Consider The Graph Of F(x) = 5x 6 5x 5 X 4.On A Coordinate Plane, A Graph Decreases Through (negative

Consider The Graph Of F(x) = 5x^6 - 5x^5 + x^4. On A Coordinate Plane, A Graph Decreases Through (negative illustrates a fascinating aspect of polynomial functions and their behavior on the coordinate plane. Understanding how this particular function behaves—especially where it decreases—is essential for students, educators, and anyone interested in graphing functions. This article delves into the detailed analysis of the function F(x) = 5x^6 - 5x^5 + x^4, exploring its key features, its decreasing and increasing intervals, critical points, and the overall shape of its graph.

Understanding the Function F(x) = 5x^6 - 5x^5 + x^4

Breaking Down the Function

The function F(x) = 5x^6 - 5x^5 + x^4 is a polynomial of degree six, which indicates it can have up to six real roots and a complex, potentially oscillating graph. Its leading term, 5x^6, dominates the behavior of the function for very large positive or negative x-values, causing the ends of the graph to rise toward infinity in both directions.

The polynomial combines three terms:

- **5x^6:** The highest degree term, influencing the end behavior.
- **-5x^5:** A lower degree term that impacts the shape and symmetry.
- **x^4:** Adds curvature, especially near the origin.

Graph Behavior at Infinity

Since the leading term is positive and of even degree, the graph rises to positive infinity as x approaches both positive and negative infinity:

- *As x → +∞, F(x) → +∞*
- *As x → -∞, F(x) → +∞*

This characteristic indicates the graph opens upward at both ends.

Analyzing the Increasing and Decreasing Intervals

Finding Critical Points

To determine where the graph decreases or increases, we need to analyze the first derivative of F(x). Critical points occur where the derivative equals zero or is undefined. Since F(x) is a polynomial, its derivative will be a polynomial as well.

Calculating the Derivative

The first derivative of F(x) is:
$$ F'(x) = \frac{d}{dx} (5x^6 - 5x^5 + x^4) = 30x^5 - 25x^4 + 4x^3 $$

Factoring the Derivative

Factoring F'(x) helps identify critical points:
$$
F'(x) = x^3 (30x^2 - 25x + 4)
$$
Set F'(x) = 0:
$$
x^3 (30x^2 - 25x + 4) = 0
$$
This yields:
$$
x = 0
$$
or solving the quadratic:
$$
30x^2 - 25x + 4 = 0
$$
Using the quadratic formula:
$$
x = \frac{25 \pm \sqrt{(-25)^2 - 4 \times 30 \times 4}}{2 \times 30} = \frac{25 \pm \sqrt{625 - 480}}{60}
$$
$$
x = \frac{25 \pm \sqrt{145}}{60}
$$
Thus, the critical points are at:
$$
x = 0, \quad x = \frac{25 + \sqrt{145}}{60}, \quad x = \frac{25 - \sqrt{145}}{60}
$$

Determining the Nature of Critical Points

To identify whether these critical points are maxima, minima, or points of inflection, we analyze the second derivative.

Calculate the second derivative:

$$
F''(x) = \frac{d}{dx} (30x^5 - 25x^4 + 4x^3) = 150x^4 - 100x^3 + 12x^2
$$

Evaluate $F''(x)$ at each critical point to determine their nature:
- If $F''(x) > 0$, the point is a local minimum.
- If $F''(x) < 0$, the point is a local maximum.
- If $F''(x) = 0$, the test is inconclusive.

Key insight: The sign of $F'(x)$ around the critical points indicates where the function increases or decreases.

Intervals of Decrease and Increase

Sign Analysis of F'(x)

Using the critical points, we analyze $F'(x)$ over the intervals:

- $(-\infty, \frac{25 - \sqrt{145}}{60})$
- $\left(\frac{25 - \sqrt{145}}{60}, 0\right)$
- $(0, \frac{25 + \sqrt{145}}{60})$
- $\left(\frac{25 + \sqrt{145}}{60}, +\infty\right)$

By selecting test points within each interval and plugging into $F'(x)$, we determine the sign of the derivative:

- Decreasing intervals: where $F'(x) < 0$
- Increasing intervals: where $F'(x) > 0$

Sample Test Points and Outcomes:

Interval	Test Point	F'(x) Sign	Behavior
$(-\infty, \frac{25 - \sqrt{145}}{60})$	$x = -1$	Negative	Decreasing
$\left(\frac{25 - \sqrt{145}}{60}, 0\right)$	$x = -0.5$	Positive	Increasing
$(0, \frac{25 + \sqrt{145}}{60})$	$x = 0.5$	Negative	Decreasing
$\left(\frac{25 + \sqrt{145}}{60}, +\infty\right)$	$x = 1$	Positive	Increasing

From this, the graph decreases on:

$$
(-\infty, \frac{25 - \sqrt{145}}{60}) \quad \text{and} \quad \left(0, \frac{25 + \sqrt{145}}{60}\right)
$$

and increases elsewhere.

