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At What Point (x,y) In The Plane Are The Functions Below Continuous? A. F(x,y)=sin(x + y) B. F(x,y) =

Understanding the continuity of functions in the plane is a fundamental aspect of multivariable calculus. When analyzing functions of two variables, (x, y), mathematicians seek to determine at which points these functions are continuous. In this article, we will explore the continuity of two specific functions:

- A. F(x,y) = sin(x + y)
- B. F(x,y) = [function not specified in the prompt]

Since the second function's definition appears incomplete, we will focus extensively on the first, providing a comprehensive analysis of its continuity. Additionally, we will discuss general principles of continuity in two-variable functions, how to determine points of continuity, and what factors can cause discontinuity.

Understanding Continuity in Functions of Two Variables

What Does Continuity Mean?

In the context of multivariable functions, a function F(x, y) is continuous at a point (a, b) if the following conditions are met:

1. F(a, b) is defined.
2. The limit of F(x, y) as (x, y) approaches (a, b) exists.
3. The limit of F(x, y) as (x, y) approaches (a, b) is equal to F(a, b).

Mathematically, F is continuous at (a, b) if:

\[
\lim_{(x,y) \to (a,b)} F(x,y) = F(a,b)
\]

This concept extends the idea of continuity from single-variable calculus into the two-variable case, but with added complexity because limits must be verified along all possible paths toward the point.

Common Causes of Discontinuity

Some typical reasons why a function may be discontinuous at a point include:

- Undefined at that point (e.g., division by zero, square root of a negative number).
- Jump discontinuities where the limit exists but is not equal to the function value.
- Removable discontinuities which can be "fixed" by redefining the function at the point.
- Essential discontinuities, which are more complex and often involve oscillations or non-limit behavior.

Understanding these causes helps in analyzing the continuity of particular functions.

Analysis of the Function F(x,y) = sin(x + y)

Definition and Basic Properties

The function F(x,y) = sin(x + y) is a composition of the sine function with the linear expression (x + y). Since sine is continuous everywhere on the real line, and the sum x + y is a polynomial (and thus continuous everywhere), their composition inherits these continuity properties.

Continuity in the Entire Plane

Because both the sine function and the addition operation are continuous everywhere, the composition:

$$
F(x,y) = \sin(x + y)
$$

is continuous at every point (x, y) in $\mathbb{R}^2$.

Why is this true?

- The sum $x + y$ is a polynomial function of two variables, which is continuous everywhere.
- The sine function is continuous everywhere on $\mathbb{R}$.
- The composition of continuous functions is continuous.

Therefore, for any point $(a, b) \in \mathbb{R}^2$:

$$
\lim_{(x,y) \to (a,b)} \sin(x + y) = \sin(a + b)
$$

and

$$
F(a, b) = \sin(a + b)
$$

which confirms continuity at every point.

Implications for Continuity

The key takeaway is that F(x,y) = sin(x + y) is continuous everywhere in the plane. There are no points of discontinuity for this function. This makes it a particularly straightforward example of a continuous multivariable function.

Analyzing the Second Function (B)

The prompt does not specify the exact form of function B, so we cannot explicitly analyze its points of continuity. However, general principles can be discussed to determine points of continuity for any two-variable function.

General Approach to Determine Continuity

To assess whether a given function $G(x, y)$ is continuous at a point (a, b), follow these steps:

1. Verify that $G(a, b)$ is defined. If not, the function is discontinuous at that point.
2. Calculate the limit $\lim_{(x,y) \to (a,b)} G(x,y)$. This often involves approaching (a, b) along various paths to check if the limit exists and is unique.
3. Compare the limit to the function's value at (a, b). If they are equal, the function is continuous at that point.

Common Examples of Discontinuous Functions

- Piecewise functions with jump discontinuities: For example, functions that change definitions over different regions.
- Functions with undefined points: Such as $G(x, y) = \frac{1}{x - a}$, which is undefined at $x = a$.
- Functions involving roots or logarithms: For example, $G(x, y) = \sqrt{y}$ is not defined for $y < 0$, leading to discontinuities along the line $y=0$.

Summary and Final Thoughts

- F(x,y) = sin(x + y): This function is continuous everywhere in $\mathbb{R}^2$ because it is composed of continuous functions (addition and sine). There are no points of discontinuity in the plane.

- The unspecified function B: To analyze its continuity, one must consider its specific form. The general method involves checking the domain, limits along various paths, and the function's value at the point.

- Why understanding continuity matters: Continuity ensures the function behaves predictably and smoothly without abrupt jumps or gaps. This is critical in applications such as optimization, modeling, and solving differential equations.

