Determine The Ratio Of The Number Of Molecules In A Gas Having A Speed Ten Times As Great As The Root

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Understanding the behavior of gases at the molecular level is fundamental in thermodynamics and kinetic theory. One intriguing aspect involves analyzing how the distribution of molecular speeds affects the number of molecules moving at specific velocities. This article delves into the problem of determining the ratio of molecules in a gas that have speeds ten times as great as the most probable speed, often referred to as the root mean square speed or the root of the average squared speed. By exploring the Maxwell-Boltzmann distribution, we will develop a comprehensive approach to calculating this ratio, providing insight into molecular dynamics and statistical mechanics.

Introduction to Molecular Speed Distribution in Gases

Maxwell-Boltzmann Distribution Overview

The Maxwell-Boltzmann distribution describes the spread of molecular speeds in a gas at thermal equilibrium. It provides the probability density that a molecule in the gas has a particular speed, taking into account the thermal energy, molecular mass, and temperature.

Key points include:
- The distribution depends on temperature (T), molecular mass (m), and Boltzmann's constant (k).
- The probability density function (pdf) for molecular speeds is given by:

$$
f(v) = \left(\frac{m}{2 \pi k T}\right)^{3/2} 4 \pi v^2 \exp\left(-\frac{m v^2}{2 k T}\right)
$$

- The distribution peaks at the most probable speed v_{mp}, which is:

$$
v_{mp} = \sqrt{\frac{2 k T}{m}}
$$

- The average (mean) speed $\bar{v}$ and root mean square (rms) speed v_{rms} are given by:

$$
\bar{v} = \sqrt{\frac{8 k T}{\pi m}}, \quad v_{rms} = \sqrt{\frac{3 k T}{m}}
$$

The focus of this analysis is on the number of molecules moving at speeds related to these characteristic speeds, especially at a speed ten times a particular reference speed.

Defining the Problem: Speed Ten Times the Root

Clarification of the "Root" in Context

The phrase "speed ten times as great as the root" is somewhat ambiguous. It could refer to:

- The root mean square speed v_{rms},
- The most probable speed v_{mp},
- The average speed $\bar{v}$.

In most kinetic theory analyses, the term "the root" often pertains to the root mean square speed, which is a common measure of molecular speed energy.

Assuming this, the problem becomes:

> Determine the ratio of the number of molecules in a gas with speeds equal to $10 v_{rms}$ to those with speeds equal to v_{rms}.

Alternatively, if the problem refers to the most probable speed, the approach remains similar, substituting v_{mp} accordingly.

Mathematical Approach to the Problem

Using the Maxwell-Boltzmann Speed Distribution

The number of molecules with speeds in the range v to $v + dv$ is proportional to the probability density $f(v)$:

$$
dN = N \times f(v) \, dv
$$

where N is the total number of molecules.

To find the ratio of molecules with speeds v_1 and v_2, we analyze:

$$
\frac{N(v_2)}{N(v_1)} = \frac{f(v_2)}{f(v_1)}
$$

since the total number N cancels out.

Deriving the Ratio for $v = 10 v_{rms}$ and $v = v_{rms}$

Given the probability density function:

$$
f(v) = A \times v^2 \exp\left(-\frac{m v^2}{2k T}\right)
$$

where $A = \left(\frac{m}{2\pi k T}\right)^{3/2} 4 \pi$ is a normalization constant, which cancels in the ratio.

The ratio becomes:

$$
\frac{f(10 v_{rms})}{f(v_{rms})} = \frac{(10 v_{rms})^2 \exp\left(-\frac{m (10 v_{rms})^2}{2k T}\right)}{v_{rms}^2 \exp\left(-\frac{m v_{rms}^2}{2k T}\right)}
$$

Simplify numerator and denominator:

$$
= 100 v_{rms}^2 \times \exp\left(-\frac{m 100 v_{rms}^2}{2k T}\right) \div v_{rms}^2 \times \exp\left(-\frac{m v_{rms}^2}{2k T}\right)
$$

which simplifies to:

$$
= 100 \times \exp\left(-\frac{m v_{rms}^2}{2k T} \times (100 - 1)\right)
$$

Recall that:

$$
v_{rms} = \sqrt{\frac{3k T}{m}}
$$

Thus,

$$
\frac{m v_{rms}^2}{2k T} = \frac{m \times \frac{3k T}{m}}{2k T} = \frac{3}{2}
$$