The Shape of the Graph and Its Key Features

End Behavior

As discussed, the graph rises to infinity on both ends, giving it an upward-opening shape at the extremes.

Critical Points and Local Extrema

The critical points at:
- $x=0$: corresponds to a local maximum or minimum depending on the second derivative.
- $x = \frac{25 \pm \sqrt{145}}{60}$: potential local minima or maxima.

By evaluating the second derivative at these points, we find:
- At $x=0$, $F''(0) = 0$, indicating a possible inflection point.
- At the other critical points, the second derivative's signs determine the nature of the extremum.

Points of Decrease

The original prompt emphasizes the graph decreasing through negative x-values. The decreasing intervals include:
- The far left portion of the graph (for large negative x).
- The interval between 0 and $\frac{25 + \sqrt{145}}{60}$.

This decreasing behavior means that as x moves from negative infinity toward these critical points, the function F(x) diminishes in value, creating the downward slopes visible on the graph.

Visualizing the Graph

To better understand the behavior, plotting the function using graphing tools or software like Desmos or GeoGebra can provide a visual representation. The key features to observe include:

- The points where the graph changes from decreasing to increasing (local minima).
- The points where the graph changes from increasing to decreasing (local maxima).
- The overall end behavior, confirming the ends rise upward.

Practical Applications of Analyzing Such Functions

Understanding the decreasing and increasing behavior of polynomial functions like $F(x) = 5x^6 - 5x^5 + x^4$ has several practical applications:

- **Optimization problems:** Finding maximum or minimum values within an interval.
- **Modeling physical phenomena:** Such as motion, where the decreasing intervals might represent deceleration.
- **Graphing and visualization skills:** Developing intuition for polynomial behavior and critical point analysis.

Conclusion

Analyzing the graph of F(x) = 5x^6 - 5x^5 + x^4 reveals a complex but understandable pattern of increasing and decreasing intervals. The function decreases notably through negative x-values, particularly on the far left of the coordinate plane, and continues to do so within specific intervals on the positive side. By calculating the

Frequently Asked Questions

What is the general behavior of the function f(x) = 5x^6 - 5x^5 + x^4 on a coordinate plane?

The function exhibits a polynomial graph that initially decreases, then increases, and may have multiple turning points due to its degree and coefficients.

At which points does the graph of f(x) = 5x^6 - 5x^5 + x^4 decrease?

The graph decreases in regions where the first derivative f'(x) is negative, which can be determined by analyzing the derivative's sign over different intervals.

How do you find where the graph of f(x) decreases on the coordinate plane?

Calculate the derivative f'(x), find its critical points, and test intervals to see where f'(x) < 0 to identify decreasing regions.

What is the significance of the point (-∞, ∞) in the context of the graph's decreasing behavior?

The notation indicates the graph extends infinitely in the negative x-direction; the decreasing trend may start or continue towards negative infinity.

What is the role of the critical points in determining where the graph decreases?

Critical points, where f'(x) = 0 or undefined, mark potential local maxima or minima; the intervals between these points determine where the function decreases.

Can the graph of f(x) = 5x^6 - 5x^5 + x^4 decrease for all x?

No, due to its polynomial nature, the graph decreases in some regions and increases in others; it does not decrease everywhere.

How does the degree of the polynomial affect the shape of its graph?

Higher-degree polynomials like degree 6 tend to have multiple turning points, resulting in complex increasing and decreasing intervals.

What tools can be used to analyze the decreasing intervals of the function?

Calculus techniques such as finding the first derivative, critical points, and testing intervals help identify where the function decreases.

Why is understanding where the function decreases important in graph analysis?

Identifying decreasing regions helps in understanding the overall shape, identifying local maxima/minima, and applying optimization or modeling techniques.

Additional Resources

Comprehensive Review of the Graph of \(F(x) = 5x^6 - 5x^5 + x^4 \)

Understanding the behavior of polynomial functions is fundamental in calculus and analytical geometry. The function \(F(x) = 5x^6 - 5x^5 + x^4 \) presents an intriguing case study due to its high degree, multiple critical points, and varying intervals of increase and decrease. This review delves deeply into the graph's characteristics, focusing especially on its decreasing behavior through specific intervals, the critical points, concavity, and the overall shape on the coordinate plane.

Introduction to the Function \(F(x) = 5x^6 - 5x^5 +

x^4

Function Overview

- Degree and Leading Coefficient: The polynomial is of degree 6, with a leading coefficient of 5, which means as $x \to \pm \infty$, $F(x) \to +\infty$. The degree indicates a "W"-shaped or "U"-shaped graph with multiple turning points.
- Polynomial Structure: The expression combines terms of x^6, x^5, and x^4, indicating complex behavior that includes potential inflection points, local maxima, and minima.

Why is this function interesting?

- The mixture of even and odd powers creates a rich structure with symmetric and asymmetric features.
- The decreasing intervals are especially notable because they reveal where the function slides downward, which is crucial in understanding the overall shape and in solving optimization problems.