Conclusion

In the realm of multivariable calculus, the pointwise continuity of functions like $F(x, y) = \sin(x + y)$ is straightforward because of the inherent continuity of elementary functions and operations involved. For this particular function, the answer is simple: it is continuous at every point in the plane.

For other functions, especially those involving division, roots, or piecewise definitions, a careful examination of the domain and limits from various directions is essential to determine points of continuity. By following the principles outlined above, mathematicians and students can systematically analyze the continuity of any function of two variables.

Remember: Continuity is a foundational concept that underpins many advanced topics in calculus and mathematical analysis. Mastering how to analyze and identify points of continuity is crucial for progressing in understanding the behavior of multivariable functions.

Note: If you provide the full definition of function B, I can include a detailed analysis of its points of continuity as well.

Frequently Asked Questions

At what points in the plane is the function F(x, y) = sin(x + y) continuous?

The function F(x, y) = sin(x + y) is continuous everywhere in $\mathbb{R}^2$ because the sine function is continuous for all real numbers, and the composition of continuous functions remains continuous.

Does the continuity of F(x, y) = sin(x + y) depend on specific points in the plane?

No, F(x, y) = sin(x + y) is continuous at every point (x, y) in $\mathbb{R}^2$ without exception.

Are there any points where F(x, y) = sin(x + y) is discontinuous?

No, since the sine function is continuous everywhere and the addition of x and y is continuous, their composition is continuous at all points in the plane.

How does the continuity of F(x, y) = sin(x + y) relate to the properties of basic functions?

It exemplifies that the composition of continuous functions, like addition and sine, results in a continuous function across the entire plane.

If a different function, such as F(x, y) = 1/(x + y), is considered, how does its continuity change?

F(x, y) = 1/(x + y) is discontinuous where x + y = 0 because of division by zero; outside this line, it is continuous.

Can the function F(x, y) = sin(x + y) be discontinuous at any point in $\mathbb{R}^2$?

No, it cannot be discontinuous at any point because both sine and addition are continuous everywhere.

What is the significance of the argument x + y in

determining the continuity of F(x, y) = sin(x + y)?

Since x + y is a polynomial (linear) function, it is continuous everywhere, ensuring the sine of this sum is also continuous everywhere.

In general, how do compositions of functions influence continuity in multivariable calculus?

The continuity of composed functions depends on the continuity of each component; composing continuous functions results in a continuous overall function.

Additional Resources

At What Point (x,y) In The Plane Are The Functions Below Continuous? A. F(x,y)=sin(x + y) B. F(x,y)=

In the vast landscape of mathematical analysis, understanding the continuity of functions in the plane is fundamental. Continuity not only determines whether a function behaves predictably but also influences the applicability of calculus tools such as derivatives and integrals. Today, we explore two intriguing functions—one involving a basic trigonometric function and the other, whose form remains to be specified—and analyze precisely where they are continuous in the Euclidean plane. This inquiry, while seemingly straightforward, opens doors to deeper topics like limits, points of discontinuity, and the nature of real-valued functions of multiple variables.

Understanding Continuity in Multiple Dimensions

Before delving into the specific functions, it’s essential to revisit what continuity means in a two-variable context. Unlike single-variable functions, where continuity at a point involves the limit approaching the function's value, multivariable functions require a more nuanced approach.

Definition of Continuity at a Point (x_0, y_0):
A function $f(x, y)$ is continuous at a point (x_0, y_0) if:

1. The function f is defined at (x_0, y_0), i.e., $f(x_0, y_0)$ exists.
2. The limit of $f(x, y)$ as (x, y) approaches (x_0, y_0) exists.
3. The limit equals the function value:
$$
\lim_{(x, y) \to (x_0, y_0)} f(x, y) = f(x_0, y_0)
$$

Key Point:
For the function to be continuous at (x_0, y_0), the limit must be the same regardless of the path taken to approach the point. If different paths yield different limits, the function is discontinuous there.

Part A: Continuity of $F(x, y) = \sin(x + y)$

Exploring the Function

The function $F(x, y) = \sin(x + y)$ is a classic example of a combination of elementary functions. Its simplicity makes it an excellent candidate to analyze for continuity.

Fundamental Properties of $\sin(z)$

- The sine function, $\sin(z)$, is continuous everywhere in the real line.
- When composed with a continuous function (here, $x + y$), the resulting function remains continuous everywhere in $\mathbb{R}^2$.

Is $F(x, y) = \sin(x + y)$ Continuous Everywhere?

Analysis:
Since the sum $x + y$ is a polynomial in two variables, it’s continuous everywhere. The sine function, being continuous everywhere on $\mathbb{R}$, preserves this property when composed.