Substituting back:

$$
\frac{f(10 v_{rms})}{f(v_{rms})} = 100 \times \exp\left(-\frac{3}{2} \times (100 - 1)\right) = 100 \times \exp\left(-\frac{3}{2} \times 99\right)
$$

Calculate the exponential:

$$
-\frac{3}{2} \times 99 = -\frac{3 \times 99}{2} = -\frac{297}{2} = -148.5
$$

So,

$$
\frac{f(10 v_{rms})}{f(v_{rms})} = 100 \times e^{-148.5}
$$

Given that $e^{-148.5}$ is an extremely small number (~10^{-64}), the ratio is effectively zero.

Conclusion: The number of molecules moving at ten times the root mean square speed is negligible compared to those moving at v_{rms}.

Physical Interpretation and Implications

Significance of the Result

The exponential decay in the Maxwell-Boltzmann distribution indicates that molecules moving at very high speeds are exceedingly rare. Specifically, molecules with speeds ten times the rms speed are virtually nonexistent in the gas sample at typical conditions.

This result emphasizes:
- The tail of the speed distribution is sparse, with probabilities decreasing exponentially.
- Most molecules have speeds clustered around the most probable or average speeds.
- The high-velocity tail, though essential for understanding phenomena like diffusion and reaction rates, contributes minimally to the total number of molecules.

Broader Context in Thermodynamics and Kinetic Theory

Understanding these ratios is crucial in:
- Calculating reaction rates, where only molecules exceeding certain energy thresholds react.
- Designing engines and turbines, where high-speed particles can impact efficiency.
- Explaining phenomena such as evaporation and escape velocity in planetary atmospheres.

Extensions and Related Calculations

Comparing Other Velocities

The same methodology applies to find ratios for other multiples of the characteristic speeds:
- At $2 v_{mp}$,
- At $3 v_{rms}$,
- Or any arbitrary multiple $n v_{rms}$.

The general expression for the ratio of molecules with speeds $v = n v_{rms}$ to those at v_{rms}:

$$
\frac{f(n v_{rms})}{f(v_{rms})} = n^2 \times \exp\left(-\frac{m v_{rms}^2}{2k T} (n^2 - 1)\right)
$$

which simplifies using the earlier substitution.

Implications for Molecular Kinetics

- The probability of molecules having extremely high speeds decreases rapidly.
- These insights guide experimental designs and theoretical models in molecular physics.

Conclusion

Determining the ratio of molecules in a gas moving at speeds ten times the root mean square speed reveals that such molecules are virtually nonexistent under typical conditions due to the exponential decay of the Maxwell-Boltzmann distribution. This profound result underscores the concentration of molecular speeds around the most probable and average speeds, with high-velocity tails contributing negligibly to the overall molecular population.

Understanding this distribution is foundational in thermodynamics, chemical kinetics, and statistical mechanics, providing critical insights into molecular behavior, reaction dynamics, and the macroscopic properties of gases.

Summary

Frequently Asked Questions

What is the significance of comparing molecular speeds in gases?

Comparing molecular speeds helps understand the kinetic energy distribution, gas diffusion rates, and reaction dynamics based on molecular motion.

How is the ratio of molecules with different speeds in a gas determined?

The ratio is determined using the Maxwell-Boltzmann distribution, which relates molecular speeds to their probability and concentration in the gas.

What does it mean when a molecule has a speed ten times as great as the root mean square speed?

It indicates that the molecule's speed is significantly higher than average, specifically ten times the root mean square speed, which represents the square root of the average of squared speeds.

How can the Maxwell-Boltzmann distribution be used to find the ratio of molecules at different speeds?

By plugging the speeds into the distribution formula, we can calculate the relative number of molecules moving at those speeds, allowing us to find the ratio.

What is the mathematical expression for the ratio of molecules with speeds v and v0 in a gas?

