (since cosine values above 1 are impossible, but small rounding errors are at play). Given the side lengths, the largest side is CA, opposite angle C, so most likely the triangle has an obtuse angle at C.

Conclusion: The triangle is scalene and obtuse-angled due to the relative side lengths, with the largest side being BC or CA.

Calculating the Area of the Triangle

Her

[A Triangle Is Defined By The Three Points 3 10 6 9 And 5 2 A 3 10 B 6 9 And C 5 2]

A Triangle Is Defined By The Three Points 3 10 6 9 And 5 2 A 3 10 B 6 9 And C 5 2

Related Articles

- Assume That A Simple Random Sample Has Been Selected From A Normally Distributed Population. Find The
- Adversaries Can Be Described As: A. People With Whom Negotiator Has Conflicting Goals And Objectives,
- A Lion With A Mass Of 190 Kg Is Chasing A Gazelle

With A Mass Of 15 Kg. The Distance Between The Lion

Back to Home