Conclusion:
$$
\boxed{
\text{The function } F(x, y) = \sin(x + y) \text{ is continuous at every point } (x, y) \in \mathbb{R}^2.
}
$$

Implication:
No matter where you are in the plane, the sine of the sum $x + y$ behaves predictably, with no abrupt jumps or points of discontinuity. This universal continuity makes $F(x, y)$ a smooth and well-behaved function across the entire plane.

Visual Interpretation

Imagine the plane as a grid, and consider the level curves where $x + y = c$ for some constant c. These are straight lines at a 45-degree angle. Along each such line, the function value $\sin(c)$ remains constant, creating a family of sinusoidal waves that are continuous everywhere.

Part B: The Unknown Function $F(x, y) =$ [To Be Specified]

(Note: Since the original prompt introduces "F(x, y) = " without a specific expression, we will explore common types of functions that are frequently analyzed for continuity in multivariable calculus. If you have a particular expression in mind, please specify. For now, we will consider a typical example:)

Hypothetical Example for Part B: $F(x, y) = \frac{1}{x^2 + y^2}$

Analyzing the Function

Suppose the second function is:

$$
F(x, y) = \frac{1}{x^2 + y^2}
$$

This function is a classic example of a rational function involving two variables.

Domain Considerations

- The denominator $x^2 + y^2$ is non-negative everywhere and equals zero only at the point $(0, 0)$.
- The function is defined everywhere except at $(0, 0)$.

Continuity Analysis

- At $(x, y) \neq (0, 0)$:
Since the denominator is non-zero, $F(x, y)$ is a quotient of continuous functions (polynomials), hence continuous everywhere except at the singular point.

- At $(0, 0)$:
The function is not defined at this point, so it cannot be continuous there unless we extend it by defining $F(0, 0)$. Even then, we need to analyze the limit:

$$
\lim_{(x, y) \to (0, 0)} \frac{1}{x^2 + y^2}
$$

As (x, y) approaches $(0, 0)$, the denominator tends to zero, making the function tend toward infinity. This indicates a vertical asymptote at the origin, and the function is discontinuous there.

Summary for this example:

$$
\boxed{
\text{The function } F(x, y) = \frac{1}{x^2 + y^2} \text{ is continuous everywhere except at } (0, 0).
}
$$

General Principles for Multivariable Continuity

The analysis above exemplifies some broader principles:

1. Continuity of Elementary Functions:
Functions like sine, cosine, exponential, and polynomials are continuous everywhere in $\mathbb{R}^2$.

2. Composition of Continuous Functions:
If $g(x, y)$ is continuous at (x_0, y_0) and $h(z)$ is continuous at $z = g(x_0, y_0)$, then the composition $h(g(x, y))$ is continuous at (x_0, y_0).

3. Rational Functions:
Continuity depends on the denominator not being zero. Points where the denominator vanishes are potential discontinuities or singularities.

4. Piecewise Functions:
Discontinuities often occur at the boundaries between pieces, especially if the pieces are defined differently or have different limits.

Path-Dependent Limits and Discontinuities

A critical aspect of multivariable analysis is examining limits along different paths. For example, consider the function:

$$
F(x, y) = \frac{xy}{x^2 + y^2}
$$

approaching $(0, 0)$.

- Along $y = x$:
$$
F(x, x) = \frac{x \cdot x}{x^2 + x^2} = \frac{x^2}{2x^2} = \frac{1}{2}
$$

- Along $y = 0$:
$$
F(x, 0) = 0
$$

Since the limits differ along different paths, the overall limit does not exist, indicating a discontinuity at $(0, 0)$.

Practical Implications of Continuity Analysis

Understanding where functions are continuous has real-world implications in physics, engineering, and data modeling. For example:

- Signal Processing: Discontinuous functions can introduce artifacts or errors.
- Optimization: Continuous functions ensure the existence of minima or maxima within a domain.
- Numerical Methods: Discontinuities can cause convergence issues or inaccuracies.

Conclusion: When Are These Functions Continuous?

To summarize:

- $F(x, y) = \sin(x + y)$:
This function is continuous everywhere in the plane. Its composition of continuous functions ensures no points of discontinuity.

- The second function (hypothetically $F(x, y) = 1/(x^2 + y^2)$):
Its continuity is restricted to points where the denominator is non-zero. It is discontinuous at the origin due to a vertical asymptote, but continuous elsewhere.

In the broader context, identifying points of discontinuity involves examining the domain, limits along various paths, and the nature of the constituent functions. Recognizing these nuances helps mathematicians and scientists ensure models behave predictably, and that calculus tools can be applied effectively.

Whether analyzing simple trigonometric compositions or complex rational functions, the principles of continuity remain a cornerstone of multivariable calculus, guiding us through the intricate tapestry of the plane's mathematical landscape.

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