

Determine The Number Of Permutations Of All The Letters Of The Word HOLLYWOOD. Show All Your Work. B.

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Understanding how to find the total number of permutations of a word, especially one with repeated letters, is a fundamental concept in combinatorics. The word HOLLYWOOD provides an excellent example because it contains both unique and repeated letters. In this article, we will systematically analyze the problem, identify the repetitions, and apply permutation formulas to determine the total number of arrangements of all the letters in HOLLYWOOD. Let's explore each step in detail to ensure clarity and thoroughness.

Understanding Permutations and Repetitions

Before diving into calculations, it's important to understand key concepts:

What Are Permutations?

Permutations refer to the different arrangements or orderings of a set of objects. When arranging items, the order matters. For example, arranging the letters A, B, and C can result in six permutations:

- ABC
- ACB
- BAC
- BCA
- CAB
- CBA

Permutations with Repeated Items

When some objects are identical, the total number of permutations decreases because swapping identical objects doesn't produce a new arrangement. The general formula for permutations of a multiset (a set with repeated elements) is:

$$\text{Number of permutations} = \frac{n!}{n_1! \times n_2! \times \dots \times n_k!}$$

Where:

- n is the total number of objects.
- $n_1, n_2, \dots, n_k$ are the counts of each repeated object.

Applying this to a word with repeated letters involves:

1. Counting total letters.
2. Identifying the letters that repeat and their counts.
3. Using the permutation formula to compute the total arrangements.

Analyzing the Word HOLLYWOOD

Let's start by examining the specific letters in HOLLYWOOD:

- H
- O
- L
- L
- Y
- W
- O
- O
- D

Step 1: Count the Total Number of Letters

The total number of letters in the word HOLLYWOOD is 9.

Step 2: Identify Repetitions

Now, determine which letters are repeated and how many times:

Letter	Count
H	1
O	3
L	2
Y	1
W	1
D	1

Note:

- The letter O appears 3 times.
- The letter L appears 2 times.
- All other letters appear once.

Step 3: Applying the Permutation Formula

Using the formula for permutations of multiset:

$$\text{Total permutations} = \frac{9!}{3! \times 2! \times 1! \times 1! \times 1! \times 1!}$$

Since factorial of 1 is 1, and dividing by 1 doesn't affect the result, we can simplify:

$$\text{Total permutations} = \frac{9!}{3! \times 2!}$$

Calculating the Permutations

Now, perform the calculations step-by-step:

Step 1: Calculate 9!

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$$

Step 2: Calculate 3! and 2!

$$3! = 3 \times 2 \times 1 = 6$$

$$2! = 2 \times 1 = 2$$

Step 3: Divide 9! by the product of 3! and 2!

$$\frac{362880}{6 \times 2} = \frac{362880}{12} = 30240$$

Final answer:

$$\boxed{30240}$$

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\text{Total permutations of all letters in HOLLYWOOD} = 30,240  
}  
\]
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Interpreting the Result

The total number of arrangements of the letters in HOLLYWOOD, considering repetitions, is 30,240. This means that if all the letters are rearranged randomly, there are 30,240 distinct ways to arrange the entire set. This number accounts for the identical letters that do not produce new arrangements when swapped.

Additional Considerations and Variations

While the above calculation addresses permutations of all the letters, several related questions often arise in combinatorics:

1. Permutations of a Subset of Letters

What if you want arrangements of only some of the letters?

2. Permutations with Restrictions

What if certain letters must or must not be adjacent?

3. Permutations of Unique Subsets

What if the word contains no repeated letters?

Each of these variations involves different formulas and approaches, but the core principles remain rooted in understanding permutations and handling repeated elements appropriately.

Summary and Key Takeaways

- The total number of permutations of a word with repeated letters can be calculated using the factorial of the total letter count divided by the factorials of each repeated letter count.
- For HOLLYWOOD, the total arrangements are 30,240.
- Recognizing repeated letters is crucial to avoid overcounting arrangements.
- The permutation formula for multiset arrangements is:

$$\frac{n!}{n_1! \times n_2! \times \dots}$$

- This method can be adapted for various similar problems involving arrangements with repetitions.

