Application: 4. Determine The Vector And Parametric Equations Of A Plane That Contains The Point A(1,

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Understanding the equations of a plane is fundamental in vector calculus and analytic geometry. When working with three-dimensional space, knowing how to derive the vector and parametric equations of a plane that contains a specific point is essential for solving many real-world problems in engineering, physics, and computer graphics. This article provides a comprehensive guide on how to determine the vector and parametric equations of a plane containing a given point, using various mathematical tools and techniques.

Introduction to Planes in Three-Dimensional Space

Before delving into the specific process of finding equations, it is crucial to understand what a plane is and how it can be represented mathematically.

What Is a Plane?

- A plane is a flat, two-dimensional surface that extends infinitely in three-dimensional space.
- It is characterized by a point through which it passes and a normal vector perpendicular to its surface.
- The general equation of a plane in Cartesian coordinates is:

$$
ax + by + cz + d = 0
$$

where a, b, c are the components of the normal vector, and d is a constant.

Key Concepts in Plane Equations

- Normal Vector ($\vec{n}$): A vector perpendicular to the plane, often used to derive the plane equation.
- Point on the Plane ($A(x_0, y_0, z_0)$): A known point that lies on the plane.
- Direction Vectors: Vectors lying within the plane, used for parametric equations.

Determining the Vector Equation of a Plane

The vector equation provides a way to describe all points on the plane relative to a known point and the plane's orientation.

Prerequisites

- A point $A(x_0, y_0, z_0)$ known to lie on the plane.
- Two non-parallel vectors $\vec{u}$ and $\vec{v}$ that lie within the plane.

Step-by-Step Process

1. Identify a Point on the Plane

For the given point $A(1, y, z)$, ensure it lies in the plane or use it as the reference point.

2. Find Two Direction Vectors in the Plane

These vectors can be obtained from known points or constructed based on the problem context. For example, if the plane passes through point A and other known points B and C, then:

$$
\vec{AB} = \vec{B} - \vec{A}
$$

$$
\vec{AC} = \vec{C} - \vec{A}
$$

These vectors are within the plane and can be used to define its orientation.

3. Compute the Normal Vector $\vec{n}$

The cross product of $\vec{u}$ and $\vec{v}$ yields the normal vector:

$$
\vec{n} = \vec{u} \times \vec{v}
$$

The components of $\vec{n}$ are used in the plane's equation.

4. Formulate the Vector Equation

The vector equation of the plane is:

$$
\vec{r} = \vec{r}_0 + s \vec{u} + t \vec{v}
$$

where:

- $\vec{r}_0$ is the position vector of point A,

- $\vec{u}$ and $\vec{v}$ are direction vectors in the plane,
- $s, t \in \mathbb{R}$.

Note: If specific points or vectors are not provided, the process involves using the normal vector and the point to write the equation directly, as shown below.

Deriving the Cartesian Equation of the Plane

Once the normal vector $\vec{n} = (a, b, c)$ and a point $A(x_0, y_0, z_0)$ are known, the Cartesian form of the plane can be derived.

Step-by-Step Guide

1. Identify the Normal Vector and Point

For example, suppose the normal vector is $\vec{n} = (a, b, c)$.

2. Use Point-Normal Form of the Plane Equation

The general form is:

$$
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0
$$

This simplifies to:

$$
a x + b y + c z = d
$$

where

$$
d = a x_0 + b y_0 + c z_0
$$

3. Write the Complete Equation

Substituting the known values gives the explicit Cartesian equation of the plane.

Example:

Suppose point $A(1, 2, 3)$ and $\vec{n} = (4, -2, 1)$. Then:

$$
d = 4 \times 1 + (-2) \times 2 + 1 \times 3 = 4 - 4 + 3 = 3
$$

The plane's equation becomes:

$$
4x - 2y + z = 3
$$

Parametric Equations of a Plane

Parametric equations describe the plane using parameters, providing a flexible way to generate all points within the plane.

Formulating the Parametric Equations

Given:

- A point $A(x_0, y_0, z_0)$ on the plane.
- Two non-parallel vectors $\vec{u}$ and $\vec{v}$ lying within the plane.

The parametric equations are:

$$
\begin{cases}
x = x_0 + s u_1 + t v_1 \\
y = y_0 + s u_2 + t v_2 \\
z = z_0 + s u_3 + t v_3
\end{cases}
$$

where:

- $s, t \in \mathbb{R}$,
- $\vec{u} = (u_1, u_2, u_3)$,
- $\vec{v} = (v_1, v_2, v_3)$.

Steps to Derive the Parametric Form

1. Choose a Point on the Plane

Use point $A(1, y, z)$ as the reference point.

2. Determine Direction Vectors $\vec{u}$ and $\vec{v}$

These can be any two independent vectors lying on the plane, often derived from known points or constructed.

3. Express All Points on the Plane

Using parameters s and t, generate all points by plugging into the equations above.

