Calculus Consider The Function 6 = Xy+yz. (a) Find Its Rate Of Change In The Direction (1,2,3) At The

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Introduction to Multivariable Calculus and Directional Derivatives

Calculus Consider The Function 6 = Xy + yz. (a) Find Its Rate Of Change In The Direction (1, 2, 3) At The. In multivariable calculus, understanding how a function changes as we move through space is fundamental. This involves concepts such as partial derivatives, gradients, and directional derivatives. These mathematical tools help analyze how functions behave in multiple dimensions, which is essential in fields like physics, engineering, economics, and more.

In this article, we will explore how to compute the rate of change of a multivariable function along a specific direction at a particular point. Specifically, we focus on the function $f(x, y, z) = xy + yz$, and we will determine its rate of change in the direction of the vector $\mathbf{v} = (1, 2, 3)$ at a specified point. This process involves calculating the gradient of the function, normalizing the direction vector, and then applying the formula for the directional derivative.

Understanding the Function and the Problem Setup

The Given Function

The function under consideration is:

$$
f(x, y, z) = xy + yz
$$

This is a scalar-valued function of three variables, which combines the products of pairs of variables. Geometrically, the surface described by this function can be complex, and analyzing its rate of change in various directions provides valuable insights into its behavior.

The Direction Vector

The direction vector provided is:

$$
\mathbf{v} = (1, 2, 3)
$$

This vector indicates the direction in three-dimensional space along which we want to find the rate of change of f.

The Point of Evaluation

To compute the directional derivative, we need to specify the point at which the rate of change is evaluated. Since the problem statement appears incomplete regarding the specific point, we will assume a general point (x_0, y_0, z_0). For demonstration purposes, let's choose $(x_0, y_0,$

z_0) = (x, y, z) \), or you can substitute your specific point once known.

Step 1: Computing the Gradient of the Function

The gradient vector ∇f points in the direction of the steepest ascent of the function and contains the partial derivatives with respect to each variable.

Partial derivatives of $f(x, y, z)$

Calculate each partial derivative:

- With respect to x:

$$
\frac{\partial f}{\partial x} = y
$$

- With respect to y:

$$
\frac{\partial f}{\partial y} = x + z
$$

- With respect to z:

$$
\frac{\partial f}{\partial z} = y
$$

The gradient vector ∇f

Putting it together, the gradient is:

$$
\nabla f(x, y, z) = \left(y, \quad x + z, \quad y \right)
$$

This vector points in the direction of the maximum rate of increase of the function at a specific point.

Step 2: Normalizing the Direction Vector

To find the directional derivative, the direction vector must be a unit vector. Therefore, normalize $\mathbf{v} = (1, 2, 3)$.

Calculating the magnitude of $\mathbf{v}$:

$$
|\mathbf{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}
$$

Unit vector in the direction of $\mathbf{v}$:

$$
\mathbf{u} = \frac{1}{|\mathbf{v}|} (1, 2, 3) = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)
$$

This unit vector will be used in the directional derivative formula.

Step 3: Computing the Directional Derivative

The directional derivative of f at point (x_0, y_0, z_0) in the direction of the unit vector $\mathbf{u}$ is given by:

$$
D_{\mathbf{u}}f(x_0, y_0, z_0) = \nabla f(x_0, y_0, z_0) \cdot \mathbf{u}
$$

where "·" denotes the dot product.

Substituting the gradient and unit vector

$$
D_{\mathbf{u}}f = \left(y_0, x_0 + z_0, y_0 \right) \cdot \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)
$$

$$
D_{\mathbf{u}}f = \frac{1}{\sqrt{14}} y_0 + \frac{2}{\sqrt{14}} (x_0 + z_0) + \frac{3}{\sqrt{14}} y_0
$$

Combining like terms:

$$
D_{\mathbf{u}}f = \frac{1}{\sqrt{14}} y_0 + \frac{2}{\sqrt{14}} x_0 + \frac{2}{\sqrt{14}} z_0 + \frac{3}{\sqrt{14}} y_0
$$

\]

$$
D_{\mathbf{u}}f = \frac{(1 + 3)}{\sqrt{14}} y_0 + \frac{2}{\sqrt{14}} x_0 + \frac{2}{\sqrt{14}} z_0
$$

$$
D_{\mathbf{u}}f = \frac{4}{\sqrt{14}} y_0 + \frac{2}{\sqrt{14}} x_0 + \frac{2}{\sqrt{14}} z_0
$$

Thus, the rate of change depends on the specific point (x_0, y_0, z_0). Once the point is known, substitute its coordinates into this formula to find the exact value.

Practical Example: Computing at a Specific Point

Suppose the point of interest is $(x_0, y_0, z_0) = (1, 2, 3)$.

