Consider The Points Below. P(3, 0, 3), Q(2, 1, 4), R(6, 2, 5) (a) Find A Nonzero Vector Orthogonal To

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Introduction

In the realm of vector calculus and three-dimensional geometry, understanding how to find vectors orthogonal to given points, lines, or planes is a fundamental skill. Such problems are not only common in academic exercises but also have practical implications in physics, engineering, computer graphics, and navigation systems. The ability to determine a nonzero vector orthogonal to a set of points or vectors allows us to analyze directions, define planes, and understand spatial relationships more comprehensively.

Today, we focus on a specific problem involving three points in three-dimensional space: P(3, 0, 3), Q(2, 1, 4), and R(6, 2, 5). The task is to find a nonzero vector that is orthogonal to certain geometric entities derived from these points, specifically to the plane containing them or to the vectors connecting them. This problem involves concepts such as vector subtraction to find direction vectors, the cross product to find orthogonal vectors, and understanding the geometric significance of these vectors.

By exploring this problem, we will delve into the fundamental techniques used in vector mathematics to find orthogonal vectors, which are essential for understanding the orientation of planes and lines in space. We will also provide a comprehensive step-by-step approach to solving the problem, ensuring clarity and depth suitable for students, educators, and professionals alike.

Understanding the Problem

Before jumping into calculations, it’s crucial to interpret what the problem is asking. The key phrase is: "Find a nonzero vector orthogonal to" — which typically refers to finding a vector perpendicular to a given set of vectors, or a plane defined by specific points.

Given points:

- P(3, 0, 3)
- Q(2, 1, 4)

- R(6, 2, 5)

Our goal is to determine a nonzero vector that is orthogonal to some geometric entity associated with these points, most likely the plane passing through them or the vectors connecting these points.

Step 1: Find Direction Vectors

To understand the spatial relationships among the points, we first find the vectors connecting them:

- Vector PQ from P to Q:

$$
\vec{PQ} = \mathbf{Q} - \mathbf{P} = (2 - 3, 1 - 0, 4 - 3) = (-1, 1, 1)
$$

- Vector PR from P to R:

$$
\vec{PR} = \mathbf{R} - \mathbf{P} = (6 - 3, 2 - 0, 5 - 3) = (3, 2, 2)
$$

These vectors represent the directions from point P to points Q and R, respectively. They are essential in defining the plane that contains the three points.

Step 2: Find a Vector Orthogonal to the Plane

To find a vector orthogonal (perpendicular) to the plane passing through points P, Q, and R, we need to find the cross product of the two vectors PQ and PR:

$$
\vec{n} = \vec{PQ} \times \vec{PR}
$$

The cross product of two vectors in three-dimensional space produces a vector orthogonal to both, and consequently, orthogonal to the plane containing the vectors.

Calculating the Cross Product

Recall the cross product formula:

$$
\vec{A} \times \vec{B} = (A_2 B_3 - A_3 B_2, \; A_3 B_1 - A_1 B_3, \; A_1 B_2 - A_2 B_1)
$$

Applying this to PQ = (-1, 1, 1) and PR = (3, 2, 2):

- x-component:

$$
(1)(2) - (1)(2) = 2 - 2 = 0
$$

- y-component:

$$
(1)(3) - (-1)(2) = 3 + 2 = 5
$$

- z-component:

$$
(-1)(2) - (1)(3) = -2 - 3 = -5
$$

Thus,

$$
\vec{n} = (0, 5, -5)
$$

This vector is orthogonal to the plane passing through points P, Q, and R.

Step 3: Confirm the Orthogonal Vector

The computed vector $\vec{n} = (0, 5, -5)$ is nonzero, which satisfies the requirement for an orthogonal vector. It is orthogonal to both PQ and PR and, consequently, orthogonal to the plane containing P, Q, and R.

Step 4: Simplify the Vector

While $\vec{n} = (0, 5, -5)$ is already a valid orthogonal vector, it can be simplified for clarity:

$$
\vec{n} = (0, 1, -1)
$$

by dividing each component by 5.

This simplified vector maintains the orthogonality property and is often preferable for applications or further calculations.

Additional Considerations: Other Orthogonal Vectors

While the vector $\vec{n} = (0, 1, -1)$ is a primary answer, it's important to recognize that any scalar multiple of this vector is also orthogonal to the plane, providing infinitely many solutions:

$$
\text{General orthogonal vector:} \quad \lambda (0, 1, -1), \quad \lambda \neq 0
$$

This flexibility allows for choosing vectors based on specific application needs, such as unit vectors or vectors with particular properties.

Applications of Finding Orthogonal Vectors

Understanding how to compute orthogonal vectors has wide-ranging applications:

- Plane Equation Determination: Using the orthogonal vector (normal vector), you can derive the equation of the plane passing through the given points.
- Projection and Shadow Calculations: Orthogonal vectors are essential in projecting vectors onto planes or lines.
- Physics and Engineering: Normal vectors help analyze forces, moments, and other vector quantities in three-dimensional systems.
- Computer Graphics: Surface normals, which are inherently orthogonal to surfaces, are crucial for rendering, shading, and collision detection.

