At What Exact Point On The Curve Y=1+2e^x 5x Is The Tangent Line Parallel To The Line 5xy=6? (x,y)=()

At What Exact Point On The Curve Y=1+2e^x 5x Is The Tangent Line Parallel To The Line 5xy=6? (x,y)=()

Understanding the relationship between a curve and a line tangent to it is a fundamental concept in calculus. This problem involves finding the specific point on the curve $Y = 1 + 2e^x + 5x$ where the tangent line is parallel to a given line $5xy = 6$. To solve this, we need to explore derivatives, find the slope of the given line, and then determine the point(s) on the curve where the tangent slope matches this value. This comprehensive guide will walk you through each step, providing a clear understanding of the concepts and calculations involved.

Understanding the Problem

The problem involves several key components:
- The curve: $Y = 1 + 2e^x + 5x$
- The line: $5xy = 6$
- The goal: Find the point(s) (x, y) on the curve where the tangent line is parallel to the given line.

Key concepts involved:
- Derivative: To find the slope of the tangent line at any point on the curve.
- Parallel lines: Lines with equal slopes.
- Implicit differentiation: To find the slope of the given line.

Step 1: Simplify and Understand the Given Line Equation

The line is given as:

$$
5xy = 6
$$

Rearranged to express y as a function of x:

$$
xy = \frac{6}{5}
$$
$$
y = \frac{6}{5x}
$$

This form helps us understand the slope of the line at any point, which is critical in comparing slopes with the tangent to the curve.

Step 2: Find the Slope of the Given Line

The slope of the line $y = \frac{6}{5x}$ can be found by differentiating with respect to x:

$$
y = \frac{6}{5x}
$$

Rewrite as:

$$
y = \frac{6}{5} \cdot x^{-1}
$$

Differentiate:

$$
\frac{dy}{dx} = \frac{6}{5} \cdot (-1) x^{-2} = -\frac{6}{5} \cdot \frac{1}{x^2}
$$

Thus, the slope of the line $y = \frac{6}{5x}$ at any point x is:

$$
m_{line} = -\frac{6}{5x^2}
$$

This slope needs to be matched by the tangent slope of the given curve at a particular x.

Step 3: Find the Derivative of the Curve $Y = 1 + 2e^x + 5x$

To find the slope of the tangent line to the curve at any point:

$$
Y = 1 + 2e^x + 5x
$$

Differentiate term-by-term:

$$
\frac{dy}{dx} = 0 + 2e^x + 5
$$

So, the derivative (slope of the tangent line to the curve at any x) is:

$$
m_{curve} = 2e^x + 5
$$

Step 4: Set the Slopes Equal to Find x

Since the tangent line to the curve is parallel to the given line, their slopes must be equal:

$$
2e^x + 5 = -\frac{6}{5x^2}
$$

This is the key equation to solve for x.

Step 5: Solve the Equation for x

The equation:

$$
2e^x + 5 = -\frac{6}{5x^2}
$$

is transcendental, involving both exponential and rational functions. To solve it:

- Multiply both sides by $5x^2$ to clear the denominator:

$$
(2e^x + 5) \times 5x^2 = -6
$$

- Expand:

$$
10x^2 e^x + 25x^2 = -6
$$

- Rearranged as:

$$
10x^2 e^x + 25x^2 + 6 = 0
$$

This is a complex equation that may not have a straightforward algebraic solution. Therefore, numerical methods, such as Newton-Raphson or graphing, are typically used to approximate solutions.

Step 6: Numerical Approximation of x

Given the difficulty of solving analytically, you can proceed with numerical approaches:

- Graphical method: Plot the function

$$
f(x) = 10x^2 e^x + 25x^2 + 6
$$

and look for roots where $f(x) = 0$.

- Using a calculator or software (e.g., Desmos, WolframAlpha, or a graphing calculator).

Sample approximate solutions:

Suppose you find that near $x \approx -0.2$ and $x \approx 0.3$, the function crosses zero (these are hypothetical; actual values require computation).

Step 7: Find Corresponding y-Values on the Curve

Once approximate x-values are identified, compute y:

$$
y = 1 + 2e^x + 5x
$$

For each \(x \):

- Calculate \(e^x \).
- Plug into the formula to find \(y \).

This gives the point \((x, y) \) where the tangent is parallel to the line.

Final Step: Verify and Present the Solution

- Verify that the slope of the tangent at each \(x \) matches the slope of the line:

\[
m_{tangent} = 2e^x + 5
\]
\[
m_{line} = -\frac{6}{5x^2}
\]

- Ensure the calculated \(y \) is consistent with the original curve.

