

An Axle Starts From Rest And Uniformly Increases Angular Speed To 0.21 Rev/s In 33 S. (a) What Is Its

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Understanding the Problem: Angular Motion Fundamentals

When analyzing rotational motion, it's essential to understand key concepts such as angular displacement, angular velocity, angular acceleration, and the relationships between these quantities. The problem at hand involves an axle that begins from rest and accelerates uniformly, reaching a specific angular velocity after a given duration.

In this context, the primary goal is to determine the angular displacement (the total angle rotated by the axle) during the period of acceleration. To do so, we need to understand the nature of uniformly accelerated rotational motion and how to calculate the relevant parameters.

Key Concepts in Rotational Kinematics

Angular Velocity (ω)

- Defined as the rate of change of angular displacement with respect to time.
- Measured in radians per second (rad/s).
- For uniform acceleration, ω changes linearly over time.

Angular Acceleration (α)

- The rate at which angular velocity changes.
- Measured in radians per second squared (rad/s²).
- For uniform angular acceleration, α is constant.

Angular Displacement (θ)

- The total angle rotated through, measured in radians.
- Related to initial and final angular velocities and acceleration.

Relationships Between Variables

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
- $\omega^2 = \omega_0^2 + 2 \alpha \theta$

where:

- ω_0 = initial angular velocity
- ω = final angular velocity
- t = time
- α = angular acceleration
- θ = angular displacement

Given Data and Objective

The problem provides the following data:

- Initial angular velocity, $\omega_0 = 0$ rad/s (since it starts from rest).
- Final angular velocity, $\omega = 0.21$ Rev/s.
- Time taken to reach this velocity, $t = 33$ seconds.

The primary task is to find the angular displacement θ during this acceleration period.

Converting Units: From Revolutions to Radians

Since angular velocity is given in revolutions per second, and standard calculations use radians, we need to convert:

- 1 revolution = 2π radians.

Thus:

$$\begin{aligned} \omega &= 0.21 \text{ Rev/s} \times 2\pi \text{ rad/Rev} = 0.21 \times 2\pi = 0.42\pi \\ &\approx 1.319 \text{ rad/s} \end{aligned}$$

This conversion is crucial for accurate calculations.

Calculating Angular Acceleration (α)

Using the kinematic equation for uniform angular acceleration:

$$\omega = \omega_0 + \alpha t$$

Given:

- $\omega_0 = 0$
- $\omega \approx 1.319$, rad/s
- $t = 33$, s

Rearranged to solve for α :

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{1.319 - 0}{33} \approx 0.040, \text{rad/s}^2$$

This value indicates the axle's angular acceleration during the period.

Calculating Angular Displacement (θ)

Using the second kinematic equation:

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Since $\omega_0 = 0$:

$$\theta = 0 + \frac{1}{2} \times 0.040 \times (33)^2$$

Calculations:

$$(33)^2 = 1089$$

$$\theta = 0.5 \times 0.040 \times 1089 = 0.02 \times 1089 = 21.78, \text{ radians}$$

Converting back to revolutions:

$$\text{Revolutions} = \frac{\theta}{2\pi} = \frac{21.78}{6.283} \approx 3.47, \text{ rev}$$

Answer: The axle rotates approximately 21.78 radians or 3.47 revolutions during the acceleration period.

Summary of Key Results

- Angular acceleration (α): approximately 0.040 rad/s².
- Total angular displacement (θ): approximately 21.78 radians or 3.47 revolutions.

Practical Applications and Significance

Understanding the angular displacement and acceleration of rotating components like axles is fundamental in various engineering and mechanical contexts. Proper analysis ensures:

1. **Design Safety:** Knowing how far an axle rotates during acceleration helps in designing systems that avoid over-rotation or mechanical failure.
2. **Efficiency Optimization:** Calculating acceleration parameters aids in optimizing motor performance and energy consumption.
3. **Control Systems:** Accurate motion profiling ensures precise control in robotics and automation applications.

Additional Considerations in Rotational

Kinematics

Constant vs. Variable Angular Acceleration

- The current problem assumes constant (uniform) acceleration, simplifying calculations.
- In real-world scenarios, acceleration may vary, requiring more complex analysis.

Angular Displacement in Different Contexts

- For example, if the axle were to decelerate or undergo oscillatory motion, different equations would apply.
- Understanding the initial conditions and boundary values is crucial for accurate modeling.

Units and Conversions

- Always convert units to SI base units (radians, seconds) for consistency.
- Be mindful of conversions, especially when dealing with revolutions, degrees, and radians.

