Determine If S Could Lie On The Perpendicular Bisector Of QR With The Given CoordinatesQ(-5 , 4), R(8

Determine If S Could Lie On The Perpendicular Bisector Of QR With The Given Coordinates Q(-5, 4), R(8, ...

Understanding the geometric principles behind perpendicular bisectors and their properties is essential in various mathematical and real-world applications, including coordinate geometry, construction problems, and geometric proofs. In this comprehensive guide, we will analyze whether a point S can lie on the perpendicular bisector of the segment QR given the coordinates Q(-5, 4) and R(8, ...). We will explore the concepts involved, step-by-step calculations, and the reasoning process to determine the positional relationship of S relative to the segment QR and its perpendicular bisector.

Understanding the Perpendicular Bisector in Coordinate Geometry

Before diving into the problem, it is crucial to understand what a perpendicular bisector is and its significance in coordinate geometry.

Definition of Perpendicular Bisector

- The perpendicular bisector of a segment is a line that cuts the segment into two equal parts at a 90-degree angle.
- It passes through the midpoint of the segment and is perpendicular to it.

Properties of the Perpendicular Bisector

- Any point lying on the perpendicular bisector is equidistant from the segment's endpoints.
- The perpendicular bisector of a segment in a coordinate plane can be derived using the coordinates of the segment's endpoints.

Relevance to the Problem

- To determine if a point S can lie on the perpendicular bisector of QR, it must satisfy the condition of being equidistant from Q and R.

Given Coordinates and Initial Data

Let's consider the given points:

- Q(-5, 4)
- R(8, ...)

Note: The coordinate of R appears incomplete in the prompt. For the purposes of this analysis, we will assume the full coordinate of R as R(8, y), where y is a real number. To proceed, we need either the full coordinate of R or an assumption.

Assumption: Suppose the coordinate of R is R(8, y), where y is a specific known value. Let's assume y= y_R for generality.

Step 1: Find the Midpoint of QR

The midpoint, M, of the segment QR is essential because the perpendicular bisector passes through M.

Formula for Midpoint:

$$
M = \left(\frac{x_Q + x_R}{2}, \frac{y_Q + y_R}{2} \right)
$$

Calculation:

$$
x_M = \frac{-5 + 8}{2} = \frac{3}{2} = 1.5
$$
$$
y_M = \frac{4 + y_R}{2}
$$

Thus, the midpoint is:

$$
M(1.5, \frac{4 + y_R}{2})
$$

Step 2: Determine the Slope of QR

The slope (m) of segment QR is:

$$
m_{QR} = \frac{y_R - 4}{8 - (-5)} = \frac{y_R - 4}{13}
$$

Step 3: Find the Slope of the Perpendicular Bisector

Since the perpendicular bisector is perpendicular to QR, its slope is the negative reciprocal of m_{QR}:

$$
m_{PB} = -\frac{1}{m_{QR}} = -\frac{13}{y_R - 4}
$$

Note: To avoid division by zero, m_{QR} must not be zero, which would happen only if $y_R = 4$.

Step 4: Equation of the Perpendicular Bisector

Using point-slope form with point M and slope m_{PB}:

$$
y - y_M = m_{PB} (x - x_M)
$$

Substituting known values:

$$
y - \frac{4 + y_R}{2} = -\frac{13}{y_R - 4} (x - 1.5)
$$

This is the general equation of the perpendicular bisector, pending the value of y_R.

Step 5: Conditions for Point S to Lie on the Perpendicular Bisector

Suppose S has coordinates (x_S, y_S). For S to lie on the perpendicular bisector:

$$
y_S - \frac{4 + y_R}{2} = -\frac{13}{y_R - 4} (x_S - 1.5)
$$

Furthermore, since S lies on the perpendicular bisector, it must be equidistant from Q and R:

$$
\text{Distance}(S, Q) = \text{Distance}(S, R)
$$

Distance formula:

$$
d = \sqrt{(x - x_0)^2 + (y - y_0)^2}
$$

Equality condition:

$$
\sqrt{(x_S + 5)^2 + (y_S - 4)^2} = \sqrt{(x_S - 8)^2 + (y_S - y_R)^2}
$$

Squaring both sides:

$$
(x_S + 5)^2 + (y_S - 4)^2 = (x_S - 8)^2 + (y_S - y_R)^2
$$

This simplifies to:

$$
(x_S + 5)^2 - (x_S - 8)^2 = (y_S - y_R)^2 - (y_S - 4)^2
$$

Step 6: Simplify the Distance Equation

Compute each difference:

$$
(x_S + 5)^2 - (x_S - 8)^2 = [(x_S)^2 + 10x_S + 25] - [x_S^2 - 16x_S + 64] = (10x_S + 25) - (-16x_S + 64) = 10x_S + 25 + 16x_S - 64 = (26x_S) - 39
$$

Similarly, for the y-coordinates:

$$
(y_S - y_R)^2 - (y_S - 4)^2 = [y_S^2 - 2 y_S y_R + y_R^2] - [y_S^2 - 8 y_S + 16] = -2 y_S y_R + y_R^2 + 8 y_S - 16
$$

Set the two expressions equal:

$$
26 x_S - 39 = -2 y_S y_R + y_R^2 + 8 y_S - 16
$$

Rearranged:

$$
26 x_S - 39 + 16 = -2 y_S y_R + y_R^2 + 8 y_S
$$

$$
26 x_S - 23 = -2 y_S y_R + y_R^2 + 8 y_S
$$

Step 7: Express y_S in Terms of x_S

Recall the equation of the perpendicular bisector:

$$
y_S = m_{PB} (x_S - 1.5) + \frac{4 + y_R}{2}
$$

Substitute y_S into the distance equation to find conditions on x_S, y_R, and whether such a point exists.

Analysis and Conclusion

- The existence of a point S lying on the perpendicular bisector and

equidistant from Q and R depends on the specific value of y_R.
- If the coordinates of R are known precisely, the above equations can be solved explicitly to find whether such an S exists and its potential coordinates.
- If $y_R = 4$, then $m_{QR} = 0$, meaning QR is horizontal, and the perpendicular bisector is vertical passing through the midpoint.

Special Case: When R is (8, 4)

Suppose R(8, 4), the same as Q's y-coordinate.

- Then, $m_{QR} = 0$, the segment QR is horizontal.
- The midpoint:

$$
M(1.5, (4 + 4)/2) = (1.5, 4)
$$

- The perpendicular bisector is vertical line $x = 1.5$.

Any point S with $x = 1.5$ and any y such that:

$$
\text{Distance}(S, Q) = \text{Distance}(S, R)
$$

can be evaluated:

$$
\text{Distance}(S, Q) = \sqrt{(1.5 + 5)^2 + (y - 4)^2} = \sqrt{6.5^2 + (y - 4)^2}
$$

$$
\text{Distance}(S, R) = \sqrt{(1.5 - 8)^2 + (y - 4)^2} = \sqrt{(-6.5)^2 + (y - 4)^2}
$$

Since both are identical:

$$
\sqrt{6.5^2 + (y - 4)^2} =
$$

Frequently Asked Questions

How can I determine if point S lies on the perpendicular bisector of segment QR with given coordinates?

To determine if S lies on the perpendicular bisector of QR, first find the midpoint of QR, then determine the slope of QR, find its negative reciprocal for the perpendicular bisector, and check if S satisfies the equation of that line.

What are the steps to find the perpendicular bisector of segment QR given points Q(-5, 4) and R(8, y)?

First, calculate the midpoint of QR: $M = ((x_Q + x_R)/2, (y_Q + y_R)/2)$. Next, find the slope of QR: $m_{QR} = (y_R - y_Q)/(x_R - x_Q)$. Then, find the negative reciprocal of m_{QR} to get the slope of the perpendicular bisector, and write its equation passing through M.

If point S has coordinates (x_s, y_s), how do I check if it lies on the perpendicular bisector of QR?

Substitute the coordinates of S into the equation of the perpendicular bisector. If the coordinates satisfy the line equation, then S lies on the perpendicular bisector; otherwise, it does not.

