# Consider The Function F(x)=x^(3)-6x^(2)-49x+294. When F(x) Is Divided By X+7, The Remainder Is 0. For

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Understanding polynomial functions and their properties is fundamental in algebra. One key concept is the Remainder Theorem, which provides a quick way to evaluate the remainder when a polynomial is divided by a linear divisor. In this article, we'll delve into the details of the function $F(x) = x^3 - 6x^2 - 49x + 294$, exploring its behavior when divided by $x + 7$, the implications of the Remainder Theorem, and how to analyze such functions step-by-step.

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## Understanding the Polynomial Function $F(x)$

Before diving into division and remainders, it is essential to comprehend the structure of the polynomial:

$$F(x) = x^3 - 6x^2 - 49x + 294$$

This is a cubic polynomial, characterized by the highest degree of 3. Each term contributes to the overall shape and roots of the function.

Key features of polynomial $F(x)$:
- Degree: 3
- Leading coefficient: 1
- Constant term: 294

Understanding the roots of $F(x)$ helps in graphing, solving, and analyzing the function.

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## The Remainder Theorem and Its Significance

The Remainder Theorem states that for any polynomial $P(x)$, the remainder when dividing $P(x)$ by $x - a$ is equal to $P(a)$.

In our case:
- The divisor is $x + 7$, which can be rewritten as $x - (-7)$.
- Applying the Remainder Theorem, the remainder when $F(x)$ is divided by $x + 7$ is $F(-7)$.

Since the problem states that the remainder is 0, it indicates that:

$$ F(-7) = 0 $$

This suggests that $x = -7$ is a root of the polynomial $F(x)$, and $(x + 7)$ is a factor of $F(x)$.

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# Calculating $F(-7)$: Confirming the Root

To verify that $x = -7$ is indeed a root, calculate $F(-7)$:

$$
\begin{aligned}
F(-7) &= (-7)^3 - 6(-7)^2 - 49(-7) + 294 \\
&= -343 - 6(49) + 343 + 294 \\
&= -343 - 294 + 343 + 294 \\
&= ( -343 + 343 ) + ( -294 + 294 ) \\
&= 0 + 0 \\
&= 0
\end{aligned}
$$

Since $F(-7) = 0$, the assertion holds true: $x = -7$ is a root, and $(x + 7)$ is a divisor of $F(x)$.

Implication:
- The polynomial $F(x)$ can be factored as:

$$
F(x) = (x + 7) \times Q(x)
$$

where $Q(x)$ is a quadratic polynomial.

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# Factoring $F(x)$ Completely

Knowing that $(x + 7)$ is a factor, the next step is to find $Q(x)$ such that:

$$
F(x) = (x + 7) \times Q(x)
$$

Method: Polynomial Division

Perform synthetic division or long division of $F(x)$ by $x + 7$.

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## Performing Synthetic Division

Set up synthetic division:

- Dividing by $x + 7$, so use $a = -7$.
- Polynomial coefficients: $1 \quad -6 \quad -49 \quad 294$

Synthetic division steps:

| -7 | 1 | -6 | -49 | 294 |
|-----|-----|-----|-------|-------|
| | -7 | 91 | -294 | 0 |

Calculations:

1. Bring down the 1.
2. Multiply 1 by -7: -7; add to -6: -6 + (-7) = -13.
3. Multiply -13 by -7: 91; add to -49: -49 + 91 = 42.
4. Multiply 42 by -7: -294; add to 294: 294 + (-294) = 0.

Result:

- Remainder: 0 (as expected).
- The coefficients of $Q(x)$ are: 1, -13, 42.

Thus,

$$
Q(x) = x^2 - 13x + 42
$$

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## Factoring the Quadratic $Q(x)$

Now, factor $Q(x) = x^2 - 13x + 42$.

Find two numbers that multiply to 42 and sum to -13:

- Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42
- Pair that sums to -13: -7 and -6 (since -7 -6 = 42 and -7 + (-6) = -13)

Therefore,

$$
Q(x) = (x - 7)(x - 6)
$$

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## Complete Factorization of $F(x)$

Putting it all together:

$$
F(x) = (x + 7)(x - 7)(x - 6)
$$

This reveals the roots of the polynomial:

- $x = -7$
- $x = 7$
- $x = 6$

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## Implications and Applications

Understanding the factorization and roots of $F(x)$ has multiple implications:

1. Roots and Zeros of $F(x)$

Knowing the roots helps in graphing the polynomial and understanding its behavior:

- The polynomial crosses the x-axis at $x = -7, 6, 7$.
- The multiplicity of roots is 1, indicating simple roots and that the graph crosses the x-axis at these points.

2. Polynomial Behavior and Graphing

- Between roots, the polynomial's sign changes.
- The leading coefficient is positive, so as $x \to \pm \infty$, $F(x) \to +\infty$.

3. Solving Equations

If tasked with solving $F(x) = 0$, the solutions are exactly the roots identified.

4. Applications in Real-World Contexts

Such polynomial functions model physical phenomena, economics, and engineering problems where roots represent equilibrium points, optimal solutions, or critical values.