Conclusion

Analyzing the angular motion of an axle that starts from rest and accelerates uniformly to a given angular velocity over a specified time involves applying fundamental kinematic equations. By converting units appropriately and methodically calculating angular acceleration and displacement, engineers and students can accurately determine the rotational behavior of mechanical components.

In this case, the axle's angular displacement during acceleration is approximately 21.78 radians, corresponding to about 3.47 revolutions, with an angular acceleration of about 0.040 rad/s^2 . Such insights are vital for designing mechanical systems, ensuring safety, and optimizing performance in various engineering applications.

FAQs

Q1: Why is it necessary to convert revolutions to radians?

A: Radians are the standard SI unit for angular measurements, enabling consistent and accurate calculation formulas derived from calculus.

Q2: What if the axle did not start from rest?

A: The initial angular velocity (ω_0) would be different, and the calculations for

displacement and acceleration would need to incorporate that value.

Q3: Can these calculations be applied to decelerating motion?

A: Yes. For deceleration, angular acceleration would be negative, and similar equations apply, with appropriate sign conventions.

Q4: How does knowing the angular displacement help in real-world applications?

A: It helps in designing systems that need specific rotation amounts, such as robotic arms, gear systems, or rotating machinery, ensuring precision and safety.

End of Content Piece

Frequently Asked Questions

What is the initial angular speed of the axle?

The initial angular speed of the axle is 0 rev/s (since it starts from rest).

What is the final angular speed of the axle after 33 seconds?

The final angular speed is 0.21 revolutions per second (rev/s).

How do you calculate the angular acceleration of the axle?

Angular acceleration can be calculated using the formula: $\alpha = (\omega_{\text{final}} - \omega_{\text{initial}}) / \text{time}$. Here, $\alpha = (0.21 \text{ rev/s} - 0) / 33 \text{ s} \approx 0.00636 \text{ rev/s}^2$.

What is the significance of uniform increase in angular speed?

A uniform increase indicates constant angular acceleration, meaning the axle accelerates at a steady rate throughout the time interval.

How is angular displacement calculated in this scenario?

Angular displacement (θ) can be found using $\theta = \omega_{\text{initial}} t + 0.5 \alpha t^2$. Since it starts from rest, $\theta = 0.5 \alpha t^2$.

Convert the final angular speed from rev/s to rad/s.

Since 1 revolution = 2π radians, $0.21 \text{ rev/s} = 0.21 \cdot 2\pi \approx 1.32 \text{ rad/s}$.

What is the total angular displacement during the 33 seconds?

Using $\theta = 0.5 \alpha t^2$, with $\alpha \approx 0.00636 \text{ rev/s}^2$, θ in revolutions = $0.5 \cdot 0.21 \cdot 33 \approx 3.465$ revolutions. In radians, $\theta \approx 3.465 \cdot 2\pi \approx 21.75$ radians.

How does the concept of angular kinematics apply to this problem?

It applies through the equations of rotational motion for constant angular acceleration, allowing us to determine angular speed, acceleration, and displacement over time.

Additional Resources

Angular Acceleration: Unveiling the Dynamics of an Accelerating Axle

Introduction: Understanding the Dynamics of Rotational Motion

In the realm of mechanical engineering and physics, rotational motion forms a fundamental aspect of numerous practical applications—from the spinning wheels of a vehicle to the intricate gears within machinery. Central to analyzing such motion is the concept of angular acceleration—a measure of how quickly an object's rotational speed changes over time.

Imagine a scenario where an axle starts from rest and gradually gains speed, reaching a specific rotational velocity after a set period. Such a situation not only exemplifies fundamental principles of rotational kinematics but also provides insights into designing efficient mechanical systems.

In this comprehensive review, we delve into the problem: An axle starts from rest and uniformly increases its angular speed to 0.21 revolutions per second in 33 seconds. Our goal is to determine its angular acceleration and understand the underlying physics governing this motion.

Foundational Concepts in Rotational Kinematics

Before tackling the problem, it's essential to familiarize ourselves with key concepts:

Angular Displacement (θ)

The total angle traversed by the axle during its rotation, measured in radians or revolutions.

Angular Velocity (ω)

The rate of change of angular displacement, indicating how fast the axle rotates. Typically expressed in radians per second (rad/s) or revolutions per second (rev/s).

Angular Acceleration (α)

The rate at which the angular velocity changes over time, measured in radians per second squared (rad/s²).

