Consider The Following Functions. G(x)=x35h(x)=x2+9f(x)=x2+9x35 Find The Derivative Of Each Function.

Consider The Following Functions. G(x)=x35h(x)=x2+9f(x)=x2+9x35 Find The Derivative Of Each Function.
Understanding how to differentiate various types of functions is fundamental in calculus. Derivatives provide insights into the rate of change of functions, slopes of curves, and are essential in optimization problems, physics, engineering, and many other fields. In this article, we will analyze three functions—G(x), h(x), and f(x)—and meticulously compute their derivatives. We will explore the rules of differentiation, including the power rule, sum rule, and the constant multiple rule, to ensure clarity and a comprehensive understanding.

Analyzing and Differentiating G(x) = x35

Understanding the Function G(x) = x35

The function G(x) appears to be a monomial of the form x raised to the 35th power. This is a classic power function where the variable x is raised to a fixed exponent.

Differentiation of G(x)

Applying the power rule, which states that if $g(x) = x^n$, then $g'(x) = n x^{n-1}$, we differentiate G(x):

$$
G(x) = x^{35}
$$

$$
G'(x) = 35 x^{35 - 1} = 35 x^{34}
$$

Result:
$$
\boxed{
\frac{d}{dx} G(x) = 35 x^{34}
}
$$

This derivative indicates that the rate of change of G(x) increases proportionally with x^{34}, scaled by the constant 35.

Analyzing and Differentiating h(x) = x^2 + 9

Understanding the Function h(x) = x^2 + 9

The function h(x) is a quadratic polynomial with a constant term. It consists of two parts: x^2 and a constant 9.

Differentiation of h(x)

Using the sum rule, which states that the derivative of a sum of functions equals the sum of their derivatives, and the power rule, we differentiate each term separately:

$$
h(x) = x^2 + 9
$$

- Derivative of x^2:

$$
\frac{d}{dx} x^2 = 2x
$$

- Derivative of the constant 9:

$$
\frac{d}{dx} 9 = 0
$$

Combined derivative:

$$
h'(x) = 2x + 0 = 2x
$$

Result:

$$
\boxed{
\frac{d}{dx} h(x) = 2x
}
$$

This derivative shows that the slope of the quadratic function at any point x is directly proportional to x, with a constant factor of 2.

Analyzing and Differentiating $f(x) = x^2 + 9x35$

Understanding the Function $f(x) = x^2 + 9x35$

The function $f(x)$ appears to combine a quadratic term x^2 and a linear term involving $9x35$. The term $9x35$ suggests multiplication of 9, x, and 35, which simplifies to a constant coefficient multiplied by x.

Clarifying the Expression

To differentiate accurately, interpret $9x35$ as $9 \times 35 \times x$:

$$
9 \times 35 = 315
$$

Therefore, the function simplifies to:

$$
f(x) = x^2 + 315x
$$

Differentiation of $f(x)$

Again, applying the sum rule and the power rule:

- Derivative of x^2:

$$
\frac{d}{dx} x^2 = 2x
$$

- Derivative of $315x$:

$$
\frac{d}{dx} 315x = 315
$$

Combined derivative:

$$
f'(x) = 2x + 315
$$

Result:
\[
\boxed{

\frac{d}{dx} f(x) = 2x + 315
}
\]

This derivative indicates that the slope of the combined quadratic-linear function varies linearly with x, with an offset of 315.

Summary of Derivatives

Let's compile the derivatives of all three functions:

1. **G(x) = x^{35}**
    - Derivative: \(35 x^{34} \)

2. **h(x) = x^2 + 9**
    - Derivative: \(2x \)

3. **f(x) = x^2 + 315** (assuming \(9x35 = 315x \))
    - Derivative: \(2x + 315 \)

Practical Applications of Derivatives in These Functions

Understanding the derivatives of these functions is crucial in various real-world contexts. Here are some examples:

1. Physics and Motion

- The derivative of G(x) could represent the velocity if G(x) describes the position of an object over time, especially when the position follows a high-degree polynomial curve.
- The derivatives of h(x) and f(x) could model velocity in scenarios where acceleration is constant or linear.

2. Economics and Optimization

- Derivatives help identify maximum and minimum points, which are essential for profit maximization or cost minimization.
- For example, the derivative of $f(x)$ could be used to find the rate of change of revenue or cost functions.

3. Engineering and Design

- Calculating derivatives allows engineers to analyze slopes, rates of change, and optimize system performance based on the functions.