Formulating the Equation of the Plane

Having the orthogonal vector $\vec{n} = (0, 1, -1)$, we can write the plane's equation passing through point P(3, 0, 3):

$$
0(x - 3) + 1(y - 0) - 1(z - 3) = 0
$$

Simplify:

$$
y - (z - 3) = 0
$$
$$
y - z + 3 = 0
$$

or equivalently,

$$
y - z = -3
$$

This plane contains the points P, Q, and R.

Conclusion

In this comprehensive exploration, we've demonstrated how to find a nonzero vector orthogonal to a set of points in three-dimensional space. Starting with the points P(3, 0, 3), Q(2, 1, 4), and R(6, 2, 5), we first identified the vectors connecting these points, then computed their cross product to find a normal vector to the plane containing them. The resulting vector, $\vec{n} = (0, 1, -1)$, is a fundamental building block in understanding spatial relationships and solving geometric problems.

This process exemplifies core principles in vector calculus, including vector subtraction, cross product calculation, and the derivation of plane equations. Mastery of these techniques is invaluable for students, educators, and professionals working in fields that require spatial analysis and three-dimensional modeling.

SEO Keywords and Phrases

- Find orthogonal vector in 3D space
- Vector calculus problems involving points
- How to compute the cross product for plane normals
- Normal vector of a plane through three points
- Geometric interpretation of cross product
- Three-dimensional geometry vector solutions
- Applications of orthogonal vectors in physics and engineering
- Deriving plane equations from points
- Vector operations in coordinate geometry
- Spatial relationships and vector analysis

By understanding and applying these concepts, you can confidently solve a wide array of problems involving vectors and geometry in three-dimensional space.

Frequently Asked Questions

How do you find a nonzero vector orthogonal to points P(3, 0, 3), Q(2, 1, 4), and R(6, 2, 5)?

First, find vectors PQ and PR by subtracting their coordinates, then compute their cross product to get a nonzero vector orthogonal to both, which will be orthogonal to the plane containing the points.

What is the step-by-step process to compute a vector orthogonal to the points P, Q, and R?

Calculate vectors PQ = Q - P and PR = R - P, then take the cross product of PQ and PR. The resulting vector is orthogonal to the plane passing through the points.

Can you provide the calculation for the orthogonal vector to points P(3, 0, 3), Q(2, 1, 4), and R(6, 2, 5)?

Yes. First, compute PQ = (-1, 1, 1) and PR = (3, 2, 2). Then, cross these vectors: PQ × PR = ((1)(2) - (1)(2), -[(-1)(2) - (1)(3)], (-1)(2) - (1)(3)) = (0, 1, -5). So, a nonzero orthogonal vector is (0, 1, -5).

What geometric significance does the orthogonal vector have in relation to the points P, Q, and R?

The orthogonal vector is normal (perpendicular) to the plane containing points P, Q, and R, and can be used to find the plane's equation or analyze spatial relationships.

Is the orthogonal vector unique for the points P, Q, and R? Why or why not?

The orthogonal vector is not unique; any scalar multiple of the cross product will also be orthogonal. Therefore, there are infinitely many orthogonal vectors differing by a scalar factor.

How can the orthogonal vector be used to determine the equation of the plane through points P, Q, and R?

Use the orthogonal vector as the normal vector and substitute any point (like P) into the plane equation $n \cdot (r - r0) = 0$, where n is the normal vector and r0 is a point on the plane, to find the plane's equation.

Additional Resources

Vector Orthogonality: Unlocking the Geometry of Space

Introduction

In the realm of three-dimensional space, vectors serve as fundamental building blocks that describe directions, magnitudes, and relationships between points. Whether you're navigating the physical world, engineering complex systems, or exploring the depths of mathematical theory, understanding how vectors interact is crucial. Among these interactions, the concept of orthogonality—or perpendicularity—stands out as a cornerstone, enabling us to decipher angles, projections, and independence within vector spaces.

In this article, we'll delve deeply into the problem of finding a nonzero vector orthogonal to a given set of points—specifically, points P(3, 0, 3), Q(2, 1, 4), and R(6, 2, 5). This exploration not only reinforces fundamental concepts in vector algebra but also demonstrates practical methods used in various fields such as computer graphics, physics, and engineering.

Understanding the Problem: What Does It Mean to Find an Orthogonal Vector?

Before diving into calculations, let's clarify the core question: "Find a nonzero vector orthogonal to the points P, Q, and R."

In vector terms, this typically means:

- Identify a vector that is perpendicular to the vectors formed by the points, particularly those that span the plane containing P, Q, and R.

- Since three points define a plane (unless they are collinear), the normal vector to this plane is orthogonal (perpendicular) to any vectors lying within it.

Thus, the task reduces to:

- Calculating vectors that represent directions within the plane.

- Using these vectors to find a normal vector, which is orthogonal to both.

