Consider The Equation Below.f(x) = X8 Ln X(a) Find The Interval On Which F Is Increasing. (Enter Your

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Understanding the behavior of functions, especially their increasing and decreasing intervals, is fundamental in calculus. In this article, we will analyze the function $f(x) = x^8 \ln x$, determine where it is increasing, and explore the methods involved in such analysis. Whether you're a student preparing for exams or someone interested in mathematical functions, this comprehensive guide will clarify the process step-by-step.

Introduction to the Function $f(x) = x^8 \ln x$

Before diving into the analysis, it's essential to understand the structure of the function:

- Domain: The natural logarithm $\ln x$ is defined only for $x > 0$. Therefore, the domain of $f(x)$ is $x > 0$.

- Components:
- The polynomial component x^8, which is always non-negative and increasing for $x > 0$.
- The logarithmic component $\ln x$, which is increasing for $x > 0$.

The combination of these two functions influences how $f(x)$ behaves — whether it increases or decreases over specific intervals.

Finding the Derivative of $f(x)$

To analyze where $f(x)$ is increasing, we need to examine its first derivative $f'(x)$:

$$
f(x) = x^8 \ln x
$$

Applying the product rule:

$$
f'(x) = \frac{d}{dx} (x^8) \cdot \ln x + x^8 \cdot \frac{d}{dx} (\ln x)
$$

Calculating derivatives:

- $\frac{d}{dx} (x^8) = 8x^7$
- $\frac{d}{dx} (\ln x) = \frac{1}{x}$

Thus,

$$
f'(x) = 8x^7 \ln x + x^8 \cdot \frac{1}{x} = 8x^7 \ln x + x^7
$$

Factor out x^7:

$$
f'(x) = x^7 (8 \ln x + 1)
$$

This derivative is valid for $x > 0$, aligning with the domain.

Determining the Sign of $f'(x)$

The critical step in identifying increasing intervals is analyzing where $f'(x) > 0$:

$$
f'(x) = x^7 (8 \ln x + 1) > 0
$$

Since $x^7 > 0$ for all $x > 0$, the sign of $f'(x)$ depends solely on the term $8 \ln x + 1$:

$$
8 \ln x + 1 > 0
$$

Solve for x:

$$
8 \ln x > -1
$$

$$
\ln x > -\frac{1}{8}
$$

Exponentiating both sides:

$$
x > e^{-\frac{1}{8}}
$$

Numerically, $e^{-\frac{1}{8}} \approx e^{-0.125} \approx 0.8825$.

Summary of the Sign Analysis:

- For $x > e^{-\frac{1}{8}}$ (approximately 0.8825), $f'(x) > 0$, so $f(x)$ is increasing.
- For $0 < x < e^{-\frac{1}{8}}$, $f'(x) < 0$, so $f(x)$ is decreasing.

Interval of Increase for $f(x)$

Based on the derivative analysis, the function $f(x) = x^8 \ln x$ is increasing on:

$$
\boxed{(e^{-\frac{1}{8}}, \infty)}
$$

or approximately,

$$
(0.8825, \infty)
$$

This means that starting just after $x \approx 0.8825$, the function begins to increase indefinitely as x grows larger.

Graphical Interpretation and Practical Implications

Visualizing the function can provide additional insights:

- At $x \to 0^+$: The $\ln x$ tends to $-\infty$, and since $x^8 \to 0$, the product $x^8 \ln x$ tends to 0 from the negative side, indicating the function sharply drops near 0.
- At $x = 1$: $\ln 1 = 0$, so $f(1) = 0$. The function crosses the x-axis here.
- For large x: Both x^8 and $\ln x$ grow, leading to a rapidly increasing function.

Understanding where the function increases helps in optimization problems, graph sketching, and understanding the behavior of complex functions involving polynomial and logarithmic components.

