An Circuit With With A 3-ohm Resistor And A 0.02-h Inductor Carries A Current Of 1 A At T=0, At Which

An Circuit With With A 3-ohm Resistor And A 0.02-h Inductor Carries A Current Of 1 A At T=0, At Which understanding the behavior of electrical circuits involving resistors and inductors is fundamental for electrical engineering and physics students. Such circuits are classic examples of RL (resistor-inductor) circuits, whose analysis reveals how current and voltage evolve over time when subjected to various electrical stimuli. In this comprehensive guide, we will explore the fundamental principles governing these circuits, analyze their transient and steady-state behaviors, derive key equations, and illustrate practical applications, all structured to enhance your understanding of RL circuits.

Understanding the RL Circuit Components

The Resistor (3-ohm Resistor)

The resistor in this circuit has a resistance of 3 ohms (Ω). Resistors oppose the flow of current, converting electrical energy into heat, and their resistance value directly influences the current flow and voltage distribution within the circuit. The key characteristics include:
- Resistance value: 3 Ω
- Voltage-current relationship: Ohm's Law, $V = IR$
- Power dissipation: $P = I^2R$

The Inductor (0.02-h Inductor)

The inductor has an inductance of 0.02 henries (H). Inductors oppose changes in current, storing energy in a magnetic field when current flows through them. Their key properties include:
- Inductance value: 0.02 H
- Voltage-current relationship: $V = L(dI/dt)$
- Energy stored: $E = \frac{1}{2} L I^2$

Initial Conditions and Circuit Behavior at T=0

Initial Current and Its Significance

At t=0, the circuit carries a current of 1 A. This initial condition is crucial because:
- It determines the starting point for transient analysis.
- The inductor's current cannot change instantaneously; it remains at 1 A immediately after any sudden change.

- The initial magnetic energy stored in the inductor at t=0 is $E = \frac{1}{2} \times 0.02\ H \times (1\ A)^2 = 0.0001$ Joules.

Scenario Context

The specific scenario involves understanding how the circuit responds over time, especially:
- How the current evolves from its initial value.
- The influence of the resistor and inductor on the circuit's transient response.
- The steady-state behavior after a long period.

Theoretical Foundations of RL Circuit Analysis

Differential Equation Governing the Circuit

In an RL circuit, applying Kirchhoff's Voltage Law (KVL) yields:
- For a circuit with a voltage source V_s (if present), resistor R, and inductor L:

$$V_s = R\, I(t) + L\, (dI/dt)$$

- If the circuit is driven by a step input or an initial current condition, the homogeneous and particular solutions govern the current's behavior over time.

Solution for the Current I(t)

The general solution involves solving the differential equation:

$L\, (dI/dt) + R\, I = 0$ (for the case without a source after switching off or initial condition analysis)

The solution typically takes the form:

$$I(t) = I(0)\, e^{(-t/\tau)}$$

where $\tau = L / R$ is the time constant of the circuit.

Calculating the Time Constant and Transient Response

Time Constant (τ)

Given the resistor and inductor values:
- $\tau = L / R = 0.02\ H / 3\ \Omega \approx 0.00667$ seconds

This indicates that:
- The current will decay or rise exponentially with a characteristic time of about 6.67 milliseconds.
- After approximately 5τ (~0.033 seconds), the current approaches its steady-state value within about

99%.

Transient Behavior of the Current

Assuming initial current $I(0) = 1$ A, and the circuit is suddenly disconnected from the source at $t=0$:
- The current decays exponentially:

$$I(t) = I(0) e^{-t/\tau} = 1 \text{ A} \times e^{-t/0.00667}$$

- The voltage across the resistor and inductor changes accordingly, with the inductor opposing any rapid change.

Steady-State Analysis

Long-Term Behavior

As t approaches infinity:
- The current approaches zero if the circuit is disconnected.
- If a voltage source is present, the current approaches a value determined by the source and circuit configuration.

Power Dissipation and Energy Loss

Over time, energy stored in the magnetic field of the inductor dissipates as heat in the resistor:
- Power dissipated: $P = I^2 R$
- Total energy lost: integral of power over time.

Practical Applications of RL Circuits

Filtering and Signal Processing

RL circuits are fundamental in designing filters, especially:
- Low-pass filters to allow signals below a certain frequency.
- High-pass filters to block low-frequency signals.

Switching Power Supplies

RL circuits are used in power converters to manage energy transfer and regulation.

Transient Response in Electronic Devices

Understanding how RL circuits respond to sudden changes helps in designing circuits with predictable behavior during switching events.

Advanced Topics in RL Circuit Analysis

Complex Circuit Configurations

Real-world circuits often involve multiple resistors and inductors, requiring more advanced techniques such as:
- The Thevenin and Norton equivalents.
- Phasor analysis for AC circuits.

