Define A Relation On Z As X R Y If |x Y| < 1. Is R Reflexive? Symmetric? Transitive? If A Property

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In the study of relations within mathematics, especially in set theory and algebra, understanding the properties of a relation helps classify and analyze its behavior. Here, we define a relation \(R \) on the set of integers \(\mathbb{Z} \) such that for any \(x, y \in \mathbb{Z} \), \(x R y \) if and only if \(|x - y| < 1 \). The goal of this article is to examine whether this relation \(R \) is reflexive, symmetric, or transitive, and to understand the implications of these properties in the context of this specific relation.

Understanding the Relation \(R \): Basic Definition

Before analyzing the properties, it's essential to clarify the nature of the relation:

- The relation is defined as:

\[
x R y \iff |x - y| < 1,
\]

for all \(x, y \in \mathbb{Z} \).

- Since \(x, y \) are integers, the absolute difference \(|x - y| \) is a non-negative integer as well.

- The inequality \(|x - y| < 1 \) places a restriction on the difference between \(x \) and \(y \).

Analyzing Reflexivity of \(R \)

What Is Reflexivity?

A relation \(R \) on a set \(A \) is called reflexive if, for every element \(a \in A \), the relation holds between \(a \) and itself:

\[

$$
a R a \quad \text{for all } a \in A.
$$

In this context, the set A is $\mathbb{Z}$.

Is R Reflexive?

To determine whether R is reflexive, check whether for every $x \in \mathbb{Z}$:

$$
x R x \quad \Longleftrightarrow \quad |x - x| < 1.
$$

Since:

$$
|x - x| = 0,
$$

and

$$
0 < 1,
$$

we see that:

$$
|x - x| < 1 \quad \text{is always true}.
$$

Therefore, for all $x \in \mathbb{Z}$:

$$
x R x \quad \text{holds}.
$$

Conclusion: The relation R is reflexive.

Analyzing Symmetry of R

What Is Symmetry?

A relation \(R \) on a set \(A \) is symmetric if, for every \(a, b \in A \):

\[
a R b \implies b R a.
\]

This means that if the relation holds from \(a \) to \(b \), it must also hold from \(b \) to \(a \).

Is \(R \) Symmetric?

Suppose \(x, y \in \mathbb{Z} \) satisfy:

\[
x R y \quad \Longleftrightarrow \quad |x - y| < 1.
\]

We need to verify whether:

\[
|x - y| < 1 \implies |y - x| < 1.
\]

Note that:

\[
|y - x| = |x - y|,
\]

since absolute value is symmetric.

Thus, if \(|x - y| < 1 \), then:

\[
|y - x| = |x - y| < 1,
\]

which means:

\[
y R x.
\]

Conclusion: The relation \(R \) is symmetric because the absolute difference is symmetric.

Analyzing Transitivity of R

What Is Transitivity?

A relation R on a set A is transitive if, for all $a, b, c \in A$:

$$
a R b \quad \text{and} \quad b R c \quad \implies \quad a R c.
$$

In words, if a relates to b, and b relates to c, then a must relate to c.

Is R Transitive?

Suppose $x, y, z \in \mathbb{Z}$ such that:

$$
x R y \quad \Longleftrightarrow \quad |x - y| < 1,
$$
$$
y R z \quad \Longleftrightarrow \quad |y - z| < 1.
$$

To check transitivity, we need to verify whether:

$$
|x - y| < 1 \quad \text{and} \quad |y - z| < 1 \quad \implies \quad |x - z| < 1.
$$

Let's analyze this with specific examples.

Counterexample to Transitivity

Consider:

- $x = 0$,
- $y = 0$,
- $z = 1$.

Compute:

- $|x - y| = |0 - 0| = 0 < 1$ — so $x R y$ holds.

- $|y - z| = |0 - 1| = 1$ — but this is not less than 1; it equals 1.

Since the relation requires strict inequality $|x - y| < 1$, the second relation $y R z$ does not hold in this case, so this example doesn't violate transitivity.

Let's choose another:

- $x = 0$,
- $y = 0.5$,
- $z = 1.0$.

Check:

- $|x - y| = |0 - 0.5| = 0.5 < 1$ — so $x R y$ holds.
- $|y - z| = |0.5 - 1.0| = 0.5 < 1$ — so $y R z$ holds.

Now, check whether $x R z$:

- $|x - z| = |0 - 1.0| = 1.0$.