The ratio is given by: (n_v / n_{v0}) = (v / v_0)^2 exp[-(v^2 - v_0^2) / (2RT / M)], where R is the gas constant, T is temperature, and M is molar mass.

If a molecule's speed is ten times the root mean

square speed, what is the approximate ratio of such molecules to the total?

The number of molecules with speed ten times the root mean square speed is extremely small compared to the total, because high-speed molecules are rare in the distribution.

Why are molecules with very high speeds, such as ten times the root mean square speed, so infrequent?

Because the Maxwell-Boltzmann distribution predicts an exponential decrease in the number of molecules as speed increases, making high speeds very rare.

Can we directly measure the ratio of molecules at different speeds in a gas?

Direct measurement is challenging, but techniques like molecular beam experiments and spectroscopic methods can estimate the distribution of molecular speeds.

How does temperature affect the ratio of molecules moving at high speeds?

Higher temperatures increase the average molecular speed, slightly increasing the number of molecules with very high speeds, but the ratio of extremely fast molecules remains small.

Additional Resources

Determine The Ratio Of The Number Of Molecules In A Gas Having A Speed Ten Times As Great As The Root

Understanding the distribution of molecular speeds in a gas is a fundamental aspect of kinetic theory and statistical mechanics. When we talk about the ratio of molecules moving at different speeds, especially at speeds differing by a specific factor, we delve into the core principles governing molecular motion. In particular, determining the ratio of molecules moving at a speed ten times greater than the root mean square (rms) speed offers profound insights into the energetic and dynamical behavior of gases. This analysis not only enhances our comprehension of molecular kinetics but also lays the groundwork for practical applications such as diffusion, effusion, and reaction rates.

Introduction to Molecular Speed Distribution

The behavior of molecules in a gas is inherently random, and their speeds follow a specific distribution known as the Maxwell-Boltzmann distribution. This distribution describes the probability of finding molecules with particular speeds at thermal equilibrium. The Maxwell-Boltzmann distribution is critical because it reveals that while most molecules have speeds around a certain average, there's a significant tail of molecules moving much faster or slower.

Key concepts include:

- Root Mean Square Speed (v_{rms}): The square root of the average of the squares of the molecular speeds. It provides a measure of the average kinetic energy of molecules.
- Most Probable Speed (v_{mp}): The speed at which the Maxwell-Boltzmann distribution peaks.
- Average Speed (v_{avg}): The mean of all molecular speeds.

Understanding these speeds is essential for analyzing molecular behavior, especially when considering molecules with exceptionally high velocities.

Maxwell-Boltzmann Speed Distribution Equation

The probability density function for molecular speeds in a gas is given by the Maxwell-Boltzmann distribution:

$$ f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{m v^2}{2 k_B T}\right) $$

where:

- v = molecular speed
- m = mass of a molecule
- k_B = Boltzmann constant
- T = absolute temperature

This function describes how the probability varies with speed. To find the ratio of molecules moving at two different speeds, especially when these speeds differ by a specific factor, we analyze the ratio of the probability density functions at those two speeds.

Determining the Speed Ratio: Theoretical Framework

The problem involves calculating the ratio of the number of molecules moving at a speed $v = 10 \times v_{rms}$ to the total number of molecules in the gas.

Step-by-step approach:

1. Identify the speeds involved:
- The reference speed: $v_{ref} = v_{rms}$
- The target speed: $v_{target} = 10 \times v_{rms}$

2. Express the probability density at these speeds:
Using Maxwell-Boltzmann distribution, compute $f(v_{ref})$ and $f(v_{target})$.

3. Calculate the ratio of molecules at these speeds:
Since the number of molecules with a specific speed is proportional to the probability density, the ratio is:

$$R = \frac{f(v_{target})}{f(v_{ref})}$$

4. Simplify the ratio:
By substituting the expressions for $f(v)$, the ratio becomes:

$$R = \left(\frac{v_{target}}{v_{ref}}\right)^2 \exp\left[-\frac{m}{2k_B T}(v_{target}^2 - v_{ref}^2)\right]$$

Given that v_{ref} corresponds to v_{rms}, and knowing that:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

we can express the ratio purely in terms of known quantities.

