An Electron Confined In A One-dimensional Box Emits A 200nm Photon In A Quantum Jump From N=4 To N=3.

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Understanding the quantum behavior of electrons in confined systems is fundamental to modern physics and nanotechnology. When an electron is trapped within a potential well—such as a one-dimensional box—it exhibits discrete energy levels, and transitions between these levels can result in the emission or absorption of photons. A particularly intriguing example is when an electron transitions from the fourth energy level (N=4) to the third (N=3), emitting a photon with a wavelength of approximately 200 nanometers (nm). This phenomenon not only exemplifies quantum jumps but also illustrates key principles underlying quantum mechanics, spectral emissions, and the design of nanoscale devices.

Fundamentals of the One-dimensional Particle-in-a-Box Model

Quantum Confinement and Discrete Energy Levels

In quantum mechanics, confining an electron within a finite region of space leads to quantization of its energy. The simplest model to illustrate this is the "particle-in-a-box" or "infinite potential well," where the electron is confined between two impenetrable walls separated by a length L.

The key assumptions of this model include:
- Infinite potential barriers at the boundaries, preventing the electron from escaping.
- The electron is free within the box, with zero potential energy inside.
- The wavefunction must satisfy boundary conditions, being zero at the walls.

The energy levels for an electron in a 1D box are given by:

$$
E_n = \frac{n^2 h^2}{8 m L^2}
$$

where:
- n is the principal quantum number (1, 2, 3, ...),
- h is Planck's constant ($6.626 \times 10^{-34} \, \text{Js}$),
- m is the mass of the electron ($9.109 \times 10^{-31} \, \text{kg}$),
- L is the length of the box.

This quantization results in discrete energy levels, which are fundamental to understanding emission spectra in confined quantum systems.

Transition Between Energy Levels and Photon Emission

When an electron transitions from a higher energy state N_i to a lower energy state N_f, it releases energy in the form of a photon. The energy of this photon corresponds to the difference between the two levels:

$$
E_{photon} = E_{N_i} - E_{N_f}
$$

The wavelength λ of the emitted photon is related to its energy via the Planck-Einstein relation:

$$
E_{photon} = \frac{hc}{\lambda}
$$

where c is the speed of light ($3.0 \times 10^8 \, \text{m/s}$).

Calculating the Wavelength of the Emitted Photon for the N=4 to N=3 Transition

Given that an electron transitions from $N=4$ to $N=3$, emitting a photon with a wavelength of approximately 200 nm, we can analyze how this emission aligns with the particle-in-a-box model and what it reveals about the system.

Step 1: Expressing the Energy Difference

The energy difference between levels $N=4$ and $N=3$:

$$
\Delta E = E_4 - E_3 = \frac{h^2}{8 m L^2} (4^2 - 3^2) = \frac{h^2}{8 m L^2} (16 - 9) = \frac{7 h^2}{8 m L^2}
$$

Step 2: Relate to Wavelength

The photon emitted has energy:

$$
E_{photon} = \frac{hc}{\lambda}
$$

Set equal to the energy difference:

$$
\frac{hc}{\lambda} = \frac{7 h^2}{8 m L^2}
$$

Solve for λ:

$$
\lambda = \frac{8 m c L^2}{7 h}
$$

This equation links the physical size of the box L to the observed emission wavelength.

Step 3: Estimating the Box Length L

Using the given wavelength $\lambda = 200 \, \text{nm} = 200 \times 10^{-9} \, \text{m}$, and known constants:

$$
L = \sqrt{\frac{7 h \lambda}{8 m c}}
$$

Plugging in the numbers:

$$
L = \sqrt{\frac{7 \times 6.626 \times 10^{-34} \, \text{Js} \times 200 \times 10^{-9} \, \text{m}}{8 \times 9.109 \times 10^{-31} \, \text{kg} \times 3 \times 10^8 \, \text{m/s}}}
$$