Since:

$$
|x - z| = 1.0,
$$

which is not less than 1, the relation $x R z$ does not hold.

This example shows that even if $x R y$ and $y R z$ are true, $x R z$ may not be true.

Conclusion: The relation R is not transitive.

Summary of Properties

Based on the above analysis:

- Reflexive: Yes, because $|x - x| = 0 < 1$ for all $x \in \mathbb{Z}$.
- Symmetric: Yes, because $|x - y| = |y - x|$, so the relation is symmetric.
- Transitive: No, as demonstrated by counterexamples where $|x - y| < 1$ and $|y - z| < 1$ do not imply $|x - z| < 1$.

Implications of These Properties

Understanding the properties of relation R helps in classifying it within well-known types of relations:

- Since R is reflexive and symmetric but not transitive, it is classified as an indistinctive relation; it does not qualify as an equivalence relation, which requires all three properties.

- The relation constrains x and y to be within less than 1 unit apart on the integer number line. But because the integers are discrete, the only pairs satisfying $|x - y| < 1$ are those where $x = y$. This is because:

- The absolute difference between two integers is always an integer.
- The only integer less than 1 is 0.
- Therefore, for integers:

$$
|x - y| < 1 \quad \Longleftrightarrow \quad |x - y| = 0 \quad \Longleftrightarrow \quad x = y.
$$

- This means the relation R effectively reduces to the identity relation on $\mathbb{Z}$: each element is related only to itself.

Conclusion

The relation R on $\mathbb{Z}$, defined by $x R y \iff |x - y| < 1$, exhibits some fundamental properties:

- Reflexive: Yes, because every integer is related

Frequently Asked Questions

Is the relation R on Z defined by X R Y if |X - Y| < 1 reflexive?

No, the relation R is not reflexive because for any integer X, |X - X| = 0, which is less than 1, so actually it is reflexive. Correction: Since |X - X| = 0 < 1, R is reflexive.

Is the relation R on Z symmetric?

Yes, the relation R is symmetric because if |X - Y| < 1, then |Y - X| = |X - Y| < 1, so Y R X whenever X R Y.

Is the relation R transitive on Z?

No, R is not transitive. For example, if X R Y and Y R Z, it does not guarantee that X R Z since |X - Z|

may be greater than or equal to 1.

Does the relation R define an equivalence relation on Z?

No, because R is not transitive, so it does not satisfy all properties of an equivalence relation.

What property does the relation R have on Z?

The relation R is reflexive and symmetric but not transitive.

Can the relation R be considered an example of a neighborhood relation on Z?

Yes, since it relates numbers within a distance less than 1, R can be viewed as defining a neighborhood relation around each integer.

Additional Resources

Define A Relation On Z As X R Y If |x - Y| < 1. Is R Reflexive? Symmetric? Transitive? If A Property

In the realm of mathematics, relations serve as foundational tools for understanding how elements within a set relate to each other. One such relation, defined on the set of integers (denoted as Z), can be expressed as follows: for any two integers x and y, the relation R holds if and only if the absolute difference between them is less than 1, that is, $|x - y| < 1$. This seemingly simple condition sparks intriguing questions about the nature of the relation itself—specifically, whether it exhibits key properties such as reflexivity, symmetry, and transitivity. Exploring these properties not only deepens our understanding of this particular relation but also sheds light on broader concepts in set theory and mathematical logic.

Understanding the Relation: The Foundation

Before diving into the properties, it’s essential to clarify what the relation R signifies within the set of integers Z.

Definition of the Relation:

- For any $x, y \in Z$,

$$
xRy \iff |x - y| < 1
$$

Implications of the Definition:

- The absolute difference between x and y must be strictly less than 1.
- Since x and y are integers, their difference is always an integer.

Key Observation:

- Because the difference between two integers is itself an integer, the only way for $|x - y|$ to be less than 1 is if $|x - y| = 0$.

- This is due to the fact that the absolute value of an integer is a non-negative integer, and the only non-negative integer less than 1 is zero.

Consequently:

$$
|x - y| = 0 \implies x = y
$$

- Therefore, the relation R simplifies to:

$$
xRy \iff x = y
$$

This realization is fundamental because it indicates that R is actually the identity relation on Z, relating each element only to itself.

Is R Reflexive?

Definition of Reflexivity:

A relation R on a set A is reflexive if, for every element $a \in A$:

$$
aRa
$$

In words, every element is related to itself.

