Determine K So That Col (A) Is A Subspace Of R^k When 7 - 2 0A = [- 2 0 - 5].0 - 5 7 -5 7 - 21. K =

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Understanding how to determine the appropriate value of k such that the column space of a matrix A is a subspace of $\mathbb{R}^k$ is a fundamental concept in linear algebra. This task involves analyzing the structure of the matrix A, its columns, and the dimensions involved. In this article, we will explore the principles behind subspaces, the significance of the column space, and the step-by-step process to find k based on the given matrix.

Introduction to Subspaces and Column Spaces

What Is a Subspace?

A subspace is a subset of a vector space that is itself a vector space under the same addition and scalar multiplication. For a subset W of a vector space V to be a subspace, it must satisfy three conditions:
- The zero vector of V is in W.
- W is closed under vector addition.
- W is closed under scalar multiplication.

Understanding the Column Space

The column space of a matrix A, denoted as $\operatorname{Col}(A)$, is the set of all linear combinations of the columns of A. It is a subspace of $\mathbb{R}^n$ if A is an $m \times n$ matrix. The dimension of this subspace is the rank of A, which indicates the maximum number of linearly independent columns.

Analysis of the Given Matrix and Its Transformation

Deciphering the Provided Data

The problem states:
Determine K So That Col (A) Is A Subspace Of R^k When 7 - 2 0A = [- 2 0 - 5].0 - 5 7 -5 7 - 21. K =

This appears to be a representation of a matrix A and some associated data. The notation suggests that the matrix A is related to the given block or augmented matrix. To interpret this correctly, let's analyze the matrix components:

- The matrix A is represented as a 3x3 matrix:

$$
A = \begin{bmatrix}
7 & -2 & 0 \\
-5 & 7 & -5 \\
7 & -21 & \text{(missing/unspecified)} \\
\end{bmatrix}
$$

Since the third row appears incomplete or possibly misformatted, we should clarify the matrix's structure. Based on the pattern:

$$
A = \begin{bmatrix}
7 & -2 & 0 \\
-5 & 7 & -5 \\
7 & -21 & -5 \\
\end{bmatrix}
$$

Alternatively, the notation might suggest that the initial data is a matrix involved in the problem, with the first line indicating some relation involving A.

Step-by-Step Approach to Find K

1. Understanding the Column Space of A

The primary goal is to determine the smallest possible k such that $\operatorname{Col}(A) \subseteq \mathbb{R}^k$. Since the columns of A are vectors in $\mathbb{R}^m$ (with m being the number of rows), the column space naturally resides in $\mathbb{R}^m$.

Thus, the dimension of the column space (the rank of A) is the key, as it indicates the minimal dimension of the subspace containing $\operatorname{Col}(A)$.

2. Computing the Rank of \(A\)

To find \(k\), we need to determine the rank of the matrix \(A\). This involves:
- Performing Gaussian elimination to reduce \(A\) to its row echelon form.
- Counting the number of non-zero rows (pivot rows).

Let's perform the row operations step-by-step:

\[
A = \begin{bmatrix}
7 & -2 & 0 \\
-5 & 7 & -5 \\
7 & -21 & -5 \\
\end{bmatrix}
\]

Step 1: Make the first pivot 1 by dividing the first row by 7 (optional for simplicity):

\[
R_1 \leftarrow \frac{1}{7} R_1
\]
\[
\Rightarrow R_1 = \begin{bmatrix}1 & -\frac{2}{7} & 0 \end{bmatrix}
\]

Step 2: Eliminate below the pivot:
- \(R_2 \leftarrow R_2 + 5 \times R_1 \)
- \(R_3 \leftarrow R_3 - 7 \times R_1 \)

Calculations:
- \(R_2 = \begin{bmatrix}-5 + 5 \times 1, 7 + 5 \times -\frac{2}{7}, -5 + 5 \times 0 \end{bmatrix} = \begin{bmatrix}0, 7 - \frac{10}{7}, -5 \end{bmatrix} \)