Summary of the Process

1. Simplify the line \(5xy=6 \) to \(y=\frac{6}{5x} \).
2. Find the slope of the line as \(-\frac{6}{5x^2} \).
3. Derive the slope of the curve as \(2e^x + 5 \).
4. Set the slopes equal to find \(x \):

\[
2e^x + 5 = -\frac{6}{5x^2}
\]

5. Solve numerically for \(x \).
6. Compute the corresponding \(y \)-coordinate on the curve:

\[
y = 1 + 2e^x + 5x
\]

7. Present solutions \((x, y) \).

Conclusion: Final Expression and Interpretation

The exact points (x, y) where the tangent to the curve $Y = 1 + 2e^x + 5x$ is parallel to the line $5xy=6$ are determined by solving the key equation:

$$
2e^x + 5 = -\frac{6}{5x^2}
$$

which generally requires numerical approximation techniques. Once the x-values are identified, calculating the corresponding y-values completes the solution.

In practical applications, tools like graphing calculators or computational software streamline the process, providing accurate approximations of the solutions. These points are significant in understanding the behavior of the curve relative to the line and are fundamental in advanced calculus and analytical geometry.

Additional Resources

- Calculus Derivatives and Tangents: Understanding how derivatives relate to slopes of tangent lines.
- Implicit Differentiation: Techniques for differentiating equations not solved explicitly for y.
- Numerical Methods: Newton-Raphson method, bisection method for solving transcendental equations.
- Graphing Tools: Desmos, GeoGebra, WolframAlpha for visualizing the functions and solutions.

Remember: Mastery of these concepts enhances your problem-solving skills in calculus, especially in applications involving tangents, slopes, and curve analysis.

Frequently Asked Questions

How do I determine the point on the curve Y=1+2e^x + 5x where the tangent line is parallel to the line 5xy=6?

First, find the derivative of Y=1+2e^x + 5x to get the slope function, then set that slope equal to the slope of the line 5xy=6 (which is constant), and solve for x to find the point(s) where the tangent is parallel.

What is the slope of the line 5xy=6, and how does it relate to

the tangent line's slope on the curve?

Rearranged as y=6/(5x), the slope of the line is the derivative of y with respect to x, which is -6/(5x^2). The tangent line to the curve must have this same slope at the point of tangency.

How do I find the derivative of Y=1+2e^x + 5x?

Differentiate each term separately: derivative of 1 is 0, of 2e^x is 2e^x, and of 5x is 5. So, the derivative Y' = 2e^x + 5.

What steps are involved in solving for the x-coordinate where the tangent line is parallel to 5xy=6?

Set the derivative Y' = 2e^x + 5 equal to the slope of the line (-6/(5x^2)), then solve the resulting equation for x.

Once I find x, how do I determine the corresponding y-coordinate on the curve?

Plug the x-value back into the original curve equation Y=1+2e^x + 5x to compute y.

Are there multiple points where the tangent line is parallel to 5xy=6 for this curve?

Potentially, yes. After solving the equation for x, verify each solution to find all points where the tangent line is parallel.

Can you provide the final coordinates (x,y) where the tangent line is parallel to 5xy=6?

Once you solve for x and compute y, the points are (x, 1+2e^x + 5x). The exact values depend on solving the resulting equations, which may involve solving a transcendental equation.

What is the overall process to find the point where the tangent line to Y=1+2e^x + 5x is parallel to 5xy=6?

Calculate the derivative of the curve, equate it to the slope of the line 5xy=6, solve for x, then substitute back into the original curve to find y, thus determining the point(s) (x, y).

Additional Resources

Understanding the Problem: When Is the Tangent to the Curve Y=1+2e^x 5x Parallel to the Line 5xy=6?

Introduction to the Problem

Mathematics often presents intriguing problems where calculus and algebra intertwine to reveal insights about curves and their tangents. The problem at hand asks: At what exact point on the curve $Y = 1 + 2e^x + 5x$ is the tangent line parallel to the line $5xy = 6$?

This question involves understanding the geometric relationship between the given curve and the line, utilizing derivatives to find tangent slopes, and solving for specific points. It combines concepts such as differentiability, slope of tangent lines, and the properties of lines and curves in the plane.

Deciphering the Given Curve and Line Equations

Understanding the Curve $Y = 1 + 2e^x + 5x$

The curve is described explicitly as a function:

$$
Y(x) = 1 + 2e^x + 5x
$$

This function is composed of:

- A constant term 1,
- An exponential component $2e^x$,
- A linear component $5x$.

The combination suggests the curve is smooth and differentiable everywhere, due to the presence of exponential and polynomial functions.

Understanding the Line $5xy = 6$

The given line is expressed implicitly:

$$
5xy = 6
$$

This is a rectangular hyperbola, not a straight line, but the problem asks about the tangent line to the curve being parallel to this line. Since the phrase "parallel to the line $5xy=6$" indicates that the tangent line's slope must match the slope of this line, we need to analyze this hyperbola's properties.

Analyzing the Slope of the Line $5xy=6$

Rewriting the Equation

To find the slope of the line or curve, it's often easier to express the equation explicitly or find derivatives accordingly.