Why is the midpoint of QR important when determining the perpendicular bisector?

The midpoint of QR is the point through which the perpendicular bisector passes. It ensures that the bisector divides the segment QR into two equal parts and helps in constructing its equation.

Can the perpendicular bisector of QR also be the line of symmetry for the segment?

Yes, the perpendicular bisector acts as the line of symmetry for the segment QR, dividing it into two equal parts at a right angle.

What additional information is needed to fully

determine if S lies on the perpendicular bisector of QR if only Q(-5, 4) and R(8, y) are given?

You need the y-coordinate of point R to find the exact equation of QR, then derive the perpendicular bisector. Without y, you cannot precisely determine the bisector's equation or test whether S lies on it.

Additional Resources

Determine If S Could Lie On The Perpendicular Bisector Of QR With The Given Coordinates Q(-5, 4), R(8, y)

Understanding the geometric relationship between points on a coordinate plane is fundamental in coordinate geometry. In particular, determining whether a point S can lie on the perpendicular bisector of segment QR involves exploring the properties of perpendicular bisectors and applying algebraic methods to analyze the given coordinates. This article aims to thoroughly examine the problem by breaking down the key concepts, step-by-step calculations, and logical reasoning necessary to arrive at a conclusion about the potential position of point S in relation to the segment QR.

Introduction to Perpendicular Bisectors in Coordinate Geometry

In coordinate geometry, the perpendicular bisector of a segment is a line that divides the segment into two equal parts at a right angle. The perpendicular bisector has several important properties:

- Equidistance: Any point lying on the perpendicular bisector is equidistant from the two endpoints of the segment.
- Geometric locus: It is the locus of all points equidistant from the segment's endpoints.

Applying these properties allows us to determine whether a specific point S can lie on this line based on its coordinates relative to Q and R.

Understanding the Given Coordinates and the Problem

The problem provides the coordinates:

- Q(-5, 4)
- R(8, y)

However, the y-coordinate of R is unspecified, which introduces an element of ambiguity. To analyze whether a point S can lie on the perpendicular bisector of QR, we need to consider the possible values of y or analyze the problem generally for any y.

Key points to consider:

- The coordinates of Q are fixed.
- The coordinate of R is partially given, with y being unknown.
- The goal is to determine if there exists a point S on the perpendicular bisector of QR, possibly with certain constraints.

Step-by-Step Approach to the Problem

To analyze whether a point S can lie on the perpendicular bisector of QR, the following steps are essential:

1. Calculate the Midpoint of QR

The midpoint M of QR is given by:

$$
M_x = \frac{x_Q + x_R}{2}, \quad M_y = \frac{y_Q + y_R}{2}
$$

Substituting Q(-5, 4) and R(8, y):

$$
M_x = \frac{-5 + 8}{2} = \frac{3}{2} = 1.5
$$

$$
M_y = \frac{4 + y}{2}
$$

The midpoint depends on y, which remains unknown.

2. Find the Slope of QR

The slope of QR is:

\[

$$
m_{QR} = \frac{y - 4}{8 - (-5)} = \frac{y - 4}{13}
$$

This slope is also dependent on y.

3. Determine the Slope of the Perpendicular Bisector

The perpendicular bisector of QR will have a slope that is the negative reciprocal of m_{QR}, provided $m_{QR} \neq 0$:

$$
m_{perp} = -\frac{1}{m_{QR}} = -\frac{13}{y - 4}
$$

This slope will be undefined if $y - 4 = 0$, i.e., when $y = 4$. This case signifies that R is horizontally aligned with Q.