Final Thoughts

Understanding how to compute permutations with repeated elements is essential in combinatorics and has practical applications in fields such as cryptography, probability, and statistical analysis. The example of HOLLYWOOD illustrates the importance of meticulous counting and applying the correct formulas to arrive at accurate results. With this knowledge, you can confidently approach similar permutation problems involving complex arrangements and repetitions.

References:

- Rosen, K. H. (2012). Discrete Mathematics and Its Applications. McGraw-Hill Education.
- Graham, R. L., Knuth, D. E., & Patashnik, O. (1994). Concrete Mathematics. Addison-Wesley.
- Online combinatorics resources and permutation calculators.

Note: Always verify counts of repeated elements carefully, and remember that factorial calculations grow rapidly, so using a calculator or computational tool can be helpful for larger numbers.

Frequently Asked Questions

How many total permutations are there of the letters in the word HOLLYWOOD?

The word HOLLYWOOD has 9 letters with some repetitions: H, O, L, L, Y, W, O, O, D. Counts: H=1, O=3, L=2, Y=1, W=1, D=1. Total permutations = $9! / (3! 2!)$ = $362880 / (6 \cdot 2) = 362880 / 12 = 30240$.

Which letters are repeated in the word HOLLYWOOD,

and how many times?

The letter O appears 3 times, and the letter L appears 2 times. All other letters H, Y, W, D appear once each.

How do you account for repeated letters when calculating permutations?

When letters are repeated, we divide the factorial of the total number of letters by the factorial of each repeated letter count to avoid overcounting. For example, total permutations = $9! / (3! 2!)$.

What is the mathematical formula used to find the number of permutations of the word HOLLYWOOD?

Number of permutations = $9! / (3! 2!)$ where $9!$ accounts for total letters, and the denominators account for repeated letters O (3 times) and L (2 times).

Calculate the value of $9! / (3! 2!)$ for the word HOLLYWOOD.

$9! = 362880$; $3! = 6$; $2! = 2$; So, permutations = $362880 / (6 \cdot 2) = 362880 / 12 = 30240$.

Are permutations of the word HOLLYWOOD distinguishable if the repeated letters are considered?

Yes, permutations are distinguishable because each arrangement differs in the order of the letters, but identical letters are only counted once per unique arrangement.

What is the significance of dividing by factorials of repeated letters in permutation calculations?

Dividing by factorials of repeated letters corrects for overcounting arrangements that are identical due to repeated letters, ensuring each unique permutation is counted once.

Can the permutation count change if the positions of repeated letters are fixed?

Yes, fixing positions of repeated letters reduces the total number of permutations, since only arrangements that keep certain letters in specific positions are considered.

How would the permutation count change if the word had all unique letters?

If all letters were unique, the total permutations would simply be $9! = 362880$, since no division for repeated letters would be necessary.

Summarize the steps to determine the number of permutations of HOLLYWOOD.

First, identify the total number of letters and count repetitions. Then, calculate factorial of total letters ($9!$). Divide this by the factorials of the counts of each repeated letter ($3!$ for O, $2!$ for L). The result gives the total number of distinct permutations: $9! / (3! 2!) = 30240$.

Additional Resources

Determine The Number Of Permutations Of All The Letters Of The Word HOLLYWOOD

Permutation problems involving words with repeated letters often seem daunting at first glance, but with methodical analysis, they become manageable. The word HOLLYWOOD offers an interesting case for combinatorial exploration because it contains both unique and repeated characters. In this comprehensive review, we will dissect the problem step-by-step, ensuring clarity in our approach and calculations, ultimately leading to an accurate count of all possible arrangements of its letters.

Understanding the Problem: Permutations with Repeated Letters

Permutations refer to the different arrangements of a set of elements where the order matters. When all elements are distinct, the total permutations are straightforward—simply the factorial of the number of elements. However, complications arise when some elements are repeated because swapping identical elements doesn't produce a new, unique permutation.

