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Consider The Following Consumers Problem: U(x,y) = X1/4 Y3/4. Prices Are Px =$10, Py = $10 And Income

This consumer problem presents a classic scenario in microeconomic theory, where an individual seeks to maximize their utility subject to a budget constraint. The utility function U(x,y) = x^{1/4} y^{3/4} reflects the consumer's preferences, indicating how they derive satisfaction from consuming quantities x and y of two goods. The prices Px = $10 and Py = $10 set the market values of these goods, and the consumer's income determines their purchasing power. Analyzing this problem involves understanding the consumer's optimization behavior, deriving the demand functions, and exploring the implications of the utility function's form and the budget constraint.

Understanding the Utility Function and Its Properties

The Cobb-Douglas Utility Function

The given utility function, U(x,y) = x^{1/4} y^{3/4}, is a form of Cobb-Douglas utility. Cobb-Douglas functions are widely used in microeconomics because they exhibit specific properties:

- **Constant Elasticity of Substitution (CES):** The elasticity of substitution between goods is constant and equal to 1.
- **Proportional Preferences:** Consumers allocate their budget proportionally to the exponents of the utility function.
- **Ease of Derivation:** The demand functions can be derived analytically using the method of Lagrangian multipliers.

In our case, the exponents are 1/4 for x and 3/4 for y, indicating that the consumer derives more satisfaction from good y than from good x. The sum of exponents is 1, which signifies that the utility function is homothetic and preferences are consistent and continuous.

Marginal Utilities and Their Significance

Understanding marginal utilities helps interpret how the consumer's satisfaction changes with small changes in consumption:

- Marginal Utility of x (MUx):
MUx = ∂U/∂x = (1/4) x^{-3/4} y^{3/4}

- Marginal Utility of y (MUy):
MUy = ∂U/∂y = (3/4) x^{1/4} y^{-1/4}

These marginal utilities indicate the additional utility gained from consuming an extra unit of each good while holding the other constant. The diminishing nature of these utilities reflects typical consumer preferences.

Formulating the Budget Constraint

Basic Budget Equation

Given the prices and income, the consumer's budget constraint is:

\[P_x \times x + P_y \times y = I \]

Substituting the known values:

\[10x + 10y = I \]

Dividing through by 10 simplifies to:

\[x + y = \frac{I}{10} \]

This linear constraint limits the combinations of x and y the consumer can afford.

Implications of the Budget Constraint

- The budget line intercepts at \(x = \frac{I}{10} \) when y = 0.
- It also intercepts at \(y = \frac{I}{10} \) when x = 0.
- The slope of the budget line is -1, indicating a trade-off between x and y at the rate of their prices.

The consumer aims to choose x and y along this line to maximize utility.

Maximizing Utility Subject to Budget Constraint

Setting Up the Optimization Problem

The consumer's goal is:

$$
\max_{x,y} U(x,y) = x^{1/4} y^{3/4}
$$

subject to:

$$
10x + 10y = I
$$

or equivalently:

$$
x + y = \frac{I}{10}
$$

To solve this, we use the method of Lagrange multipliers.

Constructing the Lagrangian

The Lagrangian function is:

$$
\mathcal{L} = x^{1/4} y^{3/4} - \lambda (10x + 10y - I)
$$

where λ is the Lagrange multiplier.

First-Order Conditions

Taking partial derivatives:

1. With respect to x:

$$
\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{4} x^{-3/4} y^{3/4} - 10 \lambda = 0
$$

2. With respect to y:

$$
\frac{\partial \mathcal{L}}{\partial y} = \frac{3}{4} x^{1/4} y^{-1/4} - 10 \lambda = 0
$$

3. With respect to λ:

$$
10x + 10y - I = 0
$$

Deriving the Demand Functions

From the first two conditions:

$$
\frac{1}{4} x^{-3/4} y^{3/4} = \frac{3}{4} x^{1/4} y^{-1/4}
$$

Dividing both sides by $\frac{1}{4}$:

$$
x^{-3/4} y^{3/4} = 3 x^{1/4} y^{-1/4}
$$

Rearranged:

$$
x^{-3/4} / x^{1/4} = 3 y^{-1/4} / y^{3/4}
$$

Simplify exponents:

$$
x^{-3/4 - 1/4} = 3 y^{-1/4 - 3/4}
$$
$$
x^{-1} = 3 y^{-1}
$$

Thus:

$$
x^{-1} = 3 y^{-1}
$$

Cross-multiplied:

$$
y = 3x
$$

This shows that the consumer prefers to consume y three times as much as x.

Finding the Quantities

Using the budget constraint:

$$
x + y = \frac{I}{10}
$$

Substitute $y = 3x$:

$$
x + 3x = \frac{I}{10}
$$
$$
4x = \frac{I}{10}
$$
$$
x = \frac{I}{40}
$$

and

$$
y = 3x = \frac{3I}{40}
$$

Demand functions:

$$
\boxed{
x^ = \frac{I}{40} \quad \text{and} \quad y^ = \frac{3I}{40}
}
$$

Analyzing the Results and Consumer Behavior

Proportional Consumption and Preferences

The derived demand functions indicate that, given the prices and utility function, the consumer will allocate their income such that:

- x constitutes \(\frac{1}{4}\) of the total budget (\(I/40 \times 10 = I/4\))
- y constitutes \(\frac{3}{4}\) of the total budget (\(3I/40 \times 10 = 3I/4\))

This aligns with the exponents in the utility function, which reflect the consumer's relative preferences.