Calculating the gradient at this point:

$$
\nabla f(1, 2, 3) = (2, 1 + 3, 2) = (2, 4, 2)
$$

Applying the directional derivative formula:

$$
D_{\mathbf{u}}f = (2, 4, 2) \cdot \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)
$$

$$
D_{\mathbf{u}}f = \frac{2}{\sqrt{14}} + \frac{8}{\sqrt{14}} + \frac{6}{\sqrt{14}} = \frac{2 + 8 + 6}{\sqrt{14}} = \frac{16}{\sqrt{14}}
$$

This is approximately:

$$
D_{\mathbf{u}}f \approx \frac{16}{3.7417} \approx 4.28
$$

This number indicates the rate at which the function f increases in the specified direction at the point $(1, 2, 3)$.

Additional Insights and Applications

Why Is the Directional Derivative Important?

The directional derivative provides critical information about the behavior of multivariable functions. It tells us how fast the function value changes as we move from a point in a particular direction. This concept is vital in optimization, physics (e.g., gradient fields), and machine learning (e.g., gradient descent algorithms).

Real-World Applications

- Engineering: Understanding stress, strain, or heat flow in materials.
- Economics: Analyzing how changing multiple factors impacts profit or cost.
- Physics: Calculating how physical quantities such as temperature or pressure change in space.

Summary and Key Takeaways

- The gradient vector \(\nabla f \) contains all the partial derivatives and indicates the direction of steepest ascent.
- To compute the rate of change in a specific direction, normalize the direction vector and take the dot product with the gradient.
- The process involves calculating partial derivatives, forming the gradient, normal

Frequently Asked Questions

What is the function given in the calculus problem?

The function given is 6 = xy + yz.

How do you interpret the vector (1, 2, 3) in the context of rate of change?

The vector (1, 2, 3) represents the direction in which we want to find the rate of change of the function.

What is the first step to find the rate of change of the function in the given direction?

The first step is to find the gradient of the function and then compute its dot product with the given direction vector.

How do you find the gradient of the function 6 = xy + yz?

Rearranged as $f(x,y,z) = xy + yz - 6$, then compute the partial derivatives with respect to x, y, and z.

What are the partial derivatives of f(x,y,z) = xy + yz - 6?

$\partial f/\partial x = y$, $\partial f/\partial y = x + z$, $\partial f/\partial z = y$.

How is the directional derivative calculated using the gradient and direction vector?

The directional derivative in the direction of vector v is given by the dot product of the gradient and the unit vector of v.

How do you normalize the direction vector (1, 2, 3) for the calculation?

Calculate its magnitude: $\sqrt{(1^2 + 2^2 + 3^2)} = \sqrt{14}$, then divide each component by $\sqrt{14}$ to get the unit vector.

What is the final step to find the rate of change in the specified direction?

Compute the dot product of the gradient at the point and the unit direction vector to obtain the rate of change.

Additional Resources

Calculus is a fundamental branch of mathematics that deals with the study of change, motion, and accumulation. It provides powerful tools to analyze how functions behave and how they change in response to variations in their variables. One of the core concepts in calculus is understanding the rate of change of multivariable functions, which plays a vital role in fields ranging from physics and engineering to economics and data science. In this article, we will explore the calculation of the rate of change for a specific function in a given direction, focusing on the example: Given the function $6 = xy + yz$, and determining its rate of change in the direction of the vector (1, 2, 3). We will break down this problem systematically, offering insights into the underlying calculus concepts, step-by-step solutions, and practical considerations.

Understanding the Function and the Problem Statement

Interpreting the Function $6 = xy + yz$

At first glance, the function is presented as an equation: $6 = xy + yz$. Typically, in calculus, when discussing rates of change, we consider functions of the form $z = f(x, y)$, or similar. The given expression resembles an implicit relation involving three variables, possibly indicating that 6 is a constant value, which suggests an implicit surface in three-dimensional space.

Rearranged, the function can be written as:

$$ xy + yz = 6 $$

This defines an implicit surface in three variables x, y, and z. Since the problem asks for the rate of change of this function in a particular direction at a certain point, our primary goal is to understand how the function changes as we move along a specified vector in space.

Note: The problem statement mentions "Given the function $6 = xy + yz$," which likely means the function is defined implicitly as:

$$ F(x, y, z) = xy + yz - 6 = 0 $$

This is a common approach in multivariable calculus, where the function $F(x, y, z) = 0$ defines a surface.

Calculating the Rate of Change in a Specific Direction

The Concept of Directional Derivative

The rate at which a function changes at a point in a specific direction is quantified by the directional derivative. For a function $F(x, y, z)$, the directional derivative at a point $\mathbf{r}_0 = (x_0, y_0, z_0)$ in the direction of a unit vector $\mathbf{u} = (u_1, u_2, u_3)$ is given by:

$$
D_{\mathbf{u}} F(\mathbf{r}_0) = \nabla F(\mathbf{r}_0) \cdot \mathbf{u}
$$

where ∇F is the gradient of F, and $\cdot$ denotes the dot product.

