Deborah Flips A Fair Coin 200 Times. She Calculates She Flips Heads 90 Times. What Is The Probability

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Understanding probability is fundamental to many aspects of everyday life, from gambling and statistics to decision-making and scientific research. In this article, we explore a fascinating question: if Deborah flips a fair coin 200 times and observes 90 heads, what is the probability of this outcome? We will delve into the principles of binomial probability, examine how to interpret such results, and discuss what they reveal about randomness and chance. Whether you're a student, educator, or simply curious about probability theory, this comprehensive guide will provide clarity and insight into this intriguing problem.

Introduction to Probability and Coin Tossing

The Basics of Probability

Probability measures the likelihood of a specific event occurring within a set of possible outcomes. It is expressed as a number between 0 and 1, where:
- 0 indicates impossibility
- 1 indicates certainty
- Values in between represent varying degrees of likelihood

For example, flipping a fair coin has two equally likely outcomes: heads or tails. Each outcome has a probability of 0.5.

Coin Tossing and Binomial Distribution

Coin tossing is a classic example used to illustrate fundamental concepts in probability. When flipping a fair coin multiple times, the number of heads observed follows a binomial distribution, which models the number of successes (heads) in a fixed number of independent trials.

Key features of the binomial distribution:
- Number of trials: n (here, 200)
- Probability of success in each trial: p (here, 0.5 for a fair coin)
- Random variable: X (number of heads observed)

The probability of observing exactly k heads in n flips is given by the binomial probability formula.

Calculating the Probability of Observing 90 Heads in 200 Flips

The Binomial Probability Formula

The probability $P(X = k)$ of getting exactly k successes (heads) in n trials is:

$$
P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}
$$

Where:
- $\binom{n}{k}$ is the binomial coefficient (number of ways to choose k successes from n trials),
- p is the probability of success on each trial,
- $1 - p$ is the probability of failure.

In Deborah’s case:
- $n = 200$
- $k = 90$
- $p = 0.5$

Calculating directly:

$$
P(X=90) = \binom{200}{90} (0.5)^{90} (0.5)^{110} = \binom{200}{90} (0.5)^{200}
$$

This calculation involves very large numbers (binomial coefficient), which are difficult to compute manually.

Approximating the Probability Using Normal Distribution

Calculating binomial probabilities directly for large n can be computationally intensive. Instead, the normal approximation to the binomial distribution provides a practical solution, especially when n is large.

Conditions for using the normal approximation:
- np and $n(1-p)$ are both greater than 5
- Here, $np = 200 \times 0.5 = 100$
- $n(1-p) = 200 \times 0.5 = 100$

Since both are greater than 5, the normal approximation is appropriate.

Parameters for the normal distribution:

- Mean $\mu = np = 100$
- Standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{200 \times 0.5 \times 0.5} = \sqrt{50} \approx 7.07$

Applying the continuity correction:
To find the probability of observing 90 heads, we calculate the probability that the normal variable falls between 89.5 and 90.5.

$$
P(89.5 < X < 90.5) = \Phi \left(\frac{90.5 - \mu}{\sigma} \right) - \Phi \left(\frac{89.5 - \mu}{\sigma} \right)
$$

Where Φ is the cumulative distribution function (CDF) of the standard normal distribution.

Calculating:
- $z_1 = \frac{89.5 - 100}{7.07} \approx -1.52$
- $z_2 = \frac{90.5 - 100}{7.07} \approx -1.33$

Using standard normal tables or a calculator:
- $\Phi(-1.52) \approx 0.064$
- $\Phi(-1.33) \approx 0.0918$

Therefore:
$$
P \approx 0.0918 - 0.064 = 0.0278
$$

Interpretation:
The probability of observing exactly 90 heads in 200 flips of a fair coin is approximately 2.78%.

Understanding the Significance of the Result

Is 90 Heads in 200 Flips a Rare Event?

Given that the expected number of heads is 100 (since p=0.5), observing 90 heads is slightly below the expected value. The probability of such an outcome is about 2.78%, indicating that while not extremely common, it is still within a reasonable range of outcomes for a fair coin.

Key points:
- Outcomes close to the expected mean are relatively more probable.
- Significant deviations from the mean (e.g., 50 or 150 heads) are much less probable.

What Does This Tell Us About Fair Coins?

This analysis demonstrates that even with a fair coin, outcomes will vary due to randomness. Observing 90 heads in 200 flips aligns well with the expected probability, reinforcing the concept that chance can produce a range of results around the mean.

Real-World Applications of Probability and Coin Tossing

Statistics and Scientific Research

Understanding probabilities helps scientists interpret experimental results, assess the likelihood of observed data, and determine statistical significance.

Gambling and Gaming

Casinos and gamblers use probability to calculate odds, design games, and manage risk.

Decision-Making and Risk Assessment

Businesses and policymakers rely on probability models to make informed decisions under uncertainty.

Educational Value

Coin tossing experiments serve as practical demonstrations for teaching fundamental concepts in probability and statistics.