Effect of Income Changes

Since the demand quantities are proportional to income:

- An increase in income \(I\) results in higher consumption of both goods.
- The ratios of consumption remain constant at \(y = 3x \).

Example:

If \(I = \$400 \):

\[
x^ = \frac{400}{40} = 10 \quad \text{units}
\]
\[
y^ = 3 \times 10 = 30 \quad \text{units}
\]

Total expenditure:

\[
(10 \times 10) + (10 \times 10) = \$200
\]

which confirms the proportional allocation.

Implications and Extensions

Price Changes and Demand Elasticities

If prices change, demand responds accordingly. For instance:

- If Px increases: The consumer may reduce x consumption.
- If Py increases: y consumption may decrease, depending on substitution effects.

The demand functions respond to price changes in predictable ways, which can be analyzed using elasticities.

Price and Income Elasticities

- Price elasticity of demand measures how quantity demanded responds to price changes.
- Income elasticity of demand measures how demand changes with income.

Given the Cobb-Douglas form, the income elasticity for both goods is equal to their exponents:

- For x: $\varepsilon_{x,I} = 1/4$
- For y: $\varepsilon_{y,I} = 3/4$

This indicates that y is more sensitive to income changes than x.

Conclusion

This consumer problem illustrates fundamental principles of consumer choice theory. The utility function's Cobb-Douglas form simplifies the derivation of demand functions, revealing that consumers allocate

Frequently Asked Questions

What is the consumer's utility function in this problem?

The consumer's utility function is $U(x,y) = x^{1/4} y^{3/4}$.

Given the prices Px = $10 and Py = $10, how can we determine the consumer's optimal bundle?

Since prices are equal, the consumer will allocate income proportionally based on the exponents in the utility function, leading to a specific ratio of x to y determined by marginal utilities and budget constraints.

What is the consumer's budget constraint in this problem?

The budget constraint is $10x + 10y = \text{Income}$, where Income is the consumer's total income, which needs to be specified to find the optimal bundle.

How do you find the optimal consumption bundle for this utility function given the prices and income?

Use the method of setting marginal utility ratios equal to price ratios and incorporate the budget constraint to solve for x and y that maximize utility.

What is the marginal utility of x and y in this utility function?

The marginal utility of x is (1/4) x^{-3/4} y^{3/4}, and the marginal utility of y is (3/4) x^{1/4} y^{-1/4}.

How does the consumer's income level affect the optimal consumption bundle?

The consumer's income determines the total amount they can spend, which influences the quantities of x and y they can afford while maximizing utility, with higher income allowing for higher consumption levels.

Additional Resources

In-Depth Analysis of the Consumer's Problem: Utility Maximization with U(x, y) = X1/4 Y3/4

Introduction

Understanding consumer choice and utility maximization is fundamental in microeconomics. It provides insights into how consumers allocate their limited resources across various goods to maximize their satisfaction. In this detailed review, we explore the problem:

> Utility Function: U(x, y) = X1/4 Y3/4
> Prices: Px = $10, Py = $10
> Income: (Assuming a given income level, which we'll denote as M; specific value to be specified later)

This problem is a classic example involving Cobb-Douglas preferences, which are characterized by their multiplicative form and specific exponents representing the consumer's relative preferences for goods X and Y.

Understanding the Utility Function

Cobb-Douglas Preferences

The utility function U(x, y) = X1/4 Y3/4 is a typical Cobb-Douglas form. Its key properties include:

- Constant Elasticity of Substitution (CES): The elasticity is 1, meaning the consumer can substitute goods at a constant rate.
- Diminishing Marginal Utility: As consumption of a good increases, the additional

satisfaction gained from consuming more diminishes.
- Ease of Optimization: The mathematical properties facilitate straightforward derivation of demand functions.

Significance of Exponents

- The exponents (1/4 for X and 3/4 for Y) represent the share of income that the consumer desires to allocate to each good in the long run.
- The sum of the exponents is 1, which aligns with the budget share interpretation: 25% of expenditure on X and 75% on Y, assuming preferences are fully satisfied.

Budget Constraint and Its Role

Formulation

Given the prices and income, the budget constraint is:

$$
P_x x + P_y y = M
$$

Substituting the known prices:

$$
10x + 10y = M
$$

or simplified:

$$
x + y = \frac{M}{10}
$$

This linear constraint restricts the consumer's feasible consumption bundles.

Significance

- It defines the feasible set of bundles the consumer can choose from.
- The consumer aims to select the bundle within this set that maximizes utility.

