Consider The Definite Integral. 13(12x1)e 6x 2xdx Let U=6x 2x. Use The Substitution Method To Rewrite

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When tackling complex integrals, especially those involving exponential functions and polynomial expressions, substitution methods serve as powerful tools to simplify and evaluate the integral efficiently. In this comprehensive guide, we will explore the process of rewriting a given definite integral using substitution. The integral in question is:

$$ \int 13(12x + 1) e^{6x} \, 2x \, dx $$

with the substitution $U = 6x + 2x$. Although this particular statement appears somewhat ambiguous at first glance, our goal is to interpret and systematically analyze it, clarify the substitution process, and demonstrate how to evaluate the integral step by step.

Understanding the Integral and the Substitution Strategy

Before diving into the substitution process, it’s essential to thoroughly understand the integral's structure and the purpose of substitution.

Analyzing the Given Integral

The integral appears to combine multiple functions:

- A polynomial expression involving x: $(12x + 1)$
- An exponential function: e^{6x}
- A linear factor: $2x$
- A constant multiplier: 13

Putting these together, the integral can be written explicitly as:

$$ \int 13 \times (12x + 1) \times e^{6x} \times 2x \, dx $$

which simplifies to:

$$ \int 13 \times 2x \times (12x + 1) \times e^{6x} \, dx $$

The constant 13 can be factored out, giving:

$$13 \int 2x (12x + 1) e^{6x} \, dx$$

Our primary focus is on the integral:

$$\int 2x (12x + 1) e^{6x} \, dx$$

Purpose of Substitution

Substitution aims to:

- Simplify the exponential component e^{6x} by choosing an appropriate substitution.
- Reduce the polynomial expression to a function of a new variable U that makes the integral more manageable.
- Eliminate complex nested expressions involving x, enabling straightforward integration.

The key is selecting a substitution that transforms the complicated integrand into a product involving a single variable, preferably with a straightforward derivative.

Choosing the Appropriate Substitution

Effective substitution relies on identifying a part of the integrand whose derivative is also present within the integral.

Analyzing the Components

The integrand contains:

- e^{6x}
- Polynomial expressions: $2x (12x + 1)$

Potential substitution candidates include:

- The exponent $6x$, as derivatives of e^{6x} involve $6 e^{6x}$.
- The polynomial $12x + 1$, which resembles the derivative of $6x^2 + x$, but not directly.

Proposed Substitution: $U = 6x + 2x$ or Variations?

The initial problem states to consider the substitution:

$$U = 6x + 2x$$

which simplifies to:

$$U = 8x$$

If this is the intended substitution, then:

$$U = 8x$$
$$\Rightarrow dU = 8 \, dx$$
$$\Rightarrow dx = \frac{dU}{8}$$

Furthermore, express all parts of the integrand in terms of U:

- $x = \frac{U}{8}$
- $12x + 1 = 12 \times \frac{U}{8} + 1 = \frac{12U}{8} + 1 = \frac{3U}{2} + 1$
- $2x = 2 \times \frac{U}{8} = \frac{U}{4}$
- $e^{6x} = e^{6 \times \frac{U}{8}} = e^{\frac{6U}{8}} = e^{\frac{3U}{4}}$

Now, rewrite the integral:

$$13 \times \int \left(\frac{U}{4} \right) \left(\frac{3U}{2} + 1 \right) e^{\frac{3U}{4}} \times \frac{dU}{8}$$

which simplifies constants:

$$\frac{13}{8} \times \int \left(\frac{U}{4} \right) \left(\frac{3U}{2} + 1 \right) e^{\frac{3U}{4}} \, dU$$

Further simplification involves algebraic expansion.

Step-by-Step Solution Using Substitution

Let's now proceed to the detailed steps of rewriting and evaluating the integral.

Step 1: Rewrite the integral with substitution $U = 8x$

As established:

$$ U = 8x $$
$$ dU = 8 dx \Rightarrow dx = \frac{dU}{8} $$

Express the integrand in U:

- $x = \frac{U}{8}$
- $12x + 1 = \frac{3U}{2} + 1$
- $2x = \frac{U}{4}$
- $e^{6x} = e^{\frac{3U}{4}}$

The integral becomes:

$$ 13 \times \int \left(\frac{U}{4} \right) \left(\frac{3U}{2} + 1 \right) e^{\frac{3U}{4}} \times \frac{dU}{8} $$

Pull constants outside:

$$ \frac{13}{8} \times \int \left(\frac{U}{4} \right) \left(\frac{3U}{2} + 1 \right) e^{\frac{3U}{4}} \, dU $$

Step 2: Simplify the algebraic expression inside the integral

Calculate the product:

$$ \left(\frac{U}{4} \right) \left(\frac{3U}{2} + 1 \right) $$

which expands to:

$$ \frac{U}{4} \times \frac{3U}{2} + \frac{U}{4} \times 1 = \frac{U}{4} \times \frac{3U}{2} + \frac{U}{4} $$

Calculate each term:

- $\frac{U}{4} \times \frac{3U}{2} = \frac{3U^2}{8}$
- $\frac{U}{4}$ remains as is.