Additional Tips for Differentiation

- Remember the power rule: $\frac{d}{dx} x^n = n x^{n-1}$.
- Use the sum rule: $\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)$.
- Apply the constant multiple rule: $\frac{d}{dx} [c \times f(x)] = c \times f'(x)$.
- Always clarify the expression, especially when dealing with combined constants or multiple variables.

Conclusion

Differentiation is a vital tool in calculus, providing insights into the behavior of functions. By applying fundamental rules such as the power rule, sum rule, and constant multiple rule, we can efficiently find derivatives of complex functions. In this case, we've examined three functions:

- $G(x) = x^{35}$, with derivative $35 x^{34}$.
- $h(x) = x^2 + 9$, with derivative $2x$.
- $f(x) = x^2 + 315$, with derivative $2x + 315$.

Understanding these derivatives equips you with the foundation to analyze more complex functions and apply calculus principles across various disciplines. Practice differentiating different types of functions to strengthen your skills and deepen your understanding of how functions change in response to variables.

Remember: Mastery of differentiation opens the door to solving real-world problems involving rates of change, optimization, and modeling complex systems.

Frequently Asked Questions

What is the derivative of the function $G(x) = x^3$?

The derivative of $G(x) = x^3$ is $G'(x) = 3x^2$.

How do you find the derivative of h(x) = x^2 + 9?

The derivative of h(x) = x^2 + 9 is h'(x) = 2x, since the derivative of a constant is zero.

What is the derivative of the function f(x) = x^2 + 9x?

The derivative of f(x) = x^2 + 9x is f'(x) = 2x + 9.

Are the derivatives of polynomial functions like these straightforward to compute?

Yes, derivatives of polynomial functions are computed using the power rule, which is simple and systematic.

What is the importance of finding derivatives of functions like these?

Finding derivatives helps analyze the rate of change, determine slopes of tangent lines, and identify local maxima or minima of the functions.

Can the derivative of G(x) = x^3 be used to find its critical points?

Yes, setting G'(x) = 3x^2 = 0 allows you to find critical points, which are x = 0 in this case.

What is the derivative of h(x) = x^2 + 9, and what does it tell us about the function?

The derivative is h'(x) = 2x, indicating the slope at any point x, and showing the function is increasing when x > 0 and decreasing when x < 0.

How does the derivative of f(x) = x^2 + 9x help in understanding its graph?

The derivative f'(x) = 2x + 9 reveals the slope at any x-value, helping identify where the function increases or decreases, and locate potential turning points.

Additional Resources

Consider The Following Functions. G(x)=x35, h(x)=x^2+9, f(x)=x^2+9x35: Find The Derivative Of Each Function.

In the world of calculus, derivatives serve as essential tools for understanding how functions behave—how they change, accelerate, or decelerate. Whether you're a student tackling your first derivative or a professional applying calculus concepts to real-world problems, understanding how to differentiate various types of functions is fundamental. Today, we'll explore three specific functions: G(x)=x^35, h(x)=x^2+9, and f(x)=x^2+9x35. Our goal is to compute their derivatives step by step, providing clarity and insight into the underlying principles of differentiation.

Understanding the Concept of Derivatives

Before diving into the specific functions, it’s important to establish a clear understanding of what a derivative is.

Definition:
A derivative of a function measures the rate at which the function's output changes with respect to its input. In simpler terms, it tells us the slope of the function at any given point.

Mathematically:
If $y = f(x)$, the derivative of f with respect to x is denoted as $f'(x)$ or $\frac{df}{dx}$. It is defined as:

$$
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
$$

This limit, when it exists, provides the instantaneous rate of change of the function at point x.

Why Derivatives Matter:
- They help identify maxima and minima (critical points).
- They are used in optimization problems.
- They describe velocity and acceleration in physics.
- They assist in understanding the behavior of functions, such as increasing or decreasing trends.

Analyzing the First Function: G(x) = x^35

The first function is $G(x) = x^{35}$. It is a polynomial function characterized by an exponent of 35, which is a high degree polynomial.

Applying Power Rule for Differentiation

The most straightforward method for differentiating this function is the power rule, which states that:

$$
\frac{d}{dx} x^n = n x^{n-1}
$$

for any real number $n \neq 0$.

Calculating $G'(x)$

Using the power rule:

$$
G'(x) = 35 \times x^{35 - 1} = 35 x^{34}
$$

Interpretation

The derivative $G'(x) = 35 x^{34}$ indicates that the rate of change of the function $G(x)$ at a point x depends on the value of x^{34}. Since 34 is an even exponent, the derivative is always non-negative for all real x, implying the function is increasing for all x, except possibly at zero where the derivative is zero.