Uniform Angular Acceleration

A scenario where the angular acceleration remains constant throughout the motion, simplifying calculations and modeling real-world systems like motors accelerating or decelerating smoothly.

Problem Breakdown: Key Data and Conversions

Let's first restate the problem with clarity:

- Initial angular velocity (ω_0): 0 rev/s (since starting from rest)
- Final angular velocity (ω): 0.21 rev/s
- Time taken (t): 33 seconds
- Type of acceleration: Uniform (constant)

Important: To perform calculations, we need to convert units consistently, especially from revolutions per second to radians per second, since SI units are in radians.

Unit Conversion: Revolutions per Second to Radians per Second

Recall:

- 1 revolution = 2π radians

Therefore:

$\frac{1 \text{ rev}}{2\pi \text{ rad}}$

$\omega = 0.21 \text{ rev/s} = 0.21 \times 2\pi \text{ rad/s} \approx 0.21 \times 6.2832 \approx 1.319 \text{ rad/s}$

Similarly, initial angular velocity:

$\omega_0 = 0 \text{ rev/s} = 0 \text{ rad/s}$

Calculating Angular Acceleration

Given the data, the problem aligns with the kinematic equations for rotational motion under constant acceleration.

The Relevant Equation:

$$\omega = \omega_0 + \alpha t$$

where:

- ω = final angular velocity in rad/s
- ω_0 = initial angular velocity in rad/s
- α = angular acceleration in rad/s^2
- t = time in seconds

Step-by-Step Solution:

1. Identify Known Values:

- $\omega_0 = 0 \text{ rad/s}$
- $\omega = 1.319 \text{ rad/s}$
- $t = 33 \text{ s}$

2. Apply the Equation:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{1.319 - 0}{33} \approx 0.040 \text{ rad/s}^2$$

Result: The axle's angular acceleration is approximately 0.040 rad/s^2 .

Interpreting the Angular Acceleration

This value indicates a moderate, steady increase in rotational speed. To appreciate its significance, consider the following:

Practical Implications:

- Design Considerations: Mechanical systems that accelerate at this rate can smoothly ramp up speed without causing excessive stress or vibrations.
- Energy Transfer: The angular acceleration directly impacts the work done on the axle, which can be analyzed further to determine the energy involved.

Visualizing the Motion:

- Starting from rest, the axle gains approximately 0.040 rad/s every second.
- After 33 seconds, it reaches 1.319 rad/s (equivalent to 0.21 rev/sec).

This gradual increase ensures controlled acceleration, vital in applications like conveyor belts, rotating machinery, or robotics where sudden starts could cause mechanical failure.

Additional Analytical Perspectives

While the primary calculation provides the angular acceleration, further insights can be obtained through supplementary analyses:

1. Angular Displacement (θ)

Using the equation:

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Since $\omega_0 = 0$:

$$\theta = \frac{1}{2} \times 0.040 \times (33)^2 \approx 0.020 \times 1089 \approx 21.78 \text{ rad}$$

Converting back to revolutions:

$$\text{Revolutions} = \frac{21.78}{2\pi} \approx \frac{21.78}{6.2832} \approx 3.47 \text{ rev}$$

\]

The axle rotates approximately 3.47 revolutions during this acceleration phase.

2. Angular Displacement in Revolutions

This can be useful in practical scenarios, like determining how far a gear or wheel rotates over a period of acceleration.

Real-World Applications and Significance

Understanding and calculating angular acceleration aren't purely academic—they're crucial in multiple engineering domains:

- Motor Control: Precise control of acceleration profiles ensures longevity and safety.
- Robotics: Smooth acceleration prevents mechanical shocks.
- Vehicle Dynamics: Accelerating wheels and axles must be managed to optimize performance.
- Manufacturing Equipment: Gears and rotating parts often accelerate uniformly to prevent wear and tear.

By mastering the calculation of angular acceleration, engineers can design systems that are both efficient and durable.

Conclusion: The Power of Rotational Kinematics

Analyzing the scenario of an axle starting from rest and reaching a specific angular velocity in a set time reveals key insights into rotational dynamics. The core calculation — determining the angular acceleration — not only reinforces fundamental physics principles but also underscores their practical relevance.

In this case, the axle's angular acceleration of approximately 0.040 rad/s^2 represents a controlled, uniform increase in rotational speed, crucial for ensuring mechanical stability and operational efficiency in real-world applications.

Understanding these fundamental concepts allows engineers and physicists to design better systems, optimize performance, and innovate with confidence, harnessing the subtle yet powerful principles of rotational motion.

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