Expressing Speeds in Terms of RMS Speed

Since $v_{rms} = \sqrt{\frac{3k_B T}{m}}$, then:

$$v_{target} = 10 v_{rms} = 10 \sqrt{\frac{3k_B T}{m}}$$

The difference in the exponent becomes:

$$
v_{target}^2 - v_{ref}^2 = (10 v_{rms})^2 - v_{rms}^2 = 100 v_{rms}^2 -
$$

v_{rms}^2 = 99 v_{rms}^2
\]

Substituting back into the ratio:

\[
R = 10^2 \times \exp\left(-\frac{m}{2k_B T} \times 99 v_{rms}^2 \right)
\]

But since:

\[
v_{rms}^2 = \frac{3k_B T}{m}
\]

we get:

\[
R = 100 \times \exp\left(-\frac{m}{2k_B T} \times 99 \times \frac{3k_B T}{m}\right)
\]

Simplify the exponent:

\[
\frac{m}{2k_B T} \times 99 \times \frac{3k_B T}{m} = \frac{1}{2} \times 99 \times 3 = \frac{1}{2} \times 297 = 148.5
\]

Therefore,

\[
R = 100 \times e^{-148.5}
\]

Final Calculation and Interpretation

The exponential term \(e^{-148.5} \) is extraordinarily small:

\[
e^{-148.5} \approx 2.36 \times 10^{-65}
\]

Multiplying by 100:

\[
R \approx 2.36 \times 10^{-63}

\]

This result indicates that the number of molecules in a gas moving at a speed ten times the RMS speed is virtually zero in practical terms.

Implications:

- The Maxwell-Boltzmann distribution predicts that molecules with such high speeds are exceedingly rare.
- The tail of the distribution decays rapidly, making extremely high velocities statistically negligible.
- This underscores the fact that most molecules in a gas at thermal equilibrium have speeds around or below the RMS speed, with very few exceeding it by large factors.

Features and Characteristics of the Result

Pros:

- Quantitative insight: Provides a precise numerical estimate demonstrating the rarity of very high-speed molecules.
- Reinforces distribution understanding: Validates the Maxwell-Boltzmann distribution's tail behavior.
- Practical applications: Helps in understanding reaction kinetics and escape probabilities (e.g., effusion, evaporation).

Cons:

- Simplified assumptions: Assumes an ideal gas with no intermolecular forces.
- Neglects quantum effects: Valid primarily at high temperatures where classical mechanics suffices.
- Limited to thermal equilibrium: Does not account for non-equilibrium situations.

Additional Considerations and Applications

While the calculation above pertains to molecules with specific speeds, similar principles are used in various fields:

- Effusion and Reaction Rates: Molecules with higher speeds have a disproportionate influence on reaction rates and escape probabilities.
- Temperature dependence: As temperature increases, \(v_{rms} \) increases,

shifting the distribution and increasing the fraction of high-speed molecules.
- Mass dependence: Lighter gases have higher v_{rms} at the same temperature, affecting the tail distribution.

Practical example:

In designing vacuum systems or understanding atmospheric escape, knowing the fraction of molecules exceeding certain speeds is crucial. For instance, lighter gases like helium have a higher proportion of molecules with velocities sufficient to escape Earth's gravity.

Conclusion

Determining the ratio of molecules in a gas moving at speeds ten times the root mean square speed vividly illustrates the statistical nature of molecular velocities in gases. The exponential decay in the Maxwell-Boltzmann distribution ensures that such high velocities are virtually nonexistent, emphasizing the dominance of the typical speed range. This analysis not only enriches our theoretical understanding but also has tangible implications across physics, chemistry, and engineering disciplines. Recognizing the rarity of extremely high-speed molecules helps in modeling processes like diffusion, effusion, and reaction kinetics, as well as in designing systems that leverage these molecular behaviors.

In essence, the ratio of molecules moving at ten times the RMS speed is approximately 2.36×10^{-63}, highlighting the extreme improbability of such high velocities under normal conditions.

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