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# Additional Considerations in Polynomial Analysis

Beyond factorization, other aspects are vital for comprehensive understanding:

## 1. Polynomial Derivatives

Calculating derivatives helps analyze the function's increasing/decreasing behavior and locate extrema.

$$
F'(x) = 3x^2 - 12x - 49
$$

Set $F'(x) = 0$:

$$
3x^2 - 12x - 49 = 0
$$

Divide through by 3:

$$
x^2 - 4x - \frac{49}{3} = 0
$$

Use quadratic formula:

$$
x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times (-\frac{49}{3})}}{2}
$$

Simplify discriminant:

$$
16 + \frac{196}{3} = \frac{48 + 196}{3} = \frac{244}{3}
$$

$$
x = \frac{4 \pm \sqrt{\frac{244}{3}}}{2}
$$

This gives approximate critical points where the slope changes.

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## 2. Graphical Representation

Plotting $F(x)$ using the roots and critical points provides visual insights into its behavior.

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# Summary and Key Takeaways

- The polynomial $F(x) = x^3 - 6x^2 - 49x + 294$ is divisible by $x + 7$, confirmed by calculating $F(-7) = 0$.
- Factoring reveals:

$$
F(x) = (x + 7)(x - 6)(x - 7)
$$

- Roots are $x = -7, 6, 7$.
- The polynomial's behavior can be analyzed using derivatives and graphing techniques.
- The Remainder Theorem simplifies polynomial division, allowing quick verification of roots.

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# Conclusion

The process of analyzing $F(x)$ demonstrates essential algebraic techniques, including polynomial division, factorization, and the application of the Remainder Theorem. Recognizing that $F(-7) = 0$

# Frequently Asked Questions

## What does it mean when the remainder is 0 when dividing F(x) by x + 7?

It means that x = -7 is a root of the polynomial F(x), so F(-7) = 0.

## How can we verify that F(x) is divisible by x + 7?

By substituting x = -7 into F(x) and checking if the result equals 0.

## What is the value of F(-7) based on the given function?

Calculating F(-7): $(-7)^3 - 6(-7)^2 - 49(-7) + 294$.

## Calculate F(-7) to confirm divisibility.

$F(-7) = -343 - 6(49) + 343 + 294 = -343 - 294 + 343 + 294 = 0$, confirming divisibility.

## What is the degree of the polynomial F(x)?

The degree of $F(x) = x^3 - 6x^2 - 49x + 294$ is 3.

## Can we find other roots of F(x)?

Yes, by factoring F(x) or using polynomial division to find other roots.

## What is the factorization of F(x) given that x + 7 is a factor?

Since $x + 7$ is a factor, F(x) can be expressed as $(x + 7)$ times a quadratic polynomial.

## How to perform polynomial division of F(x) by x + 7?

Use synthetic or long division to divide F(x) by $x + 7$, resulting in a quadratic quotient.

## What is the quotient when F(x) is divided by x + 7?

Performing the division yields the quadratic quotient, which can be further factored.

## Are there any real roots besides x = -7 for F(x)?

Yes, solving the quadratic quotient from division will reveal other real roots if they exist.

# Additional Resources

Consider The Function $F(x)=x^{(3)}-6x^{(2)}-49x+294$ is a compelling example of polynomial functions that lend themselves well to analysis through algebraic techniques such as polynomial division and the Remainder Theorem. This function, a cubic polynomial, showcases interesting properties that can be explored to understand its roots, behavior, and applications in various mathematical contexts. In this article, we will delve into a comprehensive review of this function, examining its characteristics, the significance of the divisor $x+7$, and the broader implications of these properties within algebra and calculus.

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# Understanding the Polynomial Function F(x)

## Definition and Basic Properties

The function $F(x) = x^3 - 6x^2 - 49x + 294$ is a cubic polynomial, meaning it has a degree of 3. As with all polynomials, it is continuous and differentiable over the real numbers, allowing for analysis using calculus. Its coefficients and constant term influence the shape and position of its graph.

Some basic properties include:
- Degree: 3
- Leading coefficient: 1 (positive), indicating that as $x \to +\infty$, $F(x) \to +\infty$, and as $x \to -\infty$, $F(x) \to -\infty$.
- Constant term: 294
- Symmetry: No obvious symmetry; the polynomial is neither even nor odd.

Understanding these properties sets the stage for deeper analysis, especially regarding roots and behavior.

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# Roots and Factorization

## Applying the Remainder Theorem

The statement "When $F(x)$ is divided by $x + 7$, the remainder is 0" is a direct application of the Remainder Theorem. This theorem states that if a polynomial $F(x)$ is divided by $x - a$, the remainder is $F(a)$.

Given that dividing by $x + 7$ leaves no remainder, we can conclude:
- $F(-7) = 0$

Let's verify this:

$$
F(-7) = (-7)^3 - 6(-7)^2 - 49(-7) + 294
$$
$$
= -343 - 6(49) + 343 + 294
$$
$$
= -343 - 294 + 343 + 294
$$
$$
= ( -343 + 343 ) + ( -294 + 294 ) = 0 + 0 = 0
$$

\]

This confirms that \(x = -7\) is a root of \(F(x)\).