Analyzing the Function's Behavior: Derivatives and Critical Points

First Derivative $F'(x)$

To understand where the graph decreases, identify the critical points where $F'(x) = 0$:

$$
F'(x) = \frac{d}{dx} [5x^6 - 5x^5 + x^4] = 30x^5 - 25x^4 + 4x^3
$$

Factor $F'(x)$:

$$
F'(x) = x^3 (30x^2 - 25x + 4)
$$

Thus:

$$
F'(x) = x^3 (30x^2 - 25x + 4)
$$

Critical points occur where:

$$
x^3 = 0 \quad \Rightarrow \quad x = 0
$$

and where the quadratic:

$$
30x^2 - 25x + 4 = 0
$$

has solutions.

- Quadratic solutions:

$$
x = \frac{25 \pm \sqrt{(-25)^2 - 4 \times 30 \times 4}}{2 \times 30} = \frac{25 \pm \sqrt{625 - 480}}{60} = \frac{25 \pm \sqrt{145}}{60}
$$

Calculating:

$$
\sqrt{145} \approx 12.0416
$$

Thus,

$$
x \approx \frac{25 \pm 12.0416}{60}
$$

- For the positive root:

$$
x \approx \frac{25 + 12.0416}{60} \approx \frac{37.0416}{60} \approx 0.61736
$$

- For the negative root:

$$
x \approx \frac{25 - 12.0416}{60} \approx \frac{12.9584}{60} \approx 0.216
$$

Critical points are therefore approximately at:

$$
x = 0, \quad x \approx 0.216, \quad x \approx 0.617
$$

Determining Intervals of Increase and Decrease

Sign of $F'(x)$

To classify each critical point as a local maximum, minimum, or point of inflection, analyze the sign of $F'(x)$ around these points.

- For $x < 0$:

Since $x^3 < 0$, the sign depends on the quadratic $30x^2 - 25x + 4$.

At $x = -\infty$, $F'(x) \to -\infty$. Near $x = 0^-$:

- Plug in a test value, say $x = -1$:

$$
F'(-1) = (-1)^3 (30(1) - 25(-1) + 4) = -1 (30 + 25 + 4) = -1 (59) = -59 < 0
$$

Thus, F' is negative for $x < 0$, indicating the function decreases on $(-\infty, 0)$.

- Between 0 and 0.216:

Test at $x = 0.1$:

$$
F'(0.1) = (0.1)^3 (30(0.01) - 25(0.1) + 4) = 0.001 (0.3 - 2.5 + 4) = 0.001 (1.8) = 0.0018 > 0
$$

Positive derivative indicates $F(x)$ increases on $(0, 0.216)$.

- Between 0.216 and 0.617:

Test at $x = 0.3$:

$$
F'(0.3) = (0.3)^3 (30(0.09) - 25(0.3) + 4) = 0.027 (2.7 - 7.5 + 4) = 0.027 (-0.8) \approx -0.0216 < 0
$$

Function decreases on $(0.216, 0.617)$.

- For $x > 0.617$:

Test at $x=1$:

$$
F'(1) = 1^3 (30 - 25 + 4) = 1 \times 9 = 9 > 0
$$

Function increases on $(0.617, \infty)$.

Summary of intervals:

Interval	Behavior	Explanation
$(-\infty, 0)$	Decreasing	$F'(x) < 0$ for $x < 0$
$(0, 0.216)$	Increasing	$F'(x) > 0$ between 0 and 0.216
$(0.216, 0.617)$	Decreasing	$F'(x) < 0$ between 0.216 and 0.617
$(0.617, \infty)$	Increasing	$F'(x) > 0$ after 0.617

Graphical Features: Critical Points, Extrema, and Inflection Points

Local Extrema

- At $x = 0$:

$$
F(0) = 0
$$

Since $F'(x)$ changes from negative to positive at 0, this indicates a local minimum.

- At $x \approx 0.216$:

Derivative changes from positive to negative, indicating a local maximum.

- At $x \approx 0.617$:

Derivative changes from negative to positive, indicating a local minimum.

Values at key points:

Calculating $F(x)$ at critical points:

- $F(0) = 0$
- $F(0.216)$:

$$
F(0.216) = 5(0.216)^6 - 5(0.216)^5 + (0.216)^4
$$

Approximate:

$$
(0.216)^4 \approx 0.0022
$$

$$(0.216)^5 \approx 0.00048$$

$$(0.216)^6 \approx 0.0001$$

Thus,

$$F(0.216) \approx 5(0.0001) - 5(0.00048) + 0.0022 = 0.0005 - 0.0024 + 0.0022 = 0.0003$$

Approximately at 0.0003, so a small positive value, indicating a local maximum slightly above zero.

- $F(0.617)$:

Similarly,

$$(0.617)^4 \approx 0.145$$

$$(0.617)^5 \approx 0.0897$$

$$(0.617)^6 \approx 0.0553$$

Then,

$$F(0.617) \approx 5(0.0553) - 5(0.089$$

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