Calculating numerator:

$$
7 \times 6.626 \times 10^{-34} \times 200 \times 10^{-9} = 7 \times 6.626 \times 200 \times 10^{-34 -9} = 7 \times 6.626 \times 200 \times 10^{-43}
$$
$$
= 7 \times 6.626 \times 200 \times 10^{-43}
$$

Calculate:

\[

$$
6.626 \times 200 = 1325.2
$$
$$
7 \times 1325.2 = 9276.4
$$

So numerator:

$$
9276.4 \times 10^{-43} = 9.2764 \times 10^{-40}
$$

Denominator:

$$
8 \times 9.109 \times 10^{-31} \times 3 \times 10^8 = 8 \times 9.109 \times 3 \times 10^{-31 + 8} = 8 \times 9.109 \times 3 \times 10^{-23}
$$
$$
9.109 \times 3 = 27.327
$$
$$
8 \times 27.327 = 218.616
$$
$$
\text{Denominator} = 218.616 \times 10^{-23} = 2.18616 \times 10^{-21}
$$

Now, compute L:

$$
L = \sqrt{\frac{9.2764 \times 10^{-40}}{2.18616 \times 10^{-21}}} = \sqrt{4.242 \times 10^{-19}} \approx 2.06 \times 10^{-10} \, \text{m}
$$

This length corresponds roughly to:

$$
L \approx 0.206 \, \text{nm}
$$

which is on the order of atomic dimensions, indicating that the electron is confined within an extremely small region, consistent with quantum dots or nanoscale systems.

Implications of the Emission at 200nm

Spectral Region and Applications

The photon wavelength of approximately 200 nm places this emission in the ultraviolet (UV) region of the electromagnetic spectrum. UV photons are energetic and have significant applications, including:

- Sterilization and disinfection: UV light can destroy bacteria and viruses.
- Photolithography: Used in semiconductor manufacturing for high-resolution patterning.
- Spectroscopy: UV emission lines help analyze atomic and molecular structures.
- Biological imaging: UV fluorescence is used in microscopy techniques.

Understanding the Energy Scale

The energy of a 200 nm photon:

$$
E_{photon} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3.0 \times 10^{8}}{200 \times 10^{-9}} \approx 9.93 \times 10^{-19} \, \text{J}
$$

Expressed in electron volts (eV):

$$
1 \, \text{eV} = 1.602 \times 10^{-19} \, \text{J}
$$

$$
E_{photon} \approx \frac{9.93 \times 10^{-19}}{1.602 \times 10^{-19}} \approx 6.2 \, \text{eV}
$$

This indicates that the electronic transition corresponds to an energy of about 6.2 eV, which is characteristic of deep valence shell transitions or interband transitions in semiconductors.

Quantum Mechanical Significance and Real-world Systems

Relevance to Quantum Dots and Nanoscale Devices

While the particle-in-a-box model is idealized, it provides insights into real systems such as:

- Quantum dots: Nanoscale semiconductor particles where electrons are confined in all three dimensions.
- Nanowires and quantum wells: Structures that confine electrons in

Frequently Asked Questions

What is the significance of the electron's transition from N=4 to N=3 in a one-dimensional box?

This transition represents a quantum jump where the electron moves between quantized energy levels, emitting a photon with a wavelength of 200 nm, illustrating quantum confinement effects.

How is the wavelength of 200 nm related to the energy difference between the N=4 and N=3 levels?

The wavelength of 200 nm corresponds to the energy emitted during the transition, calculated via the relation $E = hc/\lambda$, linking the photon energy to the specific energy gap between the two levels.

What does the emission of a 200 nm photon tell us about the energy levels in a quantum box?

It indicates that the energy difference between N=4 and N=3 is approximately 6.2 eV, as derived from the photon wavelength, reflecting the quantized energy spectrum in the confining potential.

How does the one-dimensional box model help in understanding the electron's behavior in this scenario?

The model simplifies the electron's confinement to a single spatial dimension, allowing calculation of energy levels and transition energies, which helps predict photon emission wavelengths during quantum jumps.

Why is the photon emitted at 200 nm considered in the ultraviolet (UV) range?

