Consider The Area In The First Quadrant Bounded Byy = 225-x9.1 (1 Mark)Firstly, Find The Exact Volume

Consider The Area In The First Quadrant Bounded By y = 225 - x^{9.1} (1 Mark) Firstly, Find The Exact Volume

Understanding how to determine the volume of a solid generated by a given region is a fundamental concept in calculus, particularly in the study of solids of revolution. In this article, we will explore the problem of finding the exact volume of a solid formed when the region bounded by the curve y = 225 - x^{9.1} in the first quadrant is revolved around a specified axis. We will systematically analyze the problem, define the boundaries, set up the integral, and compute the volume with detailed explanations to ensure clarity.

Analyzing the Region Bounded by y = 225 - x^{9.1}

Before diving into the calculation of the volume, it is crucial to understand the shape and boundaries of the region involved.

1. The Equation and Its Graph

- The curve is given by y = 225 - x^{9.1}.
- The domain of interest is the first quadrant, where x ≥ 0 and y ≥ 0.
- As x increases, x^{9.1} increases rapidly because of the high exponent, which affects the shape of the curve.

2. Boundary Points in the First Quadrant

- To find where the curve intersects the axes:

- **At y = 0:** 0 = 225 - x^{9.1} ⇒ x^{9.1} = 225 ⇒ x = (225)^{1/9.1}
- **At x = 0:** y = 225 - 0 = 225

- Therefore, the region is bounded between:

- $x = 0$ and $x = (225)^{1/9.1}$
- $y = 0$ and $y = 225 - x^{9.1}$

3. Visualizing the Region

- The curve starts at (0, 225) when $x=0$.
- It decreases as x increases, approaching $y=0$ at $x = (225)^{1/9.1}$.
- The region is the area under this curve from $x=0$ to $x=(225)^{1/9.1}$.

Setting Up the Integral for the Volume

The next step is to determine how the volume is generated. Typically, the volume of a solid of revolution is computed by revolving the region around a specified axis. The most common axes are:

- The x-axis
- The y-axis
- Any vertical or horizontal line

In this scenario, since the problem statement does not specify the axis of revolution, we will assume the region is revolved about the x-axis, which is a standard approach.

1. Method of Disks/Washers

- When revolving around the x-axis, the volume element at a given x is a disk with radius equal to $y = 225 - x^{9.1}$.
- The volume of each infinitesimal disk: $dV = \pi [\text{radius}]^2 dx = \pi [225 - x^{9.1}]^2 dx$.

2. Integral Expression for the Volume

- The total volume V is obtained by integrating the volume element over the interval $x = 0$ to $x = (225)^{1/9.1}$:

$$
V = \int_{0}^{(225)^{1/9.1}} \pi \left(225 - x^{9.1}\right)^2 dx
$$

Calculating the Exact Volume

The integral expression is:

$$
V = \pi \int_{0}^{(225)^{1/9.1}} \left(225 - x^{9.1}\right)^2 dx
$$

To evaluate this integral, we proceed step-by-step.

1. Expand the Square

$$
(225 - x^{9.1})^2 = 225^2 - 2 \times 225 \times x^{9.1} + (x^{9.1})^2
$$

Calculating each term:

- $225^2 = 50,625$
- $2 \times 225 = 450$

So,

$$
(225 - x^{9.1})^2 = 50,625 - 450 x^{9.1} + x^{18.2}
$$

2. Rewrite the Integral

$$
V = \pi \int_{0}^{(225)^{1/9.1}} \left(50,625 - 450 x^{9.1} + x^{18.2}\right) dx
$$

This can be split into three integrals:

$$
V = \pi \left[50,625 \int_{0}^{a} dx - 450 \int_{0}^{a} x^{9.1} dx + \int_{0}^{a} x^{18.2} dx \right]
$$

where $a = (225)^{1/9.1}$.

Evaluating the Integrals

Now, we compute each integral separately.

1. First integral:

$$
\int_{0}^{a} dx = a
$$

2. Second integral:

$$
\int_{0}^{a} x^{9.1} dx = \frac{x^{10.1}}{10.1} \Big|_{0}^{a} = \frac{a^{10.1}}{10.1}
$$

3. Third integral:

$$
\int_{0}^{a} x^{18.2} dx = \frac{x^{19.2}}{19.2} \Big|_{0}^{a} = \frac{a^{19.2}}{19.2}
$$

Substituting Back and Final Expression

Putting everything together:

$$
V = \pi \left[50,625 \times a - 450 \times \frac{a^{10.1}}{10.1} + \frac{a^{19.2}}{19.2} \right]
$$

Recall that:

$$
a = (225)^{1/9.1}
$$

which is a numerical value that can be approximated if needed.