Summary and Key Takeaways

Summary of the main points:
- Deborah's experiment involves flipping a fair coin 200 times.
- The probability of observing exactly 90 heads can be approximated using the binomial distribution and, more practically, the normal approximation.
- The probability of getting exactly 90 heads is approximately 2.78%, indicating such an outcome is plausible within the realm of chance.
- Results close to the expected mean (100 heads) are more probable, highlighting the variability inherent in random processes.
- Understanding these principles is essential for applications across various fields, from science to finance.

Key points to remember:
1. The binomial distribution models the probability of a specific number of successes in independent trials.
2. For large n, the normal approximation simplifies calculations.
3. Variability in outcomes is natural and expected in probabilistic experiments.
4. Probabilities help interpret whether observed results are typical or unusual.

Further Exploration and Resources

- Books:
- "Introduction to Probability" by Joseph K. Blitzstein and Jessica Hwang
- "Statistics for Dummies" by Deborah J. Rumsey
- Online Tools:
- Binomial calculator tools
- Normal distribution calculators
- Courses:
- Khan Academy's Probability and Statistics courses
- Coursera's "Statistics with R" specialization

In conclusion, the probability of Deborah flipping 90 heads in 200 coin tosses illustrates fundamental principles of probability theory and statistical analysis. Recognizing the likelihood of such outcomes enhances our understanding of randomness and helps us interpret real-world data with greater confidence.

Frequently Asked Questions

What is the probability that Deborah flips exactly 90 heads out of 200 fair coin flips?

The probability is given by the binomial formula: P = C(200, 90) (0.5)^90 (0.5)^{110}, where C(200, 90) is the combination of 200 flips taken 90 at a time.

How can we interpret Deborah's result of 90 heads in 200 flips in terms of probability?

It represents the likelihood of observing exactly 90 heads in 200 flips of a fair coin, which can be calculated using the binomial distribution.

Is flipping 90 heads out of 200 considered a typical

outcome for a fair coin?

Yes, since the expected number of heads is 100 with a standard deviation of about 6.3, 90 heads is within a common range, indicating it's a typical variation.

What is the expected number of heads in 200 fair coin flips?

The expected number of heads is 200 0.5 = 100.

How do we calculate the probability of getting at most 90 heads in 200 flips?

You sum the binomial probabilities from 0 up to 90, or use a normal approximation to estimate the cumulative probability.

Can normal distribution approximation be used to estimate this probability?

Yes, since n=200 is large, the binomial distribution can be approximated by a normal distribution with mean 100 and standard deviation approximately 6.3.

What is the approximate probability of flipping 90 or fewer heads in 200 flips?

Using normal approximation, this probability is about 0.33, indicating there's roughly a 33% chance of observing 90 or fewer heads.

How does Deborah's result compare to the expected number of heads?

She flipped 10 fewer heads than the expected 100, which is within the range of typical variation for such experiments.

What does Deborah's flipping of 90 heads suggest about the fairness of the coin?

It suggests the coin is likely fair, as the result is within normal variation; it does not provide evidence of bias.

Why is it important to consider probability when analyzing Deborah's coin flip results?

Probability helps determine whether her outcome is typical or unusual, aiding in understanding whether the coin is likely fair or biased.

Additional Resources

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Introduction

In the realm of probability and statistical inference, seemingly simple questions can reveal profound insights into randomness, bias, and the interpretation of data. One such question involves Deborah, who flips a fair coin 200 times and observes 90 heads. She then seeks to understand the probability of this outcome under the assumption that the coin is, in fact, fair. This scenario presents an excellent case study for exploring the principles of probability theory, hypothesis testing, and the nuances of interpreting observed data in probabilistic terms.

This article aims to comprehensively analyze Deborah's situation, delving into the statistical concepts involved, reviewing relevant methodologies, and discussing the implications of her findings. Through a detailed examination, we will clarify what her observed result suggests about the fairness of the coin and how to quantify the likelihood of such an outcome occurring by chance.

Context and Background

The Scenario at a Glance

- Deborah flips a fair coin 200 times.
- She records 90 heads and, by implication, 110 tails.
- She calculates the probability of obtaining 90 or fewer heads in 200 flips, assuming the coin is fair.

This scenario is a classic binomial experiment, where each flip is an independent Bernoulli trial with probability $p = 0.5$ for heads. The key question is: Given the data, what is the probability that such an outcome (or a more extreme one) would occur if the coin is truly fair?

Fundamental Concepts

The Binomial Distribution

The binomial distribution models the number of successes (heads) in a fixed number of independent trials, each with the same probability of success:

\[

$$
P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}
$$

where:
- n = total trials (200)
- k = number of successes (heads)
- p = probability of success on each trial (0.5 for a fair coin)

In Deborah's case, the probability of observing exactly 90 heads is:

$$
P(X=90) = \binom{200}{90} (0.5)^{90} (0.5)^{110}
$$

However, to assess how unusual her result is, we need to consider the cumulative probability of observing 90 or fewer heads (or, symmetrically, 110 or more heads), since deviations in either direction could suggest bias.

