

Determine The Speed At Which The Kinetic Energy Of An Electron Is Equal To 0.5 Times Its Rest Energy.

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Understanding the relationship between an electron's kinetic energy and its velocity is fundamental in both classical and modern physics. When electrons move at speeds approaching that of light, classical mechanics no longer provides accurate descriptions, and relativistic effects become significant. One intriguing scenario involves calculating the exact speed at which an electron's kinetic energy equals half of its rest energy. This problem not only exemplifies the principles of special relativity but also enhances our comprehension of energy-momentum relationships at high velocities. In this comprehensive article, we will explore the theoretical background, derive the relevant formulas, and work through the calculation step-by-step to determine this critical speed.

Understanding Rest Energy and Kinetic Energy in Relativity

Rest Energy of an Electron

In relativistic physics, the rest energy (E_0) of a particle is given by Einstein's famous equation:

$$E_0 = m c^2$$

where:

- m is the rest mass of the particle (for an electron, $m_e \approx 9.109 \times 10^{-31}$ kg)
- c is the speed of light in vacuum ($c \approx 3.00 \times 10^8$ m/s)

For an electron, the rest energy is approximately:

$$E_0 \approx 0.511 \text{ MeV}$$

which is a fundamental constant in particle physics.

Relativistic Kinetic Energy

Unlike classical kinetic energy ($\frac{1}{2} m v^2$), relativistic kinetic energy (K) accounts for effects near the speed of light and is given by:

$$K = (\gamma - 1) m c^2$$

where:

- γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Lorentz factor increases rapidly as v approaches c , highlighting the relativistic effects.

Formulating the Problem

The problem states: Determine the speed v at which the electron's kinetic energy is equal to half of its rest energy:

$$K = 0.5 \times E_0$$

Substituting the expressions:

$$(\gamma - 1) m c^2 = 0.5 \times m c^2$$

Dividing both sides by $(m c^2)$:

$$\gamma - 1 = 0.5$$

Thus:

$$\gamma = 1.5$$

Our task now simplifies to solving for v given $\gamma = 1.5$.

Calculating the Electron's Speed

Step 1: Recall the Lorentz Factor Equation

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Set $\gamma = 1.5$:

$$1.5 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Step 2: Solve for (v/c)

Invert both sides:

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{1.5} = \frac{2}{3}$$

Square both sides:

$$1 - \frac{v^2}{c^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Rearranged:

$$\frac{v^2}{c^2} = 1 - \frac{4}{9} = \frac{5}{9}$$

Finally, take the square root:

$$\frac{v}{c} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Numerically:

$$v \approx c \times 0.745$$

or approximately:

$$v \approx 3.00 \times 10^8, \text{ m/s} \times 0.745 \approx 2.24 \times 10^8, \text{ m/s}$$

Therefore, the electron must travel at about 74.5% of the speed of light for its kinetic energy to be half of its rest energy.

Implications and Significance

Understanding this specific speed has several important implications in physics:

- High-energy physics: Electrons accelerated to this velocity are common in particle accelerators, such as synchrotrons and linear accelerators.
- Relativistic effects: At 74.5% of (c) , relativistic mass increase significantly affects the electron's dynamics.
- Energy considerations: Knowing the kinetic energy at this speed helps in designing experiments and understanding particle interactions at high energies.

Additional Considerations

Total Energy of the Electron

The total energy (E) of a moving electron is:

$$E = \gamma m c^2$$

For $(\gamma = 1.5)$:

$$E \approx 1.5 \times 0.511, \text{ MeV} \approx 0.766, \text{ MeV}$$

The kinetic energy is:

$$K = E - E_0 \approx 0.766, \text{ MeV} - 0.511, \text{ MeV} \approx 0.255, \text{ MeV}$$

which confirms it is about half the rest energy.

Relativistic Momentum

The relativistic momentum (p) is given by:

$$p = \gamma m v$$

At $(\gamma = 1.5)$:

$$p = 1.5 \times m v$$

This shows how momentum increases disproportionately compared to classical expectations at high velocities.

Conclusion

In summary, when the kinetic energy of an electron reaches half of its rest energy, its speed is approximately 74.5% of the speed of light. This calculation underscores the importance of relativistic physics in accurately describing particle behavior at high velocities. As particles approach these speeds, their energy and momentum increase dramatically due to relativistic effects, which are essential considerations in modern physics experiments and applications.