Simplify:
- \(7 - \frac{10}{7} = \frac{49}{7} - \frac{10}{7} = \frac{39}{7} \)

So:
\[
R_2 = \begin{bmatrix} 0 & \frac{39}{7} & -5 \end{bmatrix}
\]

Similarly for \(R_3 \):
- \(R_3 = \begin{bmatrix}7 - 7 \times 1, -21 - 7 \times -\frac{2}{7}, -5 - 7 \times 0 \end{bmatrix} = \begin{bmatrix}0, -21 + 2, -5 \end{bmatrix} \)

Simplify:
- \(-21 + 2 = -19 \)

Thus:
\[
R_3 = \begin{bmatrix} 0 & -19 & -5 \end{bmatrix}

\]

Step 3: Now, the matrix looks like:
\[
\begin{bmatrix}
1 & -\frac{2}{7} & 0 \\
0 & \frac{39}{7} & -5 \\
0 & -19 & -5 \\
\end{bmatrix}
\]

Next, eliminate below \(R_2 \):
- Convert \(R_2 \) to have leading 1 by dividing by \(\frac{39}{7} \):

\[
R_2 \leftarrow \frac{7}{39} R_2
\]
\[
R_2 = \begin{bmatrix} 0, 1, -\frac{35}{39} \end{bmatrix}
\]

Now, eliminate the entry in \(R_3 \) in the second column:
- \(R_3 \leftarrow R_3 + 19 \times R_2 \):

Calculations:
\[
R_3 = \begin{bmatrix} 0, -19 + 19 \times 1, -5 + 19 \times -\frac{35}{39} \end{bmatrix}
\]

Simplify:
- Second element: \(-19 + 19 = 0 \)
- Third element:

\[
-5 + 19 \times -\frac{35}{39} = -5 - \frac{19 \times 35}{39} = -5 - \frac{665}{39}
\]

Express \(-5\) as \(-\frac{195}{39}\):

\[
-\frac{195}{39} - \frac{665}{39} = -\frac{860}{39}
\]

Thus, the third row becomes:

\[
R_3 = \begin{bmatrix} 0, 0, -\frac{860}{39} \end{bmatrix}
\]

Since \(R_3 \) has a non-zero entry, the rank of \(A\) is 3.

Determining the Value of k

3. Connecting Rank to the Subspace Dimension

The rank of A indicates the dimension of its column space. Because $\operatorname{Col}(A)$ is a subspace of $\mathbb{R}^m$, where m is the number of rows, the minimal k such that $\operatorname{Col}(A) \subseteq \mathbb{R}^k$ must satisfy:

$$
k \geq \operatorname{dim}(\operatorname{Col}(A)) = \text{rank}(A)
$$

Given that the rank of A is 3, the smallest such k is 3.

Frequently Asked Questions

What is the main goal when determining the value of K in the context of the column space of matrix A?

The main goal is to find the value of K such that the column space of A is a subspace of R^K, meaning all columns of A are vectors in R^K and the space they span is contained within R^K.

How do you interpret the matrix A given as [-2 0 -5; 0 -5 7; -5 7 -21] in relation to determining K?

The matrix A's columns are vectors in R^3, so initially, the column space of A is a subspace of R^3. To find K for which Col(A) is a subspace of R^K, we analyze the rank and dimension of Col(A).

What is the significance of the rank of matrix A in this problem?

The rank of matrix A indicates the dimension of its column space. To determine K, K must be at least equal to the rank, so that Col(A) can be a subspace of R^K.

How do you compute the rank of matrix A given as [-2 0 -5; 0 -5 7; -5 7 -21]?

You perform row operations to reduce A to its row echelon form and count the number of non-zero rows to find its rank. In this case, the rank is 2 because the rows are linearly dependent, but two rows are linearly independent.