4. Equation of the Perpendicular Bisector

The perpendicular bisector passes through the midpoint M, so its equation can be written as:

$$
y - M_y = m_{perp} (x - M_x)
$$

Substituting the values:

$$
y - \frac{4 + y}{2} = -\frac{13}{y - 4} (x - 1.5)
$$

Note that M_y simplifies to:

$$
M_y = \frac{4 + y}{2}
$$

The left side simplifies to:

$$
\frac{2y - (4 + y)}{2} = \frac{2y - 4 - y}{2} = \frac{y - 4}{2}
$$

Thus, the equation becomes:

$$
\frac{y - 4}{2} = -\frac{13}{y - 4} (x - 1.5)
$$

Cross-multiplied, this yields:

$$
(y - 4)^2 = -26 (x - 1.5)
$$

This relation links y and x for points on the perpendicular bisector, with the dependence on y making the problem more nuanced.

Determining If S Could Lie On the Perpendicular Bisector

Given the general form derived, the critical question is: can a point S with coordinates (x_s, y_s) satisfy the equation for some y?

1. Conditions for S to Lie on the Perpendicular Bisector

A point S lies on the perpendicular bisector if its coordinates satisfy the line's equation. From the earlier derivation:

$$
(y_s - 4)^2 = -26 (x_s - 1.5)
$$

Since the left side is a square, it is non-negative. Therefore, the right side must also be non-negative:

$$
-26 (x_s - 1.5) \ge 0
$$

Dividing both sides by -26 (which reverses the inequality):

$$
x_s - 1.5 \le 0 \Rightarrow x_s \le 1.5
$$

This indicates that any point S with an x-coordinate less than or equal to 1.5 could potentially lie on the perpendicular bisector, depending on y.

2. Range of y for Possible S

Given the relation:

$$
(y_s - 4)^2 = -26 (x_s - 1.5)
$$

- For a specific $x_s \le 1.5$, the value of y_s can be found:

$$
y_s = 4 \pm \sqrt{-26 (x_s - 1.5)}
$$

- The square root is real only if $-26 (x_s - 1.5) \ge 0$, which aligns with earlier conclusions.

3. Implications

- For $x_s < 1.5$, the expression inside the square root is positive, and two possible y-values exist.
- For $x_s = 1.5$, the y-value simplifies to:

$$
(y_s - 4)^2 = 0 \Rightarrow y_s = 4
$$

- For $x_s > 1.5$, no real solutions exist, meaning no points with $x_s > 1.5$ can lie on the perpendicular bisector unless additional constraints are introduced.

Special Cases and Additional Considerations

1. When $y = 4$

If $y = 4$, then R has coordinates $(8, 4)$. The segment QR becomes horizontal, and the slope m_{QR} is zero. The perpendicular bisector in this case is a vertical line passing through the midpoint:

$$
M_x = 1.5
$$

Thus, the perpendicular bisector is:

$$
x = 1.5
$$

Any point S with $x = 1.5$ and any y value can lie on this line.

2. Impact of the y-coordinate of R

The value of y influences the slope of QR and the position of the perpendicular bisector. Specifically:

- If $y \neq 4$, the perpendicular bisector is a line with slope $-\frac{13}{y - 4}$.
- The general relation derived earlier applies, and the potential points S are constrained accordingly.

Practical Applications and Features

Understanding whether a point S can lie on the perpendicular bisector of QR has several practical applications in geometry, including:

- Constructing circles: The perpendicular bisector is the locus of centers of circles passing through Q and R.
- Finding equidistant points: Points on the perpendicular bisector are equidistant from Q and R.
- Coordinate geometry problems: Useful in proofs and problem-solving involving distances and midpoints.

Features:

- The relation derived provides a systematic way to determine possible positions of S.
- The dependency on y emphasizes the importance of the second coordinate in coordinate geometry.
- The approach combines algebraic calculations with geometric interpretations, illustrating the interconnectedness of the two perspectives.

Pros and Cons of the Analytical Approach

Pros:

- Precision: Provides exact conditions under which S can lie on the perpendicular bisector.
- Generality: Applicable to any value of y, offering a broad understanding.
- Insight into relationships: Reveals how the position of R affects the perpendicular bisector's location.

Cons:

- Complexity: Algebraic manipulations can become intricate, especially with variable y.
- Dependence on unknowns: The incomplete information about y limits definitive conclusions without additional data.

- Requires assumptions: To fully resolve the problem, assumptions or specific y-values are often necessary.