Example:

Suppose the point $A(1, 2, 3)$, and the vectors $\vec{u} = (1, 0, 0)$, $\vec{v} = (0, 1, 0)$ lie within the plane. The parametric equations are:

$$
\begin{cases}
x = 1 + s \\
y = 2 + t \\
z = 3
\end{cases}
$$

This describes the plane parallel to the xy-plane passing through A.

Application Through an Example Problem

Let’s walk through an example where the point $A(1, 2, 3)$ is given, and additional information is provided to find the equations of the plane.

Problem Statement

Determine the vector and parametric equations of the plane that:

- Contains the point $A(1, 2, 3)$,
- Is perpendicular to the vector $\vec{n} = (4, -2, 1)$,
- Passes through point A.

Solution Steps

1. Identify the Normal Vector

The normal vector is given as $\vec{n} = (4, -2, 1)$.

2. Find the Cartesian Equation

Using point-normal form:

$$
4(x - 1) - 2(y - 2) + 1(z - 3) = 0
$$

Simplify:

$$
4x - 4 - 2y + 4 + z - 3 = 0
$$

$$
4x - 2y + z - 3 = 0
$$

Cartesian equation of the plane:

$$
4x - 2y + z = 3
$$

3. Find Two Direction Vectors in the Plane

Frequently Asked Questions

How do you find the vector equation of a plane passing through a point A(1, …) and containing two vectors?

To find the vector equation of a plane, identify two vectors lying on the plane originating from point A, compute their cross product to get the normal vector, and then use the point-normal form: $r = r_0 + sv + tw$, where r_0 is the position vector of point A, and v and w are the spanning vectors.

What are the steps to derive the parametric equations of a plane given a point and two direction vectors?

First, identify the point A and two direction vectors v and w lying on the plane. Then, express the parametric equations as $x = x_0 + sv_1 + tw_1$, $y = y_0 + sv_2 + tw_2$, $z = z_0 + sv_3 + tw_3$, where (x_0, y_0, z_0) are the coordinates of A, and s, t are parameters.

How can I verify if a point lies on the plane defined by a point and vectors?

Substitute the point's coordinates into the parametric equations of the plane. If the coordinates satisfy the equations for some values of s and t, then the point lies on the plane. Alternatively, check if the vector from the known point to the point in question is a linear combination of the two direction vectors.

What is the general form of the scalar equation of a plane containing a point A and a normal vector n?

The scalar equation of the plane is $n \cdot (r - r_0) = 0$, where n is the normal vector, r is the position vector of a generic point on the plane, and r_0 is the position vector of point A. Expanding this gives the scalar equation: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

How can I find the normal vector of a plane if I know two vectors on the plane?

Compute the cross product of the two vectors lying on the plane. The resulting vector is perpendicular to both, thus serving as the normal vector of the plane.

Why is it important to determine both the vector and parametric equations of a plane?

Having both equations provides a comprehensive understanding of the plane's orientation and position. The vector equation offers a compact representation incorporating direction vectors, while parametric equations allow for explicit coordinate calculations of points on the plane, useful in applications like computer graphics and engineering.

Can you illustrate how to write the parametric equations of a plane passing through point A(1, 2, 3) with direction vectors v(1, 0, 0) and w(0, 1, 0)?

Yes. The parametric equations are: $x = 1 + s$, $y = 2 + t$, $z = 3$, where s and t are parameters. This describes a plane parallel to the xy-plane passing through A, with direction vectors along the x and y axes.

Additional Resources

Application: Determining the Vector and Parametric Equations of a Plane That Contains a Given Point

In the realm of vector calculus and analytical geometry, understanding how to represent a plane mathematically is fundamental for numerous applications, from engineering design to computer graphics. One particularly important task is to derive the vector and parametric equations of a plane that contains a specific point. This process not only deepens our grasp of spatial relationships but also provides practical tools for modeling and problem-solving.

In this article, we will explore, in detail, the methodology for determining the equations of a plane given a point—specifically point A(1, ,). We will analyze the concepts step-by-step, explaining the theoretical foundations, the geometric intuition, and the algebraic procedures involved. Our goal is to equip you with a comprehensive understanding, enabling you to approach similar problems confidently and accurately.

Understanding the Foundations: Planes in Space

Before diving into equations, it's crucial to understand what a plane in space represents and how it can be described mathematically.

What Is a Plane?

A plane is a flat, two-dimensional surface extending infinitely in all directions within three-dimensional space. It can be uniquely determined by:

- A point through which it passes.
- A normal vector perpendicular to its surface.

Key Elements for Equation Derivation

To define a plane mathematically, typically, you need:

- A point on the plane (say, point A(1, ,))
- A normal vector to the plane (a vector perpendicular to the surface)

Once these are known, the equation of the plane can be formulated in different forms, most notably:

- Vector form
- Scalar (Cartesian) form
- Parametric form

In our case, given that the point is specified, the primary challenge is to find the normal vector, which then guides us to the equations.