Additional Considerations in Analyzing $f(x)$

While the primary focus was on increasing intervals, a comprehensive analysis also involves:

1. Critical Points:

- Found where $f'(x) = 0$:

$$
x^7 (8 \ln x + 1) = 0
$$

Since $x^7 \neq 0$ for $x > 0$, the critical point occurs at:

$$
8 \ln x + 1 = 0 \Rightarrow \ln x = -\frac{1}{8}
$$

$$
x = e^{-\frac{1}{8}} \approx 0.8825
$$

2. Second Derivative Test:

- To confirm whether this critical point is a maximum or minimum, compute $f''(x)$ and analyze its sign.

3. End Behavior:

- As $x \to 0^+$, $f(x) \to 0^-$ because $\ln x \to -\infty$, but multiplied by $x^8 \to 0$.
- As $x \to \infty$, $f(x) \to \infty$ since both x^8 and $\ln x$ grow large.

Conclusion

In summary, the function $f(x) = x^8 \ln x$ is increasing on the interval $\boxed{(e^{-\frac{1}{8}}, \infty)}$. This analysis hinges on computing the derivative, identifying the critical point, and examining the sign of the derivative around that point. Such methods are foundational in understanding the behavior of complex functions and are widely applicable in calculus, optimization, and mathematical modeling.

If you want to further explore the behavior of this function, consider plotting it to visualize the increasing interval or performing a second derivative test to confirm the nature of critical points. Mastery of these techniques enhances your ability to analyze a broad class of functions with confidence.

Frequently Asked Questions

What is the function given in the problem?

The function is $f(x) = x^8 \ln(x)$.

How do we determine the interval where f(x) is increasing?

By finding where the derivative f'(x) is positive, since a function increases where its derivative is greater than zero.

What is the first step to find the increasing interval of f(x)?

Calculate the derivative f'(x) of the function $f(x) = x^8 \ln(x)$.

What is the derivative f'(x) of the function f(x) = x^8 ln(x)?

Using the product rule, $f'(x) = 8x^7 \ln(x) + x^8 (1/x) = 8x^7 \ln(x) + x^7$.

How do we analyze the sign of f'(x) to find where f(x) is increasing?

Factor f'(x) as $x^7 (8 \ln(x) + 1)$ and determine where this expression is positive.

What is the critical point where the sign of f'(x) changes?

Set $8 \ln(x) + 1 = 0$, which gives $\ln(x) = -1/8$, so $x = e^{-1/8}$.

On which intervals is f(x) increasing?

f(x) is increasing where $x > e^{-1/8}$, since then $8 \ln(x) + 1 > 0$, making f'(x) positive.

What is the final answer for the interval where f(x) is increasing?

f(x) is increasing on the interval $(e^{-1/8}, \infty)$.

Additional Resources

Consider The Equation Below. f(x) = x^8 ln x (a) Find The Interval On Which f Is Increasing. (Enter Your Answer in Interval Notation)

Understanding where a function is increasing or decreasing is a fundamental aspect of calculus, providing insight into the behavior of the graph and the nature of the function. In this article, we will explore how to determine the intervals on which the function f(x) =

$x^8 \ln x$ is increasing. This involves calculating the derivative, analyzing its sign, and understanding the critical points that influence the function's increasing and decreasing behavior.

Introduction to the Function $f(x) = x^8 \ln x$

The function $f(x) = x^8 \ln x$ combines a polynomial component, x^8, with a logarithmic component, $\ln x$. Such a combination is typical in calculus problems that involve composite functions, and understanding their behavior requires a careful analysis of their derivatives.

Important considerations:

- The domain of $f(x)$ is $x > 0$ because $\ln x$ is only defined for positive x.
- The function's behavior depends on the interplay between the polynomial term x^8 (which is always positive for $x > 0$) and the logarithmic term $\ln x$ (which is negative for $0 < x < 1$, zero at $x=1$, and positive for $x > 1$).

Step 1: Find the Derivative of f(x)

To determine where $f(x)$ is increasing, we need to compute its first derivative $f'(x)$ and analyze its sign.