Deriving the Consumer's Demand Functions

To find the optimal consumption bundle, we need to solve the utility maximization problem:

$$
\max_{x, y} U(x, y) = x^{1/4} y^{3/4}
$$

\]
subject to

$$
10x + 10y = M
$$

or equivalently,

$$
x + y = \frac{M}{10}
$$

Step-by-Step Solution Approach

1. Set Up the Lagrangian

Define the Lagrangian:

$$
\mathcal{L} = x^{1/4} y^{3/4} + \lambda \left(\frac{M}{10} - x - y \right)
$$

where λ is the Lagrange multiplier.

---

2. Compute First-Order Conditions (FOCs)

Take partial derivatives:

- With respect to x:

$$
\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{4} x^{-3/4} y^{3/4} - \lambda = 0
$$

- With respect to y:

$$
\frac{\partial \mathcal{L}}{\partial y} = \frac{3}{4} x^{1/4} y^{-1/4} - \lambda = 0
$$

- With respect to λ:

$$
\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{M}{10} - x - y = 0
$$

---

3. Equate the Marginal Rate of Substitution (MRS) to Price Ratio

From the first two conditions, we can eliminate λ:

$$
\frac{1}{4} x^{-3/4} y^{3/4} = \frac{3}{4} x^{1/4} y^{-1/4}
$$

Simplify:

$$
x^{-3/4} y^{3/4} = 3 x^{1/4} y^{-1/4}
$$

Dividing both sides by $x^{1/4} y^{-1/4}$:

$$
x^{-3/4 - 1/4} y^{3/4 + 1/4} = 3
$$

which simplifies to:

$$
x^{-1} y^{1} = 3
$$

or:

$$
\frac{y}{x} = 3
$$

This ratio indicates the consumer prefers to allocate their consumption such that:

$$
y = 3x
$$

---

4. Demand Function Derivation

Using the budget constraint:

$$
x + y = \frac{M}{10}
$$

Substitute $y = 3x$:

\[

x + 3x = \frac{M}{10}
\]

\[
4x = \frac{M}{10}
\]

\[
x = \frac{M}{40}
\]

and

\[
y = 3x = \frac{3M}{40}
\]

Thus, the demand functions are:

\[
\boxed{
x^ = \frac{M}{40}
}
\]

\[
\boxed{
y^ = \frac{3M}{40}
}
\]

Interpreting the Demand Functions

- The consumer spends 25% of income (\(M/40\)) on good X.
- The consumer spends 75% of income (\(3M/40\)) on good Y.
- The demand ratios directly mirror the exponents in the utility function, illustrating the property of Cobb-Douglas preferences.

Implications of the Demand Functions

- The budget shares are constant and independent of income, reflecting perfectly constant expenditure proportions.
- These demands are income elastic but are unit elastic relative to prices in the sense that proportional changes in income lead to proportional changes in demand.

Effects of Price Changes

Given the demand functions, examine how a change in prices affects consumption:

- Since the demand functions depend only on income (assuming prices are fixed), a change in prices affects the consumer's real income or purchasing power, but the demand functions themselves are directly related to income, not prices.

- If prices change (say, P_x or P_y), the consumer's optimal bundle will adjust, but the derivation above assumes fixed prices.

Budget Line Adjustments and Consumer Choice Under Price Changes

Suppose prices change to:

$$
P_x' \neq 10, \quad P_y' \neq 10
$$

The new budget constraint becomes:

$$
P_x' x + P_y' y = M
$$

The demand functions under new prices can be derived similarly, but more generally, they depend on the ratio:

$$
\frac{P_x'}{P_y'}
$$

and the consumer's income.

Extension: Income Effect and Substitution Effect

- Income Effect: Changes in demand due to the change in purchasing power resulting from price change.
- Substitution Effect: Changes in demand due to relative price changes, holding utility constant.

In Cobb-Douglas preferences, the expenditure shares remain constant regardless of price or income changes, meaning:

- The demand functions adjust proportionally with income.
- The expenditure shares are unaffected by price changes, demonstrating the properties of Cobb-Douglas preferences.

Consumer Surplus and Welfare Analysis

While not explicitly asked, understanding the consumer's welfare involves:

- Calculating consumer surplus, which depends on the initial and final prices and the consumer's utility level.
- Analyzing how price changes influence the maximized utility by substituting demand functions into the utility function.

Summary of Key Findings

Aspect	Details
Utility Function	$U(x, y) = x^{1/4} y^{3/4}$
Price Levels	$P_x = P_y = \$10$
Income	M (variable, needs explicit value for numerical results)
Demand for x	$\frac{M}{40}$
Demand for y	$\frac{3M}{40}$
Budget Shares	25% on X, 75% on Y
Demand Ratios	$y / x = 3$

Practical Applications and Insights

- Consumer Behavior Modeling: The demand functions highlight how consumers allocate income based on preferences, which is fundamental in market analysis.
- Pricing Strategies: Firms can predict how price changes influence demand, especially when preferences are modeled with Cobb-Douglas functions.
- Policy Implications:

Consider The Following Consumers Problem U X Y X1 4 Y3 4 Prices Are Px 10 Py 10 And Income

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