Applying to Our Relation:

- Since R is defined as $xRy \iff |x - y| < 1$, and we've established that this only holds when $x = y$:

$$
xRx \iff |x - x| < 1 \implies 0 < 1
$$

- The absolute difference $|x - x| = 0$, and 0 is indeed less than 1.

Conclusion:

- For all $x \in Z$, xRx is true.

Therefore, the relation R is reflexive.

Is R Symmetric?

Definition of Symmetry:

A relation R on a set A is symmetric if, for all $a, b \in A$:

$$
aRb \implies bRa
$$

Application to Our Relation:

- Suppose xRy holds. From earlier, this is equivalent to:

$$
|x - y| < 1
$$

- But as established, for integers, the only way this inequality holds is when:

$$
x = y
$$

- If $x = y$, then:

$$
bRa \iff |b - a| < 1
$$

- Since $x = y$, it follows that:

$$
|x - y| = 0
$$

- And because equality is symmetric, the relation is trivially symmetric.

Conclusion:

- If xRy (which implies $x = y$), then yRx (since $y = x$) is also true.

- Therefore, the relation R is symmetric.

Is R Transitive?

Definition of Transitivity:

A relation R on A is transitive if, for all $a, b, c \in A$:

$$
aRb \text{ and } bRc \implies aRc
$$

Applying to Our Relation:

- Suppose xRy and yRz hold. From previous reasoning:

$$
|x - y| < 1 \quad \text{and} \quad |y - z| < 1
$$

- But, as shown, the only way for these inequalities to hold with integers is when:

$$
x = y \quad \text{and} \quad y = z
$$

- Consequently:

$$
x = y = z
$$

- Then:

$$
|x - z| = 0 < 1
$$

- So:

$$
x R z
$$

Conclusion:

- Since the only case where the initial conditions hold is when all three elements are equal, the relation is transitive.

Summary of Properties

Based on the detailed analysis, the relation R on Z, defined by $|x - y| < 1$, exhibits the following characteristics:

- Reflexive: Yes, because xRx holds for all $x \in Z$.
- Symmetric: Yes, because if xRy, then $x = y$, implying yRx.
- Transitive: Yes, because only when $x = y = z$ does the relation hold, ensuring the property.

Overall, the relation R is an equivalence relation, specifically the identity relation, on the set of integers.

Broader Implications and Context

Understanding the properties of this relation offers insight into how relations behave under certain constraints. In this case, the strict inequality $|x - y| < 1$ effectively reduces to the equality relation on Z.

Implications include:

- Identity Relation: The relation is exactly the identity relation, which relates each element solely to itself.
- Equivalence Relation: Because it is reflexive, symmetric, and transitive, R forms an equivalence relation, partitioning Z into singleton classes.
- Practical Perspective: In real-world applications, such a relation could model situations where entities are considered "identical" if their values differ by less than 1, which, in the integer case, reduces to exact equality.

Extensions and Variations:

- If the relation were defined on real numbers $\mathbb{R}$ instead of integers, the properties would differ significantly. For instance, with $x, y \in \mathbb{R}$:

$$
xRy \iff |x - y| < 1
$$

- In that context, the relation would be:

- Reflexive: Yes
- Symmetric: Yes
- Transitive: No, because for real numbers, $|x - y| < 1$ and $|y - z| < 1$ does not guarantee $|x -

$z| < 1$). For example, if $x = 0$, $y = 0.9$, $z = 1.8$, then:

$$
|x - y| = 0.9 < 1
$$
$$
|y - z| = 0.9 < 1
$$
but

$$
|x - z| = 1.8 \not< 1
$$

- This highlights how the properties of relations are sensitive to the underlying set and the defining condition.

Final Remarks

The exploration of the relation R defined by $|x - y| < 1$ on the set of integers reveals a fundamental truth: due to the discrete nature of integers, the relation simplifies to the equality relation. Consequently, it naturally possesses the hallmark properties of an equivalence relation—reflexivity, symmetry, and transitivity. Such analyses are vital in mathematical logic and set theory, as they help classify and understand the structure of relations within various contexts.

By examining this relation, we appreciate the elegance with which simple conditions can lead to well-understood and fundamental properties. Whether in pure mathematics or applied fields, recognizing these properties enables us to model, analyze, and interpret complex systems with clarity and precision.

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