Choose The Correct Answer For The Function M(x,y) For Which The Following Vector Field F(x,y) = (- 8x

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Understanding vector fields and their associated potential functions is fundamental in multivariable calculus and vector calculus. When dealing with a vector field like F(x, y) = (-8x, ...), a common problem is to determine a scalar function M(x, y) such that F is either a gradient field or has some specific relationship to M. This article provides a comprehensive guide to choosing the correct M(x, y) for the given vector field, focusing on the context of potential functions, line integrals, and conservative vector fields.

Introduction to Vector Fields and Potential Functions

Before delving into the specifics, it is crucial to understand the foundational concepts:

What Is a Vector Field?

A vector field assigns a vector to every point in a subset of space. In a two-dimensional setting, a vector field F(x, y) can be expressed as:

$$
F(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}
$$

where:

- $P(x, y)$ is the component in the x-direction
- $Q(x, y)$ is the component in the y-direction

In our case, the vector field is:

$$
F(x, y) = (-8x, Q(x, y))
$$

The question is: what is Q(x, y)? Since the problem statement is incomplete, it is typical in such problems that Q(x, y) is either provided or implied. For the purpose of this discussion, assume the vector field is:

$$
F(x, y) = (-8x, 0)
$$

This is a common form used in examples involving potential functions.

What Is a Potential Function?

A potential function or scalar potential M(x, y) for a vector field F is a scalar function such that:

$$
\nabla M(x, y) = F(x, y)
$$

which means:

$$
\frac{\partial M}{\partial x} = P(x, y) \quad \text{and} \quad \frac{\partial M}{\partial y} = Q(x, y)
$$

If such an M(x, y) exists, F is called a conservative vector field.

Understanding the Given Vector Field F(x, y) = (-8x, ...)

Given the vector field:

$$
F(x, y) = (-8x, Q(x, y))
$$

the typical goal is to find M(x, y) such that:

$$
\nabla M(x, y) = F(x, y)
$$

which translates into the system:

$$
\frac{\partial M}{\partial x} = -8x
$$
$$
\frac{\partial M}{\partial y} = Q(x, y)
$$

Assuming the field is conservative, the second component $Q(x, y)$ can be deduced from context or problem specifics.

Determining $M(x, y)$ for the Vector Field

The process of finding $M(x, y)$ involves integrating the components of F:

Step 1: Integrate the x-component

Since:

$$
\frac{\partial M}{\partial x} = -8x
$$

integrate with respect to x:

$$
M(x, y) = \int -8x \, dx = -4x^2 + h(y)
$$

where $h(y)$ is an arbitrary function of y (since the partial derivative with respect to x ignores $h(y)$).

Step 2: Find the y-component $Q(x, y)$

Now, differentiate $M(x, y)$ with respect to y:

$$
\frac{\partial M}{\partial y} = h'(y)
$$

which must equal $Q(x, y)$:

$$
Q(x, y) = h'(y)
$$

If the vector field's y-component is given or known, say $Q(x, y) = 0$, then:

$$
h'(y) = 0 \Rightarrow h(y) = C
$$

a constant. Therefore, the potential function simplifies to:

$$
M(x, y) = -4x^2 + C
$$

which is often presented as:

$$
\boxed{M(x, y) = -4x^2}
$$

Choosing the Correct Function M(x, y)

Given the derivations, the key is recognizing the structure of F(x, y) and the corresponding potential function.

Common Options for M(x, y):

Depending on the problem, options for M(x, y) might include:

- $-4x^2$
- $-4x^2 + y$
- $-4x^2 + y^2$
- Functions involving other terms in x and y

The correct M(x, y) must satisfy:

$$
\frac{\partial M}{\partial x} = -8x
$$

$$
\frac{\partial M}{\partial y} = Q(x, y)
$$

and be consistent with the given Q(x, y) component of the vector field.

Implications for the Vector Field and Potential Function

Understanding the relationship between the vector field and M(x, y) allows us to analyze:

- Conservative nature: Is F conservative? (If the curl of F is zero)
- Line integrals: The value of line integrals of F over paths depends only on endpoints if F is conservative.
- Applications: Potential functions are crucial in physics for energy calculations, fluid flow, and electromagnetic fields.

Additional Considerations in Choosing M(x, y)

When selecting or verifying the correct M(x, y), consider the following:

- Verify that the partial derivatives match the components of F(x, y).
- Check the domain of F to ensure the potential function exists and is well-defined.
- Ensure that the mixed partial derivatives are equal (Clairaut's theorem), i.e., $\frac{\partial^2 M}{\partial x \partial y} = \frac{\partial^2 M}{\partial y \partial x}$, confirming the field is conservative.