Summary

- Derivative of $G(x) = x^{35}$:
$$
G'(x) = 35 x^{34}
$$
- Key Takeaways:
- High-degree polynomial with rapid growth for large $|x|$.
- Derivative reflects the function's steepness at each point.

Analyzing the Second Function: h(x) = x^2 + 9

The second function is $h(x) = x^2 + 9$, a quadratic function with a constant term.

Differentiating Polynomial Terms

The derivative of x^2 using the power rule:

$$
\frac{d}{dx} x^2 = 2x
$$

Since 9 is a constant, its derivative is zero:

$$
\frac{d}{dx} 9 = 0
$$

Calculating $h'(x)$

Combining these results:

$$
h'(x) = 2x + 0 = 2x
$$

Interpretation

The derivative $h'(x) = 2x$ indicates that the slope of the parabola increases linearly with x. At $x=0$, the slope is zero, corresponding to the vertex of the parabola—its minimum point. For positive x, the function is increasing; for negative x, decreasing.

Key Points

- Critical point: at $x=0$, where the derivative is zero, indicating a potential maximum or minimum.
- Behavior: the function is increasing for $x > 0$ and decreasing for $x < 0$.

Summary

- Derivative of $h(x) = x^2 + 9$:
$$
h'(x) = 2x
$$
- Implication: The rate of change depends directly on x, with the function increasing or decreasing accordingly.

Analyzing the Third Function: $f(x) = x^2 + 9x35$

The third function, $f(x) = x^2 + 9x35$, appears to contain a notation that can be ambiguous. Typically, in mathematical expressions, a number following a variable indicates multiplication unless specified otherwise. Here, it seems the intended function is:

$$
f(x) = x^2 + 9 \times x \times 35
$$

which simplifies to:

$$
f(x) = x^2 + (9 \times 35) x
$$

Calculating 9×35:

$$
9 \times 35 = 315
$$

\]

Thus, the function simplifies to:

\[
f(x) = x^2 + 315 x
\]

Differentiating \(f(x) \)

Applying the sum rule and power rule:

\[
\frac{d}{dx} x^2 = 2x
\]

\[
\frac{d}{dx} 315 x = 315
\]

Therefore:

\[
f'(x) = 2x + 315
\]

Interpretation

The derivative \(f'(x) = 2x + 315 \) is a linear function. It indicates that the rate of change increases with \(x \), shifted by 315 units. The zero of the derivative:

\[
2x + 315 = 0 \Rightarrow x = -\frac{315}{2} = -157.5
\]

marks the critical point where the function's slope is zero, potentially indicating a maximum or minimum.

Insights and Applications

- The critical point at \(x = -157.5 \) suggests the function has a turning point there.
- For \(x > -157.5 \), the derivative is positive, so the function is increasing.
- For \(x < -157.5 \), the derivative is negative, so the function is decreasing.

Summary

- Derivative of \(f(x) = x^2 + 315 x \):
\[
f'(x) = 2x + 315
\]
- Implication: The linear derivative provides straightforward insights into the function’s

increasing/decreasing behavior and critical points.

Summary and Broader Implications

Differentiation is a cornerstone of calculus, enabling us to understand the dynamics of functions with precision. Let's briefly revisit the derivatives we computed:

- For $G(x) = x^{35}$:
$$
G'(x) = 35 x^{34}
$$
A high-degree polynomial with a derivative indicating rapid growth and steepness for large $|x|$.

- For $h(x) = x^2 + 9$:
$$
h'(x) = 2x
$$
A simple quadratic derivative illustrating linear change, with a critical point at $x=0$.

- For $f(x) = x^2 + 315 x$:
$$
f'(x) = 2x + 315
$$
A linear derivative revealing the slope's shift and critical point at $x=-157.5$.

These examples demonstrate how the power rule and basic differentiation principles enable us to analyze a wide variety of functions—from high-degree polynomials to simple quadratics. Understanding these derivatives is vital in fields ranging from physics to economics, as they quantify how systems evolve over time or space.

Practical Applications:
- In physics, derivatives determine velocity and acceleration.
- In economics, they inform marginal cost and revenue.
- In engineering, derivatives aid in stability analysis and control systems.

Final Thoughts:

Mastering the differentiation of various functions equips learners and professionals with the tools needed to interpret complex systems. Whether dealing with the rapid growth of polynomial functions or the linear shifts in simpler models, the derivative provides a window into the behavior and characteristics of functions critical for scientific and mathematical analyses.

Remember: Practice makes perfect. To deepen your understanding, try differentiating different types of functions, including trigonometric, exponential, and logarithmic functions. The more you explore, the more intuitive calculus becomes.

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