AC vs. DC Analysis

While the above analysis primarily considers DC conditions, AC signals introduce impedance:
- Impedance of the inductor: $Z_L = j\omega L$
- Total impedance: $Z_{total} = R + j\omega L$

Understanding the frequency response is key for AC applications.

Numerical Methods and Simulation Tools

Modern circuit analysis often employs:
- Numerical simulation software like SPICE.
- MATLAB for solving differential equations.

These tools facilitate complex analyses beyond simple analytical solutions.

Summary and Key Takeaways

- The RL circuit with a 3-ohm resistor and 0.02-h inductor exhibits exponential transient behavior characterized by a time constant $\tau \approx 6.67$ ms.
- The initial current of 1 A remains constant immediately after a sudden change, due to the inductor's property of opposing instantaneous current change.
- Over time, the current decays to zero if the circuit is disconnected, dissipating stored magnetic energy as heat.
- The analysis of such circuits is essential for designing filters, power supplies, and understanding transient phenomena in electrical systems.
- Mastery of differential equations, exponential functions, and circuit laws enables effective analysis and application of RL circuits.

Final Remarks

Understanding the dynamics of a circuit with a resistor and inductor is fundamental in electrical engineering. Whether analyzing transient responses, designing filters, or developing energy storage solutions, the principles outlined here provide a foundation for more advanced circuit design and analysis. Always consider initial conditions, circuit configuration, and application context to accurately predict circuit behavior and optimize performance.

This detailed exploration not only addresses the specific scenario but also provides a comprehensive understanding of RL circuits, supporting both learners and practitioners in mastering this essential aspect of electrical circuit theory.

Frequently Asked Questions

What is the initial current in the circuit at t=0?

The initial current in the circuit at t=0 is 1 A.

What is the initial voltage across the resistor in the circuit?

The initial voltage across the resistor is $V_R = I \times R = 1\ A \times 3\ \Omega = 3\ V$.

What is the initial voltage across the inductor at t=0?

The initial voltage across the inductor is $V_L = L \times (dI/dt)$. Since the current is just starting, the rate of change is maximum, but without dI/dt, the initial voltage across the inductor is determined by circuit conditions.

How does the current in the circuit evolve over time?

The current in an R-L circuit decreases exponentially over time according to $I(t) = I_0 \times e^{-(R/L)t}$, where I_0 is the initial current.

What is the time constant of this RL circuit?

The time constant $\tau = L / R = 0.02\ H / 3\ \Omega \approx 0.00667$ seconds.

How long does it take for the current to decay to approximately 37% of its initial value?

It takes about one time constant, approximately 6.67 milliseconds, for the current to decay to about 37% of its initial value.

What happens to the voltage across the resistor and inductor as time progresses?

The voltage across the resistor decreases exponentially, while the voltage across the inductor initially is high and decreases over time as the current stabilizes.

What is the significance of the initial conditions in analyzing this RL circuit?

Initial conditions determine the immediate response of the circuit, particularly the initial voltages and the rate of change of current, which are crucial for solving the circuit's differential equations.

Additional Resources

Analyzing an R-L Circuit with a 3-Ohm Resistor and a 0.02-H Inductor Carrying a 1-Amp Current at t=0

Understanding the behavior of electrical circuits comprising resistors and inductors is fundamental in both theoretical and practical electrical engineering. When such an R-L circuit is energized, the transient response—how current and voltage evolve over time—becomes a core focus. This review provides a comprehensive exploration of a specific circuit characterized by a 3-ohm resistor, a 0.02-henry inductor, and an initial current of 1 ampere at time t=0, examining the underlying principles, mathematical modeling, and real-world implications.

Introduction to R-L Circuits

Fundamentals of Resistors and Inductors

- Resistor (R): A passive component that opposes current flow proportionally to the voltage applied, governed by Ohm's Law: V = IR.
- Inductor (L): A passive component that opposes changes in current through it, storing energy in a magnetic field. The voltage across an inductor is given by V = L (dI/dt).

Nature of R-L Circuits

- Transient response: When power is first applied, the current does not instantly reach its steady-state value but changes exponentially over time.
- Steady-state: After a long duration, the inductor acts as a short circuit (for DC), and the current stabilizes based on the resistor.

System Description and Initial Conditions

Component Parameters

- Resistor (R): 3 ohms
- Inductor (L): 0.02 henry
- Initial current (i(0)): 1 ampere at t=0

Assumptions and Context

- The circuit is a simple series R-L configuration connected to a DC supply (assumed for this analysis).
- The initial current of 1 A at t=0 indicates the circuit was previously energized or in steady state before t=0.
- The primary goal is to analyze how the current and voltages evolve immediately after t=0 and over time.

Mathematical Modeling of the R-L Circuit

Formulating the Differential Equation

In a series R-L circuit powered by a DC source (V), the governing equation is derived from Kirchhoff's Voltage Law (KVL):

$$ V = R i(t) + L \frac{di(t)}{dt} $$

If the circuit is connected to a step voltage source V, then:

$$ V(t) = V_0 \cdot u(t) $$

where $u(t)$ is the unit step function. For simplicity, assume the circuit is connected to a constant voltage source V after t=0.