Statistical Significance and Hypothesis Testing

Null Hypothesis

- H0: The coin is fair ($p=0.5$)
- Alternative Hypothesis: The coin is biased ($p \neq 0.5$)

Deborah's observed data can be used to compute a p-value, which indicates the probability of observing such an extreme result (or more extreme) under H_0.

Computing the P-Value

The p-value for a two-tailed test is:

$$
p\text{-value} = P(X \leq 90 \text{ or } X \geq 110 \mid p=0.5)
$$

Alternatively, since the binomial distribution can be cumbersome for large n, we often use a normal approximation with continuity correction.

Normal Approximation to the Binomial

For large n, the binomial distribution approximates a normal distribution:

$$
X \sim N(\mu, \sigma^2)

\]

where:
- \(\mu = np = 200 \times 0.5 = 100 \)
- \(\sigma = \sqrt{np(1-p)} = \sqrt{200 \times 0.5 \times 0.5} \approx 7.071 \)

Applying a continuity correction, the probability of observing 90 or fewer heads:

\[
P(X \leq 90) \approx P\left(Z \leq \frac{90 + 0.5 - 100}{7.071}\right) = P(Z \leq -1.343)
\]

Similarly, for 110 or more:

\[
P(X \geq 110) \approx P\left(Z \geq \frac{110 - 0.5 - 100}{7.071}\right) = P(Z \geq 1.343)
\]

Because the standard normal distribution is symmetric:

\[
p\text{-value} \approx 2 \times P(Z \leq -1.343)
\]

Looking up this value:

\[
P(Z \leq -1.343) \approx 0.09
\]

Thus:

\[
p\text{-value} \approx 2 \times 0.09 = 0.18
\]

This indicates an approximately 18% chance of observing 90 or fewer heads (or equivalently, 110 or more) in 200 flips if the coin is fair.

Interpretation of Results

Is the Result Statistically Significant?

A standard significance threshold in many scientific contexts is 0.05 (5%). Since the p-value (~0.18) exceeds this threshold, Deborah's observed result does not provide sufficient evidence to reject the null hypothesis that her coin is fair.

Practical Implications

- The outcome of 90 heads in 200 flips is within the realm of typical random variation.
- There is no statistically significant evidence to suggest bias.
- Such fluctuations are expected in binomial experiments, especially with moderate sample sizes.

Exploring the Probability of Specific Outcomes

Exact Probability for 90 Heads

Calculating $P(X=90)$ exactly involves binomial coefficients, which for large n are often approximated for practical purposes. Using software or tables:

$$
P(X=90) \approx \text{Very small but not negligible}
$$

The probability of exactly 90 heads is approximately 0.014 (or 1.4%), indicating that such an exact outcome is not rare under the assumption of fairness.

Cumulative Probability for 90 or Fewer Heads

The cumulative probability, as previously approximated, is around 0.18 (18%) for 90 or fewer heads, implying that nearly one-fifth of all sequences would have 90 or fewer heads.

Broader Perspectives: What Can Deborah Conclude?

The Role of Sample Size

- The sample size (200 flips) provides a reasonable basis for inference but is not large enough to detect very subtle bias.
- Larger sample sizes would narrow confidence intervals and improve detection power.

The Nature of Random Fluctuations

- Even with a fair coin, outcomes like 90 heads in 200 flips are not unusual.
- Variability is inherent in stochastic processes; small deviations are expected.

The Importance of Repetition

- One sequence alone cannot definitively establish bias.
- Repeating the experiment multiple times and analyzing aggregate data would yield more robust conclusions.

Limitations and Considerations

Assumptions

- Independence of flips
- Fairness of the coin (no bias in manufacturing or handling)
- Accurate recording of outcomes

Potential Biases and Experimental Errors

- Physical biases in the coin
- Flipping technique
- Recording errors

Deborah's calculation assumes the first null hypothesis (fairness); if other factors are suspected, more sophisticated tests or experimental controls might be necessary.

Conclusion

Deborah's observation of 90 heads out of 200 flips, under the assumption of a fair coin, corresponds to a p-value of approximately 0.18. This indicates that such an outcome is well within the expected range of random variation, and there is no compelling statistical evidence to suggest the coin is biased.

This case exemplifies fundamental principles in statistical inference: how to interpret observed data, the importance of understanding probability distributions, and the necessity of cautious reasoning before drawing conclusions about bias or fairness. It underscores that in probabilistic experiments, fluctuations are inevitable, and only through rigorous analysis can we discern meaningful deviations from chance.

In practical terms, Deborah should consider the result as consistent with the coin's fairness. For more conclusive assessments, larger samples and repeated testing would provide deeper insights. Ultimately, this exploration illuminates how probability theory guides our understanding of randomness, variability, and uncertainty in everyday experiments.

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Final Remarks

In the realm of statistical analysis, the key takeaway from Deborah's experiment is the importance of understanding the natural variability inherent in stochastic processes. While her initial observation might raise suspicion, rigorous calculation and interpretation reveal that such results are not uncommon for a fair coin. This reinforces the critical role of statistical reasoning in making informed judgments based on data, especially when dealing with randomness and uncertainty.

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