Because 200 nm falls within the UV spectrum, which ranges approximately from 10 nm to 400 nm, indicating the photon emitted is ultraviolet light produced by the electron's energy transition.

What factors influence the energy difference between the N=4 and N=3 levels in a quantum box?

The energy difference depends on the size of the box (confinement length), the electron's mass, and the quantum numbers N, as described by the particle-in-a-box energy formula.

How can this quantum jump be experimentally observed or measured?

By detecting the emitted photon using spectroscopic techniques such as UV spectroscopy or photon counters, scientists can confirm the transition and measure its wavelength precisely.

What role does quantum confinement play in determining the emission wavelength in this system?

Quantum confinement restricts the electron's motion, leading to discrete energy levels; the specific energy difference determines the photon wavelength emitted during a transition, such as from N=4 to N=3.

How does this example illustrate the principles of quantization in nanoscale systems?

It demonstrates that electrons in confined systems can only occupy specific energy levels, and transitions between these levels result in the emission of photons with characteristic wavelengths, exemplifying quantum quantization at the nanoscale.

Additional Resources

Electron confined in a one-dimensional box emitting a 200 nm photon during a quantum jump from N=4 to N=3

Introduction

In the realm of quantum mechanics, the behavior of electrons confined within nanoscale structures offers profound insights into the fundamental nature of matter and energy. Among these phenomena, the quantum confinement effect plays a pivotal role in determining the discrete energy levels accessible to electrons in restricted geometries. A particularly illustrative example involves an electron trapped in a one-dimensional potential well—often modeled as an infinite potential box—undergoing a transition that results in the emission of a photon. When such a transition involves a change from the fourth to the third energy level, and the emitted photon has a wavelength of approximately 200 nanometers, it exemplifies the quantization principles at work and offers a window into the quantum world.

This article explores the physical principles underlying this process, elucidates the significance of the observed emission wavelength, and discusses its implications for various fields, including quantum physics, nanotechnology, and optoelectronics.

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1. The Quantum Mechanical Model of a One-Dimensional Box

1.1. The Infinite Potential Well

The fundamental model for an electron confined in a one-dimensional box assumes an infinite potential barrier at the boundaries, effectively trapping the particle within a finite region. Mathematically, this is represented as:

- Potential $V(x) = 0$ for $0 < x < L$,

- Potential $V(x) = \infty$ elsewhere,

where L is the length of the box.

This model simplifies the complex interactions in real nanostructures and allows for analytical solutions, making it invaluable for understanding the core principles of quantum confinement.

1.2. Discrete Energy Levels

Within this potential, the electron's allowed energy levels are quantized, given by:

$$
E_n = \frac{n^2 h^2}{8 m L^2}
$$

where:

- n is a positive integer (principal quantum number),
- h is Planck's constant,
- m is the electron's mass,
- L is the length of the box.

This quantization implies that electrons can only occupy specific energy states, with transitions between these states corresponding to discrete energy differences.

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2. Electron Transitions and Photon Emission

2.1. Quantum Jumps

A quantum jump occurs when an electron transitions from a higher energy level to a lower one, emitting a photon with energy equal to the difference between the initial and final states:

$$
\Delta E = E_{n_i} - E_{n_f}
$$

Here, n_i and n_f denote the initial and final quantum numbers, respectively.

2.2. Emission Wavelength Calculation

The energy of the emitted photon is related to its wavelength λ via:

$$
E_{photon} = h \nu = \frac{hc}{\lambda}
$$

where:

- c is the speed of light,

- ν is the frequency.

Given the observed wavelength of approximately 200 nm (or 200×10^{-9} m), the photon energy can be calculated:

$$
E_{photon} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \, \mathrm{Js})(3 \times 10^{8} \, \mathrm{m/s})}{200 \times 10^{-9} \, \mathrm{m}} \approx 9.94 \times 10^{-19} \, \mathrm{J}
$$

In electronvolts (eV):

$$
E_{photon} \approx \frac{9.94 \times 10^{-19}}{1.602 \times 10^{-19}} \approx 6.2 \, \mathrm{eV}
$$

This energy corresponds to the difference between the $N=4$ and $N=3$ levels.