Given:
$$ f(x) = x^8 \ln x $$

Using the product rule:
$$ f'(x) = \frac{d}{dx} (x^8) \cdot \ln x + x^8 \cdot \frac{d}{dx} (\ln x) $$

Calculations:
- $\frac{d}{dx} (x^8) = 8x^7$
- $\frac{d}{dx} (\ln x) = \frac{1}{x}$

Therefore:
$$
f'(x) = 8x^7 \ln x + x^8 \cdot \frac{1}{x} = 8x^7 \ln x + x^7
$$

Factor out x^7:
$$
f'(x) = x^7 (8 \ln x + 1)
$$

Step 2: Analyze the Sign of f'(x)

Since $x^7 > 0$ for all $x > 0$, the sign of f'(x) depends solely on the term (8 ln x + 1).

Set:
$$
8 \ln x + 1 = 0
$$
to find critical points.

Solve for x:
$$
8 \ln x = -1 \Rightarrow \ln x = -\frac{1}{8} \Rightarrow x = e^{-1/8}
$$

Recall:
- For $x > 0$, $x = e^{-1/8} \approx 0.882$.

Now, analyze the sign:

- When $x < e^{-1/8}$:
$$
\ln x < -\frac{1}{8} \Rightarrow 8 \ln x + 1 < 0
$$
So, $f'(x) < 0$.

- When $x > e^{-1/8}$:
$$
\ln x > -\frac{1}{8} \Rightarrow 8 \ln x + 1 > 0
$$
So, $f'(x) > 0$.

Step 3: Determine the Intervals of Increase

Since f'(x) is positive where the function is increasing and negative where it is decreasing:

- f(x) is decreasing on: $(0, e^{-1/8})$
- f(x) is increasing on: $(e^{-1/8}, \infty)$

Summary of the Results

Interval	Behavior of f(x)
$(0, e^{-1/8})$	Decreasing
$(e^{-1/8}, \infty)$	Increasing

Answer in interval notation:

> The function f(x) = x^8 ln x is increasing on $(e^{-1/8}, \infty)$.

Additional Insights and Context

Why is this important?

Knowing where a function is increasing helps in graphing, optimization, and understanding real-world phenomena modeled by the function. For example, if $f(x)$ represented profit over units sold, the interval where it increases shows the range where the profit is rising.

Critical points and their significance:

- The point at $x = e^{-1/8} \approx 0.882$ is a critical point where the function transitions from decreasing to increasing.
- Because $f(x)$ involves $\ln x$, the only critical point in the domain occurs at this value, indicating a potential minimum point.

Visualizing the Behavior

While an actual graph isn't provided here, visualizing $f(x) = x^8 \ln x$ can reinforce understanding:

- For x close to 0 from the right, $\ln x$ is large negative, so $f(x)$ is negative and decreasing.
- At $x = e^{-1/8}$, $f'(x) = 0$, indicating a local minimum.
- For larger x, $f(x)$ grows rapidly since x^8 dominates, and the function is increasing.

Practical Tips for Analyzing Similar Functions

1. Identify the domain: For functions involving logarithms, ensure the domain is properly defined.
2. Differentiate carefully: Use the product rule when functions are products of two functions.
3. Factor the derivative: Simplify to identify critical points easily.
4. Solve for critical points: Set the derivative equal to zero and solve for the variable.
5. Analyze the sign of the derivative: Use test points or sign analysis to determine increasing/decreasing intervals.
6. Express intervals clearly: Use interval notation for clarity.

Conclusion

By computing the derivative of $f(x) = x^8 \ln x$ and analyzing its sign, we've determined that the function is increasing on the interval $(e^{-1/8}, \infty)$. This process underscores the power of calculus in understanding the behavior of complex functions and highlights the importance of critical points in analyzing where a function increases or decreases.

Remember: Always consider the domain of the function, perform derivative tests, and interpret the results within the context of the problem for comprehensive analysis.

Happy Calculating!

[Consider The Equation Below F X X8 Ln X A Find The Interval On Which F Is Increasing Enter Your]

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