The implicit equation:

$$
5xy = 6
$$

can be rearranged to express y in terms of x:

$$
y = \frac{6}{5x}
$$

This is a hyperbolic function, not a linear function. The key is understanding that the problem asks about the tangent parallel to this hyperbola, meaning:

- The tangent line to the curve at some point must have the same slope as the tangent to the hyperbola at some point.

Calculating the Slope of the Hyperbola $y = \frac{6}{5x}$

Differentiate $y = \frac{6}{5x}$ with respect to x:

$$
\frac{dy}{dx} = \frac{d}{dx} \left(\frac{6}{5x} \right) = \frac{6}{5} \cdot \frac{d}{dx} \left(x^{-1} \right)
$$

$$
\frac{dy}{dx} = \frac{6}{5} \times (-1) x^{-2} = - \frac{6}{5x^2}
$$

Thus, the slope of the hyperbola at any point (x, y) on it is:

$$
m_{hyperbola} = - \frac{6}{5x^2}
$$

Finding the Slope of the Tangent to the Curve $Y = 1 + 2e^x + 5x$

Calculating the Derivative $Y'(x)$

Differentiate $Y(x)$:

$$
Y'(x) = \frac{d}{dx} \left(1 + 2e^x + 5x \right)
$$

$$
Y'(x) = 0 + 2e^x + 5
$$

This derivative gives the slope of the tangent line to the curve at any point x:

$$
m_{curve} = 2e^x + 5
$$

Setting the Slopes Equal for Parallelism

The core of the problem is:

> When is the tangent to the curve parallel to the hyperbola $5xy = 6$?

Since parallel lines have equal slopes, set:

$$
Y'(x) = - \frac{6}{5x^2}
$$

which leads to:

$$
2e^x + 5 = - \frac{6}{5x^2}
$$

This is the fundamental equation to solve for x.

Solving the Key Equation

$$
2e^x + 5 = - \frac{6}{5x^2}
$$

Rearranged as:

$$
2e^x + 5 + \frac{6}{5x^2} = 0
$$

or equivalently:

$$
2e^x + 5 = - \frac{6}{5x^2}
$$

Now, consider the following points:

- The left side $2e^x + 5$ is always positive for all real x because $e^x > 0$, so $2e^x > 0$, thus $2e^x + 5 > 0$.
- The right side $- \frac{6}{5x^2}$ is always less than or equal to zero for all $x \neq 0$, since $x^2 > 0$.

Therefore, the equation becomes:

$$
\text{Positive} = \text{Negative}
$$

which cannot be true unless both sides are zero, but the left side cannot be zero (since $2e^x + 5 > 0$), and the right side cannot be positive.

This suggests no real solution exists under this direct equality unless we consider the possibility of the equation being satisfied at some specific points where the functions could intersect under certain considerations.

But wait, the key is that the slopes are equal (for the tangent to be parallel to the hyperbola), so:

$$
2e^x + 5 = - \frac{6}{5x^2}
$$

Given the analysis above, the only possibility for equality might be at points where the right side is positive, but it's always negative for $x \neq 0$.

Conclusion: There appears to be no real x satisfying this condition, implying no point on the curve where the tangent is parallel to the hyperbola $5xy=6$.

Re-evaluating the Problem: Is There a Misinterpretation?

Given the above, perhaps the problem expects us to interpret "parallel to the line $5xy=6$" differently.

- The line $5xy=6$ is not a line but a hyperbola.
- The phrase might mean the tangent line to the curve is parallel to the tangent line of the hyperbola $5xy=6$ at some point.

This leads us to revisit the earlier derivative:

- For the hyperbola $5xy=6$, the derivative $dy/dx = - \frac{6}{5x^2}$.

- For the curve, the derivative is $Y' = 2e^x + 5$.

Thus, the tangent line to the curve is parallel to the hyperbola's tangent when:

$$
Y' = - \frac{6}{5x^2}
$$

which, as shown, cannot be satisfied for any real x, because:

- $Y' = 2e^x + 5 > 0$,
- and $- \frac{6}{5x^2} \leq 0$.

Therefore, the derivative of the curve is always positive, while the derivative of the hyperbola's tangent at any point is always negative or zero (at $x \to \infty$).

Implication: The slopes cannot be equal; hence, no such point exists where the tangent to the given curve is parallel to the hyperbola $5xy=6$.

Alternative Interpretation: Is the Question About the Tangent Line to the Curve Being Parallel to the Chord or the Asymptote?

Given the previous conclusion, perhaps the question is asking:

> At what point on the curve \(Y=1+2e^x+5x\) is the tangent line parallel to the line \(5xy=6\) considered as a straight line?

If we interpret \(5xy=6\) as a line, then:

\[
5xy=6 \implies y = \frac{6}{5x}
\]

which is not a line, but a hyperbola. So, the only potential line for comparison is the tangent line to the hyperbola at some point.

Hence, the problem reduces to:

- Find \(x\) such that the tangent line to the curve has the same slope as the tangent to the hyperbola at some point.

Given the previous analysis, the slopes are:

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