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3. Linking Energy Levels to the Observed Transition

3.1. Calculating Energy Difference for the Transition

Using the energy level formula:

$$
E_n = \frac{n^2 h^2}{8 m L^2}
$$

The energy difference between levels $N=4$ and $N=3$ is:

$$
\Delta E = E_4 - E_3 = \frac{h^2}{8 m L^2} (4^2 - 3^2) = \frac{h^2}{8 m L^2} (16 - 9) = \frac{7 h^2}{8 m L^2}
$$

Matching this to the emitted photon energy:

$$
\frac{7 h^2}{8 m L^2} = E_{photon} \approx 6.2 \, \mathrm{eV}
$$

Expressed in SI units, this allows us to solve for the box length L:

$$
L = \sqrt{\frac{7 h^2}{8 m E_{photon}}}
$$

Plugging in constants:

- $h = 6.626 \times 10^{-34} \, \mathrm{Js}$,
- $m = 9.109 \times 10^{-31} \, \mathrm{kg}$,
- $E_{photon} = 6.2 \times 1.602 \times 10^{-19} \, \mathrm{J}$.

Calculating:

$$
L = \sqrt{\frac{7 \times (6.626 \times 10^{-34})^2}{8 \times 9.109 \times 10^{-31} \times 6.2 \times 1.602 \times 10^{-19}}}
$$

This yields an approximate value for the box length L, revealing the scale at which quantum confinement occurs.

3.2. Significance of the Box Size

The calculated L indicates the physical size of the potential well necessary to produce the observed photon energy during the transition. Typically, for photon energies in the UV range (~6 eV), the confinement length is on the order of a few nanometers, aligning with the dimensions of quantum dots or nanowires.

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4. Implications and Applications

4.1. Quantum Confinement and Material Properties

The process described exemplifies how electrons in nanostructures can be manipulated to produce specific emission wavelengths. This forms the basis for quantum dot technology, where size control enables tuning of optical properties for applications in:

- Light-emitting diodes (LEDs),
- lasers,
- biological imaging.

4.2. Relevance to Nanotechnology and Device Engineering

Understanding the quantum transitions in a one-dimensional box is crucial for designing nanoscale devices that rely on precise energy level engineering. For example, in quantum computing, controlling electron transitions and photon emissions can facilitate quantum information processing and secure communication.

4.3. Spectroscopic Techniques

The emitted photon wavelength provides a spectroscopic signature that can be exploited to characterize nanostructures. By analyzing emission spectra, researchers can infer dimensions, material composition, and electronic properties of quantum-confined systems.

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5. Broader Context and Future Directions

5.1. Extending the Model

While the infinite potential well offers a clear theoretical framework, real-world systems involve finite potential barriers, electron-electron interactions, and environmental effects. Incorporating these complexities leads to more accurate models, such as finite potential wells, quantum wells with variable depths, and multi-electron systems.

5.2. Advances in Material Synthesis

Nanofabrication techniques continue to improve, enabling precise control over the size and shape of quantum structures. This precision allows for the tailored design of emission properties, facilitating next-generation optoelectronic devices.

5.3. Quantum Information and Communication

Harnessing quantum jumps and photon emissions at specific wavelengths opens pathways for quantum cryptography, single-photon sources, and quantum networks. Understanding fundamental processes like the transition from N=4 to N=3 in confined electrons is foundational for these advancements.

Conclusion

The emission of a 200 nm photon during an electron transition from the fourth to the third energy level within a one-dimensional quantum box encapsulates the essence of quantum confinement and energy quantization. This phenomenon not only exemplifies fundamental principles of quantum mechanics but also underpins technological innovations across photonics, nanotechnology, and quantum information science. As research progresses, refining our understanding of such quantum jumps will continue to unlock new capabilities in manipulating matter and light at the nanoscale, shaping the future of science and technology.

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