Therefore, the integrand simplifies to:

$$ \left(\frac{3U^2}{8} + \frac{U}{4} \right) e^{\frac{3U}{4}} $$

The entire integral:

$$ \frac{13}{8} \times \int \left(\frac{3U^2}{8} + \frac{U}{4} \right) e^{\frac{3U}{4}} \, dU $$

Step 3: Factor constants and set up for integration

Pull constants outside:

$$ \frac{13}{8} \times \left(\int \frac{3U^2}{8} e^{\frac{3U}{4}} \, dU + \int \frac{U}{4} e^{\frac{3U}{4}} \, dU \right) $$

which simplifies to:

$$ \frac{13}{8} \times \left(\frac{3}{8} \int U^2 e^{\frac{3U}{4}} \, dU + \frac{1}{4} \int U e^{\frac{3U}{4}} \, dU \right) $$

Expressed as:

$$ \frac{13}{8} \times \left(\frac{3}{8} I_1 + \frac{1}{4} I_2 \right) $$

where

$$ I_1 = \int U^2 e^{\frac{3U}{4}} \, dU $$
$$ I_2 = \int U e^{\frac{3U}{4}} \, dU $$

Evaluating the Integrals I_1 and I_2

These integrals involve polynomial times exponential functions, which are classic candidates for integration by parts.

Calculating $I_2 = \int U e^{\frac{3U}{4}} \, dU$

Method: Integration by Parts

- Let:

$u = U \Rightarrow du = dU$

\(dv = e^{\frac{3U

Frequently Asked Questions

What is the main goal when considering the definite

integral of 13(12x1)e^{6x} 2x dx using substitution?

The main goal is to simplify the integral by choosing an appropriate substitution, such as U = 6x + 2x, to make the integral easier to evaluate.

How do you define the substitution U = 6x + 2x in the context of this integral?

You combine like terms to get U = 8x, which simplifies the integral by replacing the expressions involving x with U, making integration more straightforward.

What is the next step after setting U = 8x in rewriting the integral?

Differentiate U with respect to x to find dU = 8 dx, then express dx in terms of dU to change the integral variables accordingly.

How does substitution help in evaluating the integral of 13(12x + 1) e^{6x + 2x} dx?

Substitution transforms the integral into a form involving U and dU, often converting it into a basic exponential integral that is easier to evaluate.

What are the limits of integration after substituting U = 8x for a definite integral?

You substitute the original limits x = a and x = b into U = 8x to find the new limits U_a = 8a and U_b = 8b, ensuring the integral is correctly evaluated over the new bounds.

Why is the substitution method particularly useful for integrals involving exponential functions like e^{6x + 2x}?

Because it simplifies the exponential expression by converting it into a single variable U, making the integral more manageable and straightforward to evaluate.

Additional Resources

Consider The Definite Integral. 13(12x1)e 6x 2xdx Let U=6x 2x. Use The Substitution Method To Rewrite is a complex mathematical problem that offers an excellent opportunity to explore the power of substitution in calculus, especially in evaluating definite integrals. This problem encapsulates core concepts that are vital for students and professionals working with integration techniques. In this comprehensive review, we will delve into the details of the problem, understand the substitution method, analyze step-by-step solutions, and discuss the broader implications and features of this approach.

Understanding the Problem: A Closer Look

The problem statement appears to be a combination of integral notation and algebraic expressions, which can initially seem confusing. To clarify, it involves evaluating a definite integral that involves exponential functions and algebraic expressions, with a hint for substitution.

The core components are:

- The integral expression: $\int 13(12x + 1) e^{6x} 2x \, dx$
- The substitution variable: $U = 6x + 2x$

While the notation in the original problem may be slightly ambiguous, the key idea is to simplify the integral by substitution, making it more manageable.

Key observations:

- The integral involves a product of polynomial expressions and an exponential function.
- The substitution $U = 6x + 2x$ suggests combining similar terms to simplify the integral.
- The goal is to rewrite the integral in terms of U, perform the integration, and then back-substitute to return to the original variable.

The Substitution Method in Calculus

The substitution method, also known as u-substitution, is a fundamental technique in calculus used to evaluate integrals that are difficult to compute directly. It involves replacing a complicated expression with a single variable, simplifying the integral into a basic form.

Features of the Substitution Method

- Simplification of integrals: Transforms complex integrals into more manageable forms.
- Reverse substitution: After integration, substitute back to original variables.
- Applicability: Particularly useful when the integral contains composite functions or products whose derivatives are present within the integral.

Pros and Cons

Pros:

- Significantly simplifies the process of integration.
- Helps recognize patterns and parts of the integrand that are derivatives of other parts.
- Essential in solving complex integrals involving exponential, logarithmic, or trigonometric functions.