In the case of HOLLYWOOD, the key is to identify which letters are repeated and how many times, then apply the permutation formula that accounts for these repetitions.

Analyzing the Word HOLLYWOOD

Breakdown of Letter Frequencies

Let's first examine the individual letters in HOLLYWOOD:

- H: appears 1 time
- O: appears 2 times
- L: appears 2 times
- Y: appears 1 time
- W: appears 1 time
- D: appears 1 time

Adding these up:

- Total letters: 9
- Repetitions: O (2), L (2)

This analysis is crucial because the total number of permutations considering repeated elements is calculated via the formula:

$$\text{Number of permutations} = \frac{n!}{n_1! \times n_2! \times \dots}$$

where:

- n is the total number of letters,
- $n_1, n_2, \dots$ are the counts of each repeated letter.

Calculating Total Permutations

Step 1: Total arrangements if all letters were distinct

If all 9 letters were unique, the total permutations would be:

$$9! = 362,880$$

Step 2: Adjust for repeated letters

Since O and L are repeated twice, we need to divide by the factorial of their counts to avoid overcounting identical arrangements:

$$\text{Total permutations} = \frac{9!}{2! \times 2!}$$

\]

Calculating:

\[
\frac{362,880}{(2 \times 1) \times (2 \times 1)} = \frac{362,880}{4} = 90,720
\]

Final Result:

\[
\boxed{90,720}
\]

There are 90,720 unique arrangements of all the letters in HOLLYWOOD considering the repetitions.

Deeper Analysis: Permutations with Constraints and Variations

While the primary question revolves around permutations of all the letters, it's worth exploring related variations and constraints to deepen understanding.

Permutations of Subsets of Letters

Suppose we are interested in permutations of:

- Only some of the letters (e.g., 5-letter arrangements),
- Or arrangements where certain letters are fixed or in specific positions.

These variations often require combinatorial formulas such as permutations with restrictions or combinations, but they go beyond the scope of the original problem.

Permutations with Additional Constraints

In more complex scenarios, constraints might include:

- No two identical letters are adjacent,
- Certain positions are fixed,
- Specific letters must appear in particular positions.

Addressing these requires advanced combinatorial methods like Inclusion-Exclusion, recursive counting, or generating functions, which are more sophisticated than the straightforward calculation above.

Implications and Insights From the Calculation

Significance of the Result

The total of 90,720 permutations highlights the rich combinatorial landscape even within a relatively short word like HOLLYWOOD. This figure underscores how duplicate letters significantly reduce the total permutations compared to all unique arrangements (which would have been $9!$ or 362,880).

Practical Applications

Understanding such permutation calculations is vital in fields like:

- Cryptography: where permutations of characters form the basis of encryption algorithms,
- Data analysis: where permutations help in understanding possible arrangements,
- Puzzle design: in creating or solving word-based puzzles,
- Linguistics: analyzing possible word formations.

Summary and Conclusion

The problem of determining the number of permutations of the letters in HOLLYWOOD is a classic example of applying combinatorial principles to words with repeated elements. By carefully analyzing the frequency of each letter and applying the permutation formula that accounts for repetitions, we arrive at the total:

$$\boxed{90,720}$$

This comprehensive breakdown not only answers the specific question but also provides a framework for tackling similar permutation problems involving repeated elements. The methodical approach—identifying repetitions, calculating factorials, and applying the division—serves as a foundational technique in combinatorics and discrete mathematics. Understanding these calculations is essential for students, educators, and professionals working in mathematical, computational, or linguistic fields where permutations play a critical role.

In summary, the permutation of all letters in HOLLYWOOD, considering the repetitions of O and L, yields 90,720 unique arrangements. This exercise demonstrates the power of factorial calculations combined with division for repeated elements, illustrating fundamental combinatorial principles with practical relevance across various disciplines.

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