Numerical Approximation of the Volume

To obtain a numerical value, proceed as follows:

1. Calculate $a = (225)^{1/9.1}$

- Take natural logarithm:

$$
\ln a = \frac{1}{9.1} \ln 225
$$

- Since $\ln 225 \approx 5.4161$,

$$
\ln a \approx \frac{5.4161}{9.1} \approx 0.5955
$$

- Exponentiating:

$$
a \approx e^{0.5955} \approx 1.813
$$

2. Calculate $a^{10.1}$ and $a^{19.2}$

- $a^{10.1} = e^{10.1 \times \ln a} = e^{10.1 \times 0.5955} \approx e^{6.006} \approx 403.4$

- $a^{19.2} = e^{19.2 \times 0.5955} \approx e^{11.44} \approx 92,972$

3. Compute the volume

$$
V \approx \pi \left[50,625 \times 1.813 - 450 \times \frac{403.4}{10.1} + \frac{92,972}{19.2} \right]
$$

Calculations:

- $50,625 \times 1.813 \approx 91,747.125$

- $450 \times \frac{403.4}{10.1} \approx 450 \times 39.94 \approx 17,973.6$

- $\frac{92,972}{19.2} \approx 4,846.76$

Putting it all together:

$$
V \approx \pi (91,747.125 - 17,973.6 + 4,846.76) \approx \pi (78,620.285)
$$

Finally, multiply by π (~3.1416):

$$
V \approx 3.1416 \times 78,620.285 \approx 246,964.8
$$

Summary and Final Results

- The exact volume of the solid generated by revolving the region bounded by $y = 225 - x^{9.1}$ in the first quadrant about the x-axis is given by the integral:

$$
V = \pi \left[50,625 \times a - 450 \times \frac{a^{10.1}}{10.1} + \frac{a^{19.2}}
$$

Frequently Asked Questions

What is the first step to find the exact volume of the area bounded by y = 225 - x^2 in the first quadrant?

The first step is to identify the limits of integration by finding the points where $y = 0$, which occur at $x = 15$, and to set up the integral for the volume calculation accordingly.

How do you interpret the area bounded by y = 225 - x^2 in the context of volume calculation?

This area represents the region under the parabola $y = 225 - x^2$ in the first quadrant, which can be revolved around an axis to find the volume or integrated directly to find the area, depending on the problem's specifics.

What is the formula for finding the exact volume when

revolving the area bounded by $y = 225 - x^2$ about the x-axis?

The volume V is given by the integral $V = \pi \int$ from 0 to 15 of $[225 - x^2]^2\,dx$, where the limits are determined by the x-intercepts.

How do you evaluate the integral to find the exact volume for the given area?

You expand the integrand $[225 - x^2]^2$ to $(225)^2 - 2\times225\times x^2 + x^4$, then integrate term-by-term from 0 to 15 and multiply by π to obtain the volume.

What is the significance of the bounds x=0 and x=15 in calculating the volume?

They represent the points where the parabola intersects the x-axis, defining the limits of integration for calculating the volume of the bounded region in the first quadrant.

Can the exact volume be expressed in a simplified closed form after integration?

Yes, after performing the integration and simplification, the volume can be expressed as a combination of polynomial and constant terms involving powers of 15, multiplied by π, providing an exact closed-form expression.

Additional Resources

Consider the Area in the First Quadrant Bounded by $y = 225 - x^{9.1}$: An In-Depth Analysis and Volume Calculation

Understanding the geometrical and mathematical properties of complex functions is a cornerstone of calculus and analytical geometry. In this detailed exploration, we examine the bounded area in the first quadrant under the curve $y = 225 - x^{9.1}$, and proceed to determine the exact volume generated when this area is revolved about a specified axis. This comprehensive review aims to clarify the process step-by-step, offering insights into the underlying principles and methods involved.

Introduction to the Problem and Its Significance

Before diving into calculations, it is crucial to understand the significance of the problem:

- Relevance in Calculus: Calculating areas and volumes bounded by curves is foundational in integral calculus, with applications spanning physics, engineering, and computer

graphics.
- Complex Functions: The function y = 225 - x^9.1 is non-standard, involving a fractional exponent greater than 1, which influences the shape and properties of the graph.
- First Quadrant Focus: Limiting the analysis to the first quadrant (x ≥ 0, y ≥ 0) simplifies the domain considerations and ensures the bounded region is well-defined and finite.

Graphical Interpretation of y = 225 - x^9.1

Visualizing the curve aids in understanding the bounded region:

- Shape of the Curve:
- At x = 0: y = 225, the maximum y-value.
- As x increases, y decreases sharply due to the high power of 9.1.
- For large x, y approaches negative infinity theoretically, but since we're focusing on the first quadrant, the region of interest is where y ≥ 0.

- Intercepts:
- Y-intercept: When x = 0, y = 225.
- X-intercept: Solve 225 - x^9.1 = 0 → x^9.1 = 225 → x = (225)^{1/9.1}.

- Domain in the First Quadrant:
- x values range from 0 up to x = (225)^{1/9.1}.

Understanding the shape and bounds of this region sets the stage for calculating its area and the subsequent volume.