Summary and Best Practices in Identifying M(x, y)

To accurately choose M(x, y) for a given F(x, y):

1. Identify the components: Extract $P(x, y)$ and $Q(x, y)$.
2. Integrate the x-component: Find $M(x, y) = \int P(x, y) dx + h(y)$.
3. Determine $h(y)$: Use the y-component to find $h'(y) = Q(x, y)$.
4. Verify consistency: Confirm that mixed partial derivatives are consistent.
5. Simplify: Present M(x, y) in the simplest form.

Conclusion

Choosing the correct function M(x, y) for a vector field like F(x, y) = (-8x, ...) involves understanding the connection between the vector field components and potential functions. By integrating the partial derivatives and ensuring consistency, one can determine a valid scalar potential. In typical problems, the potential function often takes the form:

\[

\boxed{M(x, y) = -4x^2 + \text{(terms involving y)}}
\]

which satisfies the necessary conditions if the y-component aligns accordingly. Mastery of these concepts enables precise analysis of vector fields, line integrals, and applications across physics and engineering.

Note: For specific multiple-choice questions, always verify that your selected M(x, y) matches the derivatives of the given vector field components and satisfies the conditions for conservative fields.

Frequently Asked Questions

What is the most likely form of the scalar function M(x, y) if the vector field F(x, y) = (-8x, y) is its gradient?

$M(x, y) = -4x^2 + y^2 + C$

Given the vector field F(x, y) = (-8x, y), which of the following functions could be its potential function M(x, y)?

$M(x, y) = -4x^2 + y^2 + C$

If F(x, y) = (-8x, y) is a gradient field, what is the potential function M(x, y)?

$M(x, y) = -4x^2 + y^2 + \text{constant}$

Which function correctly represents the potential function M(x, y) for the vector field F(x, y) = (-8x, y)?

$M(x, y) = -4x^2 + y^2 + C$

For the vector field F(x, y) = (-8x, y), what is the potential function M(x, y) such that $F = \nabla M$?

$M(x, y) = -4x^2 + y^2 + C$

Additional Resources

Choosing the Correct Function M(x, y) for the Given Vector Field F(x, y) = (-8x)

Introduction

In the realm of vector calculus, understanding the relationship between vector fields and their potential functions is fundamental. When analyzing vector fields such as F(x, y) = (-8x), one of the principal objectives is to determine if the field is conservative—that is, whether there exists a scalar potential function M(x, y) such that its gradient reproduces the vector field. Correctly identifying this potential function is crucial for tasks ranging from computing line integrals to understanding the underlying physics of conservative force fields.

This article delves into the process of selecting the correct function M(x, y) corresponding to the vector field F(x, y) = (-8x). We will explore the properties of conservative vector fields, the mathematical conditions they satisfy, and the step-by-step procedure to find the potential function. Additionally, the article discusses common pitfalls, interpretations, and applications, providing a comprehensive understanding suitable for students, educators, and professionals in mathematics, physics, or engineering.

Understanding the Vector Field F(x, y) = (-8x)

Nature of the Vector Field

The vector field in question, F(x, y) = (-8x), appears to be a one-dimensional vector field with respect to the x-component, and no y-component specified. Typically, a vector field in two variables is expressed as:

$$
\mathbf{F}(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}
$$

where:

- $P(x, y)$ is the component along the x-axis.
- $Q(x, y)$ is the component along the y-axis.

Given the notation, F(x, y) = (-8x) suggests that the vector field has:

$$
\mathbf{F}(x, y) = -8x \mathbf{i}
$$

with

$$
P(x, y) = -8x, \quad Q(x, y) = 0
$$

This implies that the vector field points along the x-axis, with magnitude proportional to x, and no component in the y-direction.

Geometric and Physical Interpretations

Such a vector field can model various phenomena:

- Electric or gravitational fields in simplified models where the force acts only along one axis.
- Flow fields where movement occurs purely in the x-direction, with strength depending on the x-coordinate.

Understanding whether this field is conservative—meaning it can be derived from a potential function—is vital for calculating work done or energy stored within the field.

Conditions for a Vector Field to Be Conservative

The Concept of Conservative Fields

A vector field $\mathbf{F} = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}$ is called conservative if:

- There exists a scalar function $M(x, y)$ such that

$$
\nabla M(x, y) = \left(\frac{\partial M}{\partial x}, \frac{\partial M}{\partial y} \right) = \mathbf{F}(x, y)
$$

- Equivalently,

$$
\frac{\partial M}{\partial x} = P(x, y), \quad \frac{\partial M}{\partial y} = Q(x, y)
$$

- An important condition, especially in simply connected regions, is that the curl of $\mathbf{F}$ must be zero:

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0
$$

This is known as the curl test for conservative fields in two dimensions.