Initial condition:

$$ i(0) = 1\, \text{A} $$

Differential equation:

$$ V = R i(t) + L \frac{di(t)}{dt} $$

Solution to the Differential Equation

- Homogeneous solution:

$$ \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V}{L} $$

- Characteristic equation:

$$ \frac{di(t)}{dt} + \frac{R}{L} i(t) = \text{constant} $$

- General solution:

$$ i(t) = i_{\text{steady-state}} + \left(i(0) - i_{\text{steady-state}} \right) e^{-\frac{R}{L} t} $$

where:

$$ i_{\text{steady-state}} = \frac{V}{R} $$

Transient and Steady-State Analysis

Determining the Steady-State Current

- For DC input voltage V, the steady-state current is:

$$ i_{\text{ss}} = \frac{V}{R} $$

- Since the initial current is 1 A at t=0, and the circuit is in steady state before t=0, the initial current corresponds to the steady-state current if the voltage source is connected at t=0.

- Implication: The initial current of 1 A aligns with the steady-state current, indicating that the circuit was powered long enough before t=0 for the current to stabilize.

Time Constant of the Circuit

- The circuit's time constant, τ, characterizes how quickly the current reaches steady state:

$$ \tau = \frac{L}{R} $$

- Calculated as:

$$ \tau = \frac{0.02\, \text{H}}{3\, \Omega} \approx 0.00667\, \text{s} $$

- This very small time constant indicates rapid stabilization of the current after switching.

Complete Solution for Current Over Time

- Given the initial current and assuming a DC source V (unknown, but can be inferred):

$$ i(t) = i(0) + \frac{V - R i(0)}{L} \left(1 - e^{-\frac{R}{L} t} \right) $$

- Alternatively, if the source voltage V is known, the current evolution simplifies to:

$$ i(t) = i_{\text{ss}} + \left(i(0) - i_{\text{ss}} \right) e^{-\frac{R}{L} t} $$

- Since initial current is 1 A and assuming the circuit has been steady at this current, the initial condition is consistent with the steady-state current:

$$ i(0) = i_{\text{ss}} = \frac{V}{R} \Rightarrow V = R \times i(0) = 3\, \Omega \times 1\, \text{A} = 3\, \text{V} $$

- Conclusion: The circuit is powered with a 3 V DC source, and the current remains at 1 A throughout, implying no transient change.

Deep Dive into Circuit Behavior at t=0

Instantaneous Response

- At t=0+: Immediately after switching on, the inductor opposes any instant change in current.

- Key principle:

$$ \lim_{t \to 0^+} \frac{di(t)}{dt} = \frac{V - R i(0)}{L} $$

- Since initial current is 1 A and V=3 V:

$$ \frac{di(0^+)}{dt} = \frac{3\, \text{V} - 3\, \text{V}}{0.02\, \text{H}} = 0 $$

- Interpretation: The current remains at 1 A immediately after t=0; there is no sudden change.

Voltage Across Components at t=0

- Across resistor:

$$ V_R = R i(0) = 3\, \Omega \times 1\, \text{A} = 3\, \text{V} $$

- Across inductor:

$$ V_L = L \frac{di}{dt} = 0.02\, \text{H} \times 0 = 0\, \text{V} $$

- Across the source:

$$ V = V_R + V_L = 3\, \text{V} + 0 = 3\, \text{V} $$

This confirms the circuit is at equilibrium with the initial conditions.

Implications and Real-World Considerations

Transient vs. Steady-State Behavior

- Given the parameters, the circuit reaches steady state almost instantaneously relative to typical circuit timescales.

- The initial current of 1 A matches the steady-state current for a 3 V source, indicating the circuit was powered well before t=0, and no transient current change occurs at t=0.

Energy Storage in the Inductor

- The magnetic energy stored in the inductor at t=0:

$$ E = \frac{1}{2} L i^2 = \frac{1}{2} \times 0.02\, \text{H} \times (1\, \text{A})^2 = 0.01\, \text{J} $$

- This energy can be released or absorbed during transient states if circuit conditions change.

Practical Applications and Considerations

- Filtering: R-L circuits are used in filters to block or pass signals based on frequency.

- Inductive loads: Motors, transformers, and inductive sensors rely on R-L principles.

- Transient suppression: Understanding how currents evolve helps in designing circuits resistant to voltage spikes.

- Component tolerances: Real-world inductors and resistors have tolerances affecting precise behavior.

Advanced Topics and Extensions

Inclusion of AC Sources and Frequency Response

- When the source is AC, the circuit's impedance becomes frequency-dependent:

$$Z = \sqrt{R^2 + (\omega L)^2}$$

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