Calculating the Area in the First Quadrant Bounded by the Curve

The initial step involves finding the exact area under the curve between x = 0 and the x-intercept:

Step 1: Determine the Limits of Integration

- The upper limit, x = a, where a = (225)^{1/9.1}.
- The lower limit is x = 0.

Step 2: Set Up the Integral for Area

$$
A = \int_{0}^{a} y \, dx = \int_{0}^{a} (225 - x^{9.1}) \, dx
$$

Step 3: Compute the Integral

- Break down the integral into two parts:

$$
A = \int_{0}^{a} 225 \, dx - \int_{0}^{a} x^{9.1} \, dx
$$

- Evaluate each:

1. $\int_{0}^{a} 225 \, dx = 225a$

2. $\int_{0}^{a} x^{9.1} \, dx = \frac{x^{10.1}}{10.1} \bigg|_{0}^{a} = \frac{a^{10.1}}{10.1}$

- Therefore,

$$
A = 225a - \frac{a^{10.1}}{10.1}
$$

Step 4: Express the Limit a Explicitly

- Recall that:

$$
a = (225)^{1/9.1}
$$

- For precise calculations, leave a in this form or compute numerically as needed.

Exact Numerical Computation of Area

To obtain the numerical value:

- Calculate $a = (225)^{1/9.1}$:

Using logarithms:

$$
a = e^{\frac{\ln 225}{9.1}}
$$

Approximate:

$$
\ln 225 \approx 5.416
$$

$$
a \approx e^{5.416/9.1} \approx e^{0.595} \approx 1.813
$$

- Compute $a^{10.1}$:

$$
a^{10.1} = e^{10.1 \times \ln a} \approx e^{10.1 \times 0.595} = e^{6.005} \approx 403.7
$$

- Now, the area:

$$
A = 225 \times 1.813 - \frac{403.7}{10.1} \approx 407.925 - 40.0 \approx 367.925
$$

Thus, the exact area under the curve in the first quadrant is approximately 368 square units.

Extending to Volume Calculation: Revolution About an Axis

The problem's next phase involves computing the volume generated by revolving this region about an axis. Common axes are the x-axis, y-axis, or other lines; assuming the typical case, we consider revolution about the x-axis.

Why Calculate Volume?

- Real-World Applications: Volumes of revolved shapes model objects like vases, bottles, or mechanical parts.
- Methodology: The disk/washer method is a classical approach to finding such volumes.

Step 1: Set Up the Volume Integral

- When revolving around the x-axis, each cross-section perpendicular to the x-axis resembles a disk with radius $y(x)$.
- The volume element:

$$
dV = \pi [y(x)]^2 dx
$$

- Total volume:

$$
V = \int_{0}^{a} \pi [225 - x^{9.1}]^2 dx
$$

Step 2: Expand the Square

$$
V = \pi \int_{0}^{a} \left(225^2 - 2 \times 225 x^{9.1} + x^{18.2}\right) dx
$$

$$
V = \pi \left[\int_{0}^{a} 225^2 dx - 2 \times 225 \int_{0}^{a} x^{9.1} dx + \int_{0}^{a} x^{18.2} dx \right]
$$

- Calculate each integral separately:

1. $\int_{0}^{a} 225^2 dx = 225^2 a$

2. $\int_{0}^{a} x^{9.1} dx = \frac{a^{10.1}}{10.1}$

3. $\int_{0}^{a} x^{18.2} dx = \frac{a^{19.2}}{19.2}$

- The volume expression:

$$
V = \pi \left[225^2 a - 2 \times 225 \times \frac{a^{10.1}}{10.1} + \frac{a^{19.2}}{19.2} \right]
$$

Numerical Calculation of the Volume

Using previous approximations:

- $a \approx 1.813$

- Compute $a^{10.1} \approx 403.7$ (from earlier)

- Compute $a^{19.2}$:

$$
a^{19.2} = e^{19.2 \times 0.595} \approx e^{11.436} \approx 92,707
$$

- Constants:

$$
225^2 = 50,625
$$

- Now, substitute:

$$
V \approx \pi \left[50,625 \times 1.813 - 2 \times 225 \times \frac{403.7}{10.1} + \frac{92,707}{19.2} \right]
$$

Calculate each component:

1. $50,625 \times 1.813 \approx 91,690$

2. $2 \times 225 = 450$

3. $\frac{403.7}{10.1} \approx 40.0$

4. $450 \times 40.0 = 1,600$

5. $\frac{92,707}{19.2} \approx 4,829$

Putting it all together:

$$
V \approx \pi (91,690 - 1,600 + 4,829) = \pi (94,919) \approx 3.1416 \times 94,919 \approx 297,872
$$

Final approximate volume: ~297,872 cubic units

Summary and Insights

This extensive analysis demonstrates the process of:

- Determining the bounded area under a complex curve in the first quadrant.
- Applying definite

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