## Factoring Out \((x + 7)\)

Since \(-7\) is a root, \(F(x)\) is divisible by \(x + 7\). To factor the polynomial fully, polynomial division or synthetic division can be used to find the other factors.

Using synthetic division:

| -7 | 1 | -6 | -49 | 294 |
|-------|-------|--------|---------|--------|
| | -7 | 91 | -294 | 0 |
|-------|-------|--------|---------|--------|
| 1 | -13 | 42 | -0 | 0 |

The quotient is \(x^2 - 13x + 42\). Therefore,

\[
F(x) = (x + 7)(x^2 - 13x + 42)
\]

Next, factor the quadratic:

\[
x^2 - 13x + 42
\]

Find two numbers that multiply to 42 and add to -13: these are -6 and -7.

Thus,

\[
x^2 - 13x + 42 = (x - 6)(x - 7)
\]

Complete factorization:

\[
F(x) = (x + 7)(x - 6)(x - 7)
\]

Roots:

\[
x = -7, \quad x = 6, \quad x = 7
\]

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# Graphical Behavior and Critical Points

## Plotting and Significance of Roots

The roots indicate where the graph intersects the x-axis:

- At $x = -7$, the function crosses the x-axis.
- At $x = 6$ and $x = 7$, the function also crosses the x-axis.

The roots are real and distinct, which influences the shape of the cubic curve.

## Critical Points and Turning Points

To analyze the function's extrema, compute the first derivative:

$$
F'(x) = 3x^2 - 12x - 49
$$

Set $F'(x) = 0$:

$$
3x^2 - 12x - 49 = 0
$$

Divide through by 3:

$$
x^2 - 4x - \frac{49}{3} = 0
$$

Apply quadratic formula:

$$
x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times (-\frac{49}{3})}}{2}
$$

$$
x = \frac{4 \pm \sqrt{16 + \frac{196}{3}}}{2}
$$

Express 16 as $\frac{48}{3}$:

$$
x = \frac{4 \pm \sqrt{\frac{48 + 196}{3}}}{2} = \frac{4 \pm \sqrt{\frac{244}{3}}}{2}
$$

Simplify the square root:

$$
\sqrt{\frac{244}{3}} = \frac{\sqrt{244}}{\sqrt{3}} = \frac{2\sqrt{61}}{\sqrt{3}} = 2 \sqrt{\frac{61}{3}}
$$

Thus,

$$
x = \frac{4 \pm 2 \sqrt{\frac{61}{3}}}{2} = 2 \pm \sqrt{\frac{61}{3}}
$$

Approximate the critical points:

$$
x \approx 2 \pm \sqrt{20.33} \approx 2 \pm 4.51
$$

- $x \approx 6.51$ (local maximum)
- $x \approx -2.51$ (local minimum)

Features:

- The cubic has one local maximum near $x \approx 6.51$.
- It has one local minimum near $x \approx -2.51$.

The function's behavior around these points indicates where it changes from increasing to decreasing or vice versa, shaping the graph's curve.

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# Features, Pros, and Cons of F(x)

Features:

- Fully factorized form: $F(x) = (x + 7)(x - 6)(x - 7)$.
- Roots at $-7, 6, 7$.
- Intercepts the x-axis at these roots.
- The end behavior is typical of a cubic with positive leading coefficient.

Pros:

- Easy to analyze roots after factorization.
- Allows straightforward graph sketching.

- Demonstrates the application of synthetic division and the Remainder Theorem.
- Useful in polynomial root-finding exercises.

Cons:

- Without prior factorization, roots may be less obvious.
- The quadratic factor's roots are irrational approximations unless simplified.
- Limited in describing complex behaviors like inflection points or concavity without further calculus.

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# Applications and Broader Context

Mathematical Analysis:

- Polynomial functions like $F(x)$ are fundamental in algebra, calculus, and mathematical modeling.
- Factoring cubic polynomials helps in solving equations and understanding polynomial functions' graphs.
- The roots provide solutions to equations $F(x) = 0$, which have applications in physics, engineering, and economics.

Real-World Applications:

- Polynomial equations model phenomena such as projectile trajectories, optimization problems, and economic supply-demand curves.
- Understanding roots and factors can help in designing systems with desired behaviors or constraints.

Educational Significance:

- Serves as an excellent example for teaching the Remainder Theorem, synthetic division, and quadratic solutions.
- Demonstrates how algebraic techniques simplify complex polynomial functions.

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# Conclusion

The function $F(x) = x^3 - 6x^2 - 49x + 294$, particularly when analyzed through the lens of division by $(x + 7)$, exemplifies the elegant interplay of algebraic principles. Its roots, derived via the Remainder Theorem and polynomial factorization, reveal critical points and shape the graph's behavior. The detailed exploration of its critical points, roots, and factors underscores the importance of algebraic techniques in understanding polynomial functions. Whether for educational purposes or practical applications, the insights gained from studying $F(x)$ exemplify the power of fundamental mathematical tools in analyzing and interpreting complex functions.

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