Based on the matrix A, what is the rank and how does it influence the value of K?

The rank of matrix A is 2, indicating the column space is 2-dimensional. Therefore, K must be at least 2 for Col(A) to be a subspace of R^K.

Is the subspace Col(A) always contained within R^K for K equal to the rank of A?

Yes, the column space of A is a subspace of R^K when K is equal to or greater than the rank of A. For minimal K, K equals the rank of A.

What is the final value of K that ensures Col(A) is a subspace of R^K?

Since the rank of A is 2, the smallest K such that Col(A) is a subspace of R^K is K = 2.

Can Col(A) be a subspace of R^K for K less than the rank of A?

No, because the dimension of the column space cannot be contained within a space of smaller dimension, so K must be at least equal to the rank of A.

Additional Resources

Determine K So That Col(A) Is A Subspace Of R^k When 7 - 2 0A = [- 2 0 - 5].0 - 5 7 -5 7 - 21. K =

Understanding the problem of determining the appropriate dimension k such that the column space of a matrix A is a subspace of $\mathbb{R}^k$ is fundamental in linear algebra. This process involves analyzing the structure of the matrix A, its columns, and the underlying properties of vector spaces. The given matrix and the associated expressions suggest an exploration of the rank, column space, and the dimensionality of the subspace that contains $\operatorname{Col}(A)$. In this review, we will delve into the key concepts involved in this problem, providing a detailed step-by-step approach, theoretical insights, and practical considerations to determine the suitable value of k.

Understanding the Context and the Given Data

Before we proceed to the core calculations, it’s essential to interpret the problem statement and the provided data accurately.

Deciphering the Mathematical Expression

The expression:

> 7 - 2 0A = [- 2 0 - 5].0 - 5 7 -5 7 - 21. K =

appears somewhat ambiguous at first glance, but breaking it down clarifies the intent:

- Matrix \(A\): It seems the expression involves a matrix \(A\) and some linear combinations or transformations involving it.

- The matrix \([- 2 0 - 5]\): Likely a vector or a matrix block.

- The sequence "7 - 2 0A" and subsequent entries: Possibly indicating operations involving \(A\), or perhaps a typo or formatting issue.

Given the context, and typical linear algebra problems, it’s reasonable to interpret the core of the problem as follows:

> Find the smallest integer \(k\) such that the column space of matrix \(A\) is a subspace of \(\mathbb{R}^k\).

To proceed, we need to reconstruct the matrix \(A\) from the given data.

Reconstructing the Matrix \(A\)

The key to solving the problem lies in understanding the structure of matrix \(A\). Based on the data:

> 7 - 2 0A = [- 2 0 - 5].0 - 5 7 -5 7 - 21.

This suggests that \(A\) is a matrix with specific entries, possibly:

\[
A = \begin{bmatrix}
7 & -2 & 0 \\
-5 & 7 & -5 \\
7 & -21 & \dots
\end{bmatrix}
\]

Alternatively, it could be that the matrix is constructed from these entries, or that the expression indicates linear relations involving \(A\).

Given the ambiguity, the most logical assumption is that \(A\) is a matrix with columns or rows involving the numbers provided, and the goal is to find \(k\) such that \(\operatorname{Col}(A) \subseteq \mathbb{R}^k\).

Key Concepts in Determining \(k\)

To find \(k\), it is crucial to understand the relationship between the matrix \(A\) and its column space.

Column Space (\(\operatorname{Col}(A)\))

The column space of a matrix \(A\) is the subspace of \(\mathbb{R}^n\) spanned by its columns. Its dimension, known as the rank of \(A\), indicates how many linearly independent columns \(A\) has.

Features:

- The dimension of \(\operatorname{Col}(A)\) is equal to the rank of \(A\).

- \(\operatorname{Col}(A)\) is always a subspace of \(\mathbb{R}^n\), where \(n\) is the number of rows in \(A\).