Cons:

- Requires careful algebraic manipulation.
- Mistakes in substitution or back-substitution can lead to incorrect answers.
- Not always straightforward; some integrals may require multiple substitutions or advanced techniques.

Step-by-Step Solution Using Substitution

Let's now walk through the process of rewriting and evaluating the integral based on the problem statement.

Step 1: Clarify the Integral Expression

Assuming the integral is:

$$
\int 13 (12x + 1) e^{6x} 2x \, dx
$$

The terms are:

- Polynomial part: $13 (12x + 1) 2x$
- Exponential part: e^{6x}

The integral becomes:

$$
\int 13 (12x + 1) 2x e^{6x} \, dx
$$

which simplifies to:

$$
\int 26 (12x + 1) x e^{6x} \, dx
$$

This is a complex integral involving a product of polynomial and exponential functions.

Step 2: Choose the Substitution

Given the exponential term e^{6x}, it makes sense to set:

$$
U = 6x
$$

Then,

$$
dU = 6 \, dx \quad \Rightarrow \quad dx = \frac{dU}{6}
$$

Express the polynomial parts in terms of U. Note:

$$
12x + 1 = 2U + 1
$$

and

$$
x = \frac{U}{6}
$$

Now, the integrand becomes:

$$
26 \times (2U + 1) \times \frac{U}{6} \times e^{U} \times \frac{dU}{6}
$$

Simplify constants:

$$
\frac{26}{6 \times 6} = \frac{26}{36} = \frac{13}{18}
$$

So, the integral in terms of U:

$$
\frac{13}{18} \int (2U + 1) U e^{U} \, dU
$$

Evaluating the Transformed Integral

The integral simplifies to:

$$
\frac{13}{18} \int (2U + 1) U e^{U} \, dU
$$

which can be expanded as:

$$
\frac{13}{18} \int (2U^2 + U) e^{U} \, dU
$$

This integral can now be separated into two parts:

$$
\frac{13}{18} \left[2 \int U^2 e^{U} \, dU + \int U e^{U} \, dU \right]
$$

Step 3: Integration by Parts

To evaluate these integrals, apply integration by parts:

$$
\int U^n e^{U} \, dU
$$

which generally involves recursive reduction.

First, evaluate $\int U^2 e^{U} \, dU$:

Let:

- $v = U^2 \Rightarrow dv = 2U \, dU$
- $dw = e^{U} \, dU \Rightarrow w = e^{U}$

Applying integration by parts:

$$
\int U^2 e^{U} \, dU = U^2 e^{U} - \int 2U e^{U} \, dU
$$

Now, evaluate $\int U e^{U} \, dU$:

Let:

- $v = U \Rightarrow dv = dU$
- $dw = e^{U} \, dU \Rightarrow w = e^{U}$

Then:

$$
\int U e^{U} \, dU = U e^{U} - \int e^{U} \, dU = U e^{U} - e^{U} + C
$$

Returning to the previous expression:

$$
\int U^2 e^{U} \, dU = U^2 e^{U} - 2 [U e^{U} - e^{U}] = U^2 e^{U} - 2U e^{U} + 2 e^{U}
$$

Similarly, the integral $\int U e^{U} \, dU$ is:

$$
U e^{U} - e^{U}
$$

Step 4: Assemble the Results

Putting everything together:

$$
\frac{13}{18} \left[2 (U^2 e^{U} - 2U e^{U} + 2 e^{U}) + (U e^{U} - e^{U}) \right]
$$

Simplify inside the brackets:

$$
2 U^2 e^{U} - 4 U e^{U} + 4 e^{U} + U e^{U} - e^{U}
$$

Combine like terms:

$$
2 U^2 e^{U} - 3 U e^{U} + 3 e^{U}
$$

Factor out e^{U}:

$$
e^{U} (2 U^2 - 3 U + 3)
$$

Now, multiply by the constant $\frac{13}{18}$:

$$
\frac{13}{18} e^{U} (2 U^2 - 3 U + 3)
$$

Finally, substitute back $U = 6x$:

$$
\boxed{
\frac{13}{18} e^{6x} \left[2 (6x)^2 - 3 (6x) + 3 \right]
}
$$

which simplifies as:

$$
\frac{13}{18} e^{6x} \left[2 \times 36 x^2 - 18 x + 3 \right] = \frac{13}{18} e^{6x} (72 x^2 - 18 x + 3)
$$

Broader Implications and Features of the Substitution Method

This example illustrates how substitution transforms a complicated integral into a more straightforward form, enabling easier evaluation. The key features and implications include:

- Efficiency in solving complex integrals: Substitution reduces the need for multiple integration techniques.
- Pattern recognition: Recognizing parts of the integrand that are derivatives of other parts simplifies the process.
- Versatility: Applicable across various types of functions, particularly exponential, polynomial, and logarithmic functions.

Features:

- Simplifies polynomial-exponential integrals
- Reduces multi-step calculations into manageable parts
- Enhances understanding of the relationship between functions and their derivatives

Limitations:

- Requires insight

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