Applying Conditions to Our Field

Given:

$$
P(x, y) = -8x, \quad Q(x, y) = 0
$$

Compute the curl:

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - \frac{\partial (-8x)}{\partial y} = 0 - 0
$$

$$
= 0
$$

Since the curl is zero everywhere and $\mathbf{F}$ is defined over a simply connected region (the entire plane, for example), the field is conservative, and a potential function $M(x, y)$ exists.

Finding the Potential Function M(x, y)

Step-by-Step Process

Given that:

$$
\frac{\partial M}{\partial x} = P(x, y) = -8x
$$

and

$$
\frac{\partial M}{\partial y} = Q(x, y) = 0
$$

we can proceed to find $M(x, y)$:

1. Integrate $P(x, y)$ with respect to x:

$$
M(x, y) = \int -8x \, dx + h(y)
$$

where $h(y)$ is an arbitrary function of y because the partial differentiation with respect to x eliminates terms solely in y.

Performing the integration:

$$
M(x, y) = -4x^2 + h(y)
$$

2. Differentiate $M(x, y)$ with respect to y:

$$
\frac{\partial M}{\partial y} = h'(y)
$$

But from earlier, the partial derivative with respect to y must equal $Q(x, y) = 0$:

$$
h'(y) = 0
$$

\]

which implies:

$$
h(y) = C
$$

where C is a constant.

3. Final form of $M(x, y)$:

$$
\boxed{
M(x, y) = -4x^2 + C
}
$$

The constant C can be chosen arbitrarily, often set to zero for simplicity.

Interpretation and Significance of the Result

The Potential Function

The potential function:

$$
\boxed{
\text{M}(x, y) = -4x^2
}
$$

serves as a scalar field from which the original vector field can be derived via gradient:

$$
\nabla M = \left(\frac{\partial M}{\partial x}, \frac{\partial M}{\partial y} \right) = (-8x, 0)
$$

This confirms that the original vector field $F(x, y) = (-8x, 0)$ is indeed conservative, with $M(x, y)$ acting as the potential.

Practical Implications

- Work Calculation: In physics, the work done by a conservative force field along a path is determined solely by the endpoints, and the potential function supports straightforward calculations.
- Energy Storage: The potential function indicates how energy is stored within the field, with the quadratic dependence on x suggesting a parabolic potential profile.
- Flow and Dynamics: In fluid dynamics, such a potential may describe potential flows with specific boundary conditions.

Common Pitfalls and Clarifications

Misinterpretation of the Vector Field Components

- Ensure clarity whether the vector field has only an x-component or both x and y components. The problem context suggests the latter, but sometimes notation can be ambiguous.
- If the vector field had a non-zero y-component, the potential function would differ, and curl conditions would need reevaluation.

Region of Definition

- The derivation assumes a simply connected region (e.g., the entire plane). If the domain were restricted or had holes, the field might not be globally conservative.

Constant of Integration

- The additive constant C in potential functions is often omitted or set to zero, but it can be important when considering potential differences or boundary conditions.

Broader Context and Applications

In Physics

- Electrostatics: Conservative electric fields derive from potentials, and understanding the potential function simplifies calculations of forces and energy.
- Gravity: Gravitational fields near Earth are conservative and described by potential functions similar in form to the quadratic function found here.

In Engineering

- Fluid Mechanics: Potential flow theory uses scalar potentials to analyze irrotational and incompressible flows.
- Electromagnetism: Scalar potentials provide a means to simplify Maxwell's equations under certain conditions.

In Mathematics and Education

- The process exemplifies key concepts in vector calculus, such as gradient fields, curl, and potential functions.
- It demonstrates the importance of verifying field properties before attempting to find potential functions.

Summary and Final Remarks

Choosing the correct potential function $M(x, y)$ for the vector field $\mathbf{F}(x, y) = (-8x, 0)$

involves verifying the field's properties, particularly its conservativeness. The curl condition confirms that the field is conservative, allowing us to find \(M(x, y) \) by integrating the components and ensuring consistency across derivatives.

The derived potential function:

$$
\boxed{
M(x, y) = -4x^2 + C
}
$$

encapsulates the essence of the vector field, providing a scalar measure from which the field can be reconstructed. This process exemplifies fundamental principles in vector calculus, with wide-ranging applications in physics, engineering, and mathematics.

By understanding these concepts thoroughly, students and professionals can analyze similar vector

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