Key points:

- The gradient vector ∇F points in the direction of the steepest ascent.
- The magnitude of the gradient gives the rate of maximum increase.
- To find the rate of change in an arbitrary direction, we project the gradient onto that direction.

Given Data and Approach

The problem specifies:

- The function: \(F(x, y, z) = xy + yz - 6\)
- The direction vector: \(\mathbf{v} = (1, 2, 3)\)

To compute the rate of change in the direction of \(\mathbf{v}\), we need:

1. The point at which to evaluate the rate of change (this is not explicitly specified in the prompt; in typical problems, a point \((x_0, y_0, z_0)\) is given or assumed).
2. The unit vector in the direction of \(\mathbf{v}\): \(\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}\).

Assuming the point of interest is \((x_0, y_0, z_0)\) on the surface \(F=0\), the process involves:

- Calculating \(\nabla F\) at this point.
- Normalizing the direction vector.
- Computing the dot product to find the directional derivative.

Step-by-Step Solution

Step 1: Find the Gradient \(\nabla F\)

Given:

\[

F(x, y, z) = xy + yz - 6

\]

Calculate partial derivatives:

\[

\frac{\partial F}{\partial x} = y

\]

\[

\frac{\partial F}{\partial y} = x + z

\]

\[

\frac{\partial F}{\partial z} = y

\]

Thus,

\[

\nabla F(x, y, z) = (y, x + z, y)

\]

Step 2: Determine the Point \((x_0, y_0, z_0)\)

The problem doesn't specify the point, but typically, such problems evaluate the rate of change at a known point on the surface. For illustration, suppose we choose a point that satisfies the surface equation:

\[

$$
xy + yz = 6
$$

Let's pick a simple point, for example:

$$
x_0 = 1, \quad y_0 = 2
$$

Calculate z_0:

$$
(1)(2) + (2)z = 6 \Rightarrow 2 + 2z = 6 \Rightarrow 2z = 4 \Rightarrow z = 2
$$

Therefore, the point is $(1, 2, 2)$.

Step 3: Compute the Gradient at $(1, 2, 2)$

Plug in the point:

$$
\nabla F(1, 2, 2) = (2, 1 + 2, 2) = (2, 3, 2)
$$

Step 4: Normalize the Direction Vector $\mathbf{v} = (1, 2, 3)$

Calculate its magnitude:

$$
\|\mathbf{v}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}
$$

The unit vector:

$$
\mathbf{u} = \frac{1}{\sqrt{14}} (1, 2, 3)
$$

Step 5: Compute the Directional Derivative

Dot product:

$$
\nabla F(1, 2, 2) \cdot \mathbf{u} = (2, 3, 2) \cdot \frac{1}{\sqrt{14}} (1, 2, 3)
$$

$$
= \frac{1}{\sqrt{14}} (2 \times 1 + 3 \times 2 + 2 \times 3) = \frac{1}{\sqrt{14}} (2 + 6 + 6) = \frac{14}{\sqrt{14}} = \sqrt{14}
$$

Result:

$$
\boxed{
D_{\mathbf{u}} F = \sqrt{14} \approx 3.7417
}
$$

This value indicates the rate at which the function F increases at the point $(1,2,2)$ in the direction of $\mathbf{v}$.

Analysis and Insights

Features of the Calculated Rate of Change

- Positive value indicates the function increases in the direction of the vector $(1, 2, 3)$.
- The magnitude depends on both the gradient and the chosen direction vector.
- The calculation demonstrates the importance of the gradient in determining directional change.

Pros and Cons of the Approach

Pros:

- Provides a precise measure of how the function behaves locally in a specified direction.
- Uses fundamental calculus tools (gradient, dot product) that are straightforward to compute.

- Applicable to any point and direction in multivariable functions.

Cons:

- Requires knowledge of the point of interest; assumptions may not always be valid.
- Assumes the function is differentiable at the point.
- The implicit nature of the surface sometimes complicates the interpretation.

Additional Considerations and Generalizations

Handling Different Points or Directions

- The process is the same regardless of the point or direction vector chosen.
- For points not on the surface, the gradient calculation remains valid, but the interpretation of the rate of change may differ.
- For directions not normalized, always convert to a unit vector to obtain the correct rate.

Applications in Real-World Contexts

- Physics: Calculating the rate of change of potential or fields along a certain direction.
- Engineering: Understanding how a system responds to perturbations in specific directions.
- Economics: Analyzing how a multivariable function (like profit or cost) changes with multiple factors.

Conclusion

Calculus provides robust tools for

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