Establishing the Foundations: Vectors in Space

Let's recall some key definitions:

- A vector in 3D space is an ordered triplet, e.g., $\vec{v} = (v_x, v_y, v_z)$.

- The vector difference between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by:

$$
\vec{AB} = \vec{B} - \vec{A} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)
$$

- The dot product of vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$ is:

$$
\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z
$$

- Two vectors are orthogonal if their dot product is zero.

Step 1: Forming Vectors from the Points

Given the points:

- $P(3, 0, 3)$
- $Q(2, 1, 4)$
- $R(6, 2, 5)$

Let's form vectors that lie within the plane containing these points:

- $\vec{PQ}$: from P to Q

$$
\vec{PQ} = (2 - 3, 1 - 0, 4 - 3) = (-1, 1, 1)
$$

- $\vec{PR}$: from P to R

$$
\vec{PR} = (6 - 3, 2 - 0, 5 - 3) = (3, 2, 2)
$$

Step 2: Finding the Normal Vector via Cross Product

The key to finding a vector orthogonal to the plane (and thus orthogonal to both $\vec{PQ}$ and $\vec{PR}$) is to compute their cross product.

The cross product of two vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$ is:

$$
\vec{u} \times \vec{v} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
u_x & u_y & u_z \\
v_x & v_y & v_z \\
\end{vmatrix}
$$

which expands to:

$$
\vec{u} \times \vec{v} = (u_y v_z - u_z v_y)\mathbf{i} - (u_x v_z - u_z v_x)\mathbf{j} + (u_x v_y - u_y v_x)\mathbf{k}
$$

Applying this to our vectors:

- $\vec{PQ} = (-1, 1, 1)$
- $\vec{PR} = (3, 2, 2)$

Calculating the cross product:

$$
\vec{n} = \vec{PQ} \times \vec{PR} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-1 & 1 & 1 \\
3 & 2 & 2 \\
\end{vmatrix}
$$

Expanding:

$$
\vec{n} = \left((1)(2) - (1)(2) \right) \mathbf{i} - \left((-1)(2) - (1)(3) \right) \mathbf{j} + \left((-1)(2) - (1)(3) \right) \mathbf{k}

\]

Calculating each component:

- \(i\)-component:

\[
(1)(2) - (1)(2) = 2 - 2 = 0
\]

- \(j\)-component:

\[
- [(-1)(2) - (1)(3)] = - [-2 - 3] = - (-5) = 5
\]

- \(k\)-component:

\[
(-1)(2) - (1)(3) = -2 - 3 = -5
\]

So, the normal vector is:

\[
\boxed{
\vec{n} = (0, 5, -5)
}
\]

Step 3: Simplifying and Interpreting the Normal Vector

The vector \(\vec{n} = (0, 5, -5)\) is orthogonal to the plane containing points P, Q, R. To make it more elegant, we can factor out common factors:

\[
\vec{n} = (0, 5, -5) = 5 \times (0, 1, -1)
\]

Since scalar multiples of vectors preserve orthogonality, the unit or simplified normal vector can be:

\[
\boxed{
\mathbf{\hat{n}} = (0, 1, -1)
}
\]

This vector is nonzero (as required), and it is perpendicular to the plane defined by the three points.

Additional Insights: Verifying Orthogonality

To confirm, we can verify that $\mathbf{\hat{n}}$ is orthogonal to $\vec{PQ}$ and $\vec{PR}$:

- Dot product with $\vec{PQ} = (-1, 1, 1)$:

$$
(0)(-1) + (1)(1) + (-1)(1) = 0 + 1 - 1 = 0
$$

- Dot product with $\vec{PR} = (3, 2, 2)$:

$$
(0)(3) + (1)(2) + (-1)(2) = 0 + 2 - 2 = 0
$$

Both are zero, confirming orthogonality.

Practical Applications and Implications

Finding a normal vector to a plane defined by three points isn't just an academic exercise; it has tangible applications:

- Computer Graphics: Normal vectors are essential for lighting calculations, shading, and rendering realistic scenes.

- Physics: Determining planes and their normals helps analyze forces, reflections, and electromagnetic fields.

- Engineering: Structural analysis often involves calculating perpendicular components to understand load distributions.

- Mathematics & Geometry: The process exemplifies how algebra and calculus tools elucidate spatial relationships.

Summary of the Procedure

To encapsulate the process:

1. Identify the points defining the plane.
2. Construct vectors within the plane by subtracting coordinates.
3. Compute the cross product of these vectors to find the normal vector.

4. Simplify the resulting vector if possible.
5. Verify orthogonality via dot product calculations.

This systematic approach is robust and widely applicable, providing a foundational skill in spatial analysis and vector calculus.

Final Remarks

The problem of finding a nonzero vector orthogonal to three given points underscores the elegance of vector algebra in solving geometric problems. The normal vector $\boxed{(0, 1, -1)}$ not only satisfies the mathematical criteria but also exemplifies the power of cross product operations in revealing the hidden symmetries and orientations within three-dimensional space.

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