- To find the smallest \(k\) such that \(\operatorname{Col}(A) \subseteq \mathbb{R}^k\), we need to determine the dimension of \(\operatorname{Col}(A)\), i.e., its rank.

The Concept of Subspaces in \(\mathbb{R}^k\)

Any subspace of \(\mathbb{R}^k\) can only have a maximum dimension of \(k\). Conversely, any subspace of \(\mathbb{R}^k\) must have a dimension less than or equal to \(k\).

Implication:

- If \(\operatorname{Col}(A)\) is a subspace of \(\mathbb{R}^k\), then \(\operatorname{dim}(\operatorname{Col}(A)) \leq k\).

- To find the minimal such \(k\), we set \(k = \operatorname{dim}(\operatorname{Col}(A))\).

Step-by-Step Approach to Determine \(k\)

Given the above understanding, the main steps are:

1. Identify the matrix \(A\).
2. Compute the rank of \(A\).
3. Set \(k\) equal to the rank of \(A\).

Let’s elaborate on each step.

Step 1: Constructing the Matrix \(A\)

Based on the information, suppose the matrix \(A\) has columns represented by the vectors:

\[
A = \begin{bmatrix}
7 & -2 & 0 \\
-5 & 7 & -5 \\
7 & -21 & \dots
\end{bmatrix}
\]

However, the third entry seems incomplete. To proceed, assume that the core data involves two columns:

\[
A = \begin{bmatrix}
7 & -2 \\
-5 & 7 \\
7 & -21
\end{bmatrix}
\]

This is a plausible reconstruction based on the pattern of entries, and such a matrix is typical in linear algebra exercises.

Step 2: Computing the Rank of \(A\)

The rank of \(A\) can be determined via row operations or by computing the determinant of the largest square submatrix.

Method 1: Row reduction

Starting with:

$$
A = \begin{bmatrix}
7 & -2 \\
-5 & 7 \\
7 & -21
\end{bmatrix}
$$

Perform row operations to check for linear independence:

- Use R_1 as a pivot.

- Eliminate entries below and above the pivot to find the number of pivot positions.

Alternatively, form the matrix's augmented form and perform Gaussian elimination.

Method 2: Determinant method

Since A is 3×2, the rank is at most 2. To determine whether the columns are linearly independent, check if one column is a scalar multiple of the other, or compute the rank directly.

Calculate the determinant of the 2×2 minors:

- Minor involving first two rows:

$$
\det \begin{bmatrix}
7 & -2 \\
-5 & 7
\end{bmatrix} = (7)(7) - (-2)(-5) = 49 - 10 = 39 \neq 0
$$

- Minor involving first and third rows:

$$
\det \begin{bmatrix}
7 & -2 \\
7 & -21
\end{bmatrix} = (7)(-21) - (-2)(7) = -147 + 14 = -133 \neq 0
$$

- Minor involving second and third rows:

$$
\det \begin{bmatrix}
-5 & 7 \\

7 & -21
\end{bmatrix} = (-5)(-21) - (7)(7) = 105 - 49 = 56 \neq 0
\]

Since at least one \(2 \times 2\) minor is non-zero, the columns are linearly independent, and \(\operatorname{rank}(A) = 2\).

Step 3: Determining \(k\)

Given that the rank of \(A\) is 2, the column space \(\operatorname{Col}(A)\) is a subspace of \(\mathbb{R}^n\) (with \(n=3\) in this case), of dimension 2.

Therefore:

\[
\boxed{
k = \operatorname{rank}(A) = 2
}
\]

The subspace \(\operatorname{Col}(A)\) can be embedded in \(\mathbb{R}^2\). Hence, the minimal \(k\) such that \(\operatorname{Col}(A) \subseteq \mathbb{R}^k\) is 2.

Implications and Practical Considerations

Understanding how to determine \(k\) has several practical implications in various fields like data science, engineering, and computer graphics, where dimensionality reduction and sub

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