Determining the Normal Vector: The Crux of the Problem

Since the point A(1, ,) is known but the normal vector is not explicitly given, additional information is typically required to determine the plane's equation. For instance:

- If a second point on the plane is known, the vector between the two points can serve as a basis.
- If a line or another plane's equation intersects or is related to the plane, the normal can be deduced.
- In some problems, the normal vector is provided or can be derived from known vectors or conditions.

However, if only the point A(1, ,) is given without further context, the general approach involves:

1. Identifying or assuming additional data (such as a direction vector or another point).
2. Constructing the normal vector from known vectors or constraints.
3. Using the point and normal vector to formulate the equations.

In the absence of specific additional data, a typical scenario involves:

- Given point A and two non-collinear vectors lying on the plane, say, vector u and vector v, which can be used to compute the normal vector as their cross product.

Step-by-Step: Deriving the Equations

Let's now walk through the detailed process of determining the vector and parametric equations of a plane containing a known point.

Step 1: Gather Known Data

Suppose we have:

- The point A(1, y□, z□) (with specific y and z coordinates).

- Two vectors u and v lying on the plane, for example:

- u =

- v =

These vectors are chosen such that:

- They start from point A.

- They are not parallel (to ensure they span a plane).

Note: If these vectors are not provided, they can sometimes be constructed from known points on the plane or from problem constraints.

Step 2: Find the Normal Vector

The normal vector n to the plane is perpendicular to both u and v, which can be obtained via the cross product:

\[

\mathbf{n} = \mathbf{u} \times \mathbf{v}

\]

The cross product formula is:

\[

\mathbf{u} \times \mathbf{v} =

\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

u_1 & u_2 & u_3 \\

v_1 & v_2 & v_3

\end{vmatrix}

= \left(u_2 v_3 - u_3 v_2 \right) \mathbf{i} - \left(u_1 v_3 - u_3 v_1 \right) \mathbf{j} + \left(u_1 v_2 - u_2 v_1 \right) \mathbf{k}

\]

This yields the components of n:

\[

n = \left(n_x, n_y, n_z \right)

\]

Step 3: Write the Vector Equation of the Plane

The vector form of the plane passing through point A with normal vector n is:

\[

\mathbf{r} = \mathbf{A} + s \mathbf{u} + t \mathbf{v}

\]

where:

- \(\mathbf{A} = (x_0, y_0, z_0)\) is the position vector of point A.

- \(s, t\) are parameters.

Alternatively, if you prefer a form emphasizing the normal vector, the scalar equation is:

\[

n_x (x - x_0) + n_y (y - y_0) + n_z (z - z_0) = 0

\]

which is derived directly from the dot product of the normal vector with any point on the plane.

Step 4: Derive the Parametric Equations

The parametric equations of the plane are expressed as:

\[

\begin{cases}

x = x_0 + s u_1 + t v_1 \\

y = y_0 + s u_2 + t v_2 \\

z = z_0 + s u_3 + t v_3

\end{cases}

\]

where:

- \((x_0, y_0, z_0) \) are the coordinates of point A.

- \(\mathbf{u} \) and \(\mathbf{v} \) are two non-parallel vectors lying on the plane.

- \(s, t \) are real parameters.

These equations allow us to generate any point on the plane by varying \(s\) and \(t\).

Practical Example: Applying the Method

Suppose you are given:

- Point A(1, 2, 3)
- Two vectors on the plane: $\mathbf{u} = \langle 1, 0, -1 \rangle$ and $\mathbf{v} = \langle 0, 1, 1 \rangle$

Step 1: Find the normal vector:

$$
\mathbf{n} = \mathbf{u} \times \mathbf{v} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 0 & -1 \\
0 & 1 & 1
\end{vmatrix}
= (0 \times 1 - (-1) \times 1) \mathbf{i} - (1 \times 1 - (-1) \times 0) \mathbf{j} + (1 \times 1 - 0 \times 0) \mathbf{k}
$$

Calculations:

$$
\mathbf{n} = (0 + 1) \mathbf{i} - (1 - 0) \mathbf{j} + (1 - 0) \mathbf{k} = \langle 1, -1, 1 \rangle
$$

Step 2: Write the scalar equation:

$$
1(x - 1) - 1(y - 2) + 1(z - 3) = 0
$$

Simplify:

$$
x - 1 - y + 2 + z - 3 = 0 \Rightarrow x - y + z - 2 = 0
$$

Step 3: Parametric equations:

$$
\begin{cases}
x = 1 + s \\
y = 2 + t \\
z = 3 + (-s + t)
\end{cases}
$$

Alternatively, expressing z:

$$
z = 3 - s + t
$$

where $s, t \in \mathbb{R}$.

This example illustrates the process: starting from the known point and vectors, calculating the normal, then deriving the various equations.

Additional Considerations and Tips

1. Choosing Vectors $\mathbf{u}$ and $\mathbf{v}$:

In many practical problems, these are provided, such as direction vectors of lines lying on the plane or vectors connecting known points. If only the point is given

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