Key takeaways:

- The Lorentz factor (γ) relates velocity and energy in special relativity.
- At $(\gamma = 1.5)$, the kinetic energy is half the rest energy.
- The corresponding velocity is approximately $(0.745 c)$.

Understanding these principles is fundamental for physicists working in high-energy particle physics, astrophysics, and related fields, where particles routinely reach relativistic speeds.

Frequently Asked Questions

How do you find the speed of an electron when its kinetic energy equals half of its rest energy?

You use the relativistic kinetic energy formula $KE = (\gamma - 1)mc^2$, set KE to $0.5 mc^2$, and solve for the Lorentz factor γ , then find the velocity v from $\gamma = 1 / \sqrt{1 - v^2/c^2}$.

What is the approximate speed of an electron when its kinetic energy is 50% of its rest energy?

The electron's speed is approximately $0.707c$ (about 70.7% of the speed of light) when its kinetic energy equals half of its rest energy.

Why is relativistic mechanics necessary to determine the electron's speed at high kinetic energies?

Because at high energies approaching the electron's rest energy, classical (non-relativistic) physics fails, and relativistic formulas are required to accurately relate energy and velocity.

How does the Lorentz factor relate to the kinetic energy of an electron in this scenario?

The Lorentz factor γ equals $1 + (KE / mc^2)$; for $KE = 0.5 mc^2$, $\gamma = 1.5$, which then helps determine the electron's speed via $v = c\sqrt{1 - 1/\gamma^2}$.

Can you derive the speed of an electron when its kinetic energy is half its rest energy using relativistic formulas?

Yes. Set $KE = (\gamma - 1)mc^2 = 0.5 mc^2$, so $\gamma = 1.5$. Then, $v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/2.25} \approx 0.745c$, indicating the electron moves at about 74.5% of the speed of light.

Additional Resources

Kinetic Energy and Electron Dynamics: Understanding the Speed at Which Kinetic Energy Equals Half Rest Energy

In the realm of modern physics, particularly in the study of relativistic particles, understanding the relationship between an electron's kinetic energy and its velocity is fundamental. When examining high-energy phenomena, such as particle accelerators or astrophysical processes, it becomes essential to determine the precise speed at which an electron's kinetic energy reaches a significant fraction of its rest energy. Specifically, this article aims to explore the speed at which the kinetic energy of an electron equals 0.5 times its rest energy, providing an in-depth analysis rooted in special relativity.

Understanding the Concepts: Rest Energy and Kinetic Energy in Relativity

Before delving into calculations, it's crucial to clarify the core concepts involved:

Rest Energy of an Electron

An electron possesses an intrinsic energy known as its rest energy, which is the energy it has by virtue of its mass alone, independent of its motion. This is given by Einstein's famous relation:

$$E_0 = m c^2$$

where:

- E_0 is the rest energy,
- m is the rest mass of the electron ($\sim 9.10938356 \times 10^{-31}$ kg),
- c is the speed of light in vacuum ($\sim 3.00 \times 10^8$ m/s).

Numerically, the rest energy is approximately:

$$E_0 \approx 0.511 \text{ MeV}$$

which is a standard value in particle physics.

Kinetic Energy in Relativity

In classical mechanics, kinetic energy is simply $\frac{1}{2} m v^2$, but at relativistic speeds (approaching the speed of light), it must be described differently. The relativistic kinetic energy K is:

$$K = (\gamma - 1) m c^2$$

where γ is the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

This factor accounts for time dilation and length contraction effects at high velocities.

Formulating the Problem: When Does Kinetic Energy Equal Half Rest Energy?

The problem asks: At what speed v does the electron's kinetic energy K equal 0.5 times E_0 ?

Expressed mathematically:

$$K = 0.5 \times E_0$$

Substituting the relativistic kinetic energy expression:

$$(\gamma - 1) m c^2 = 0.5 \times m c^2$$

Dividing both sides by $(m c^2)$:

$$\gamma - 1 = 0.5$$

Therefore:

$$\boxed{\gamma = 1.5}$$

This simplifies our problem to finding the velocity (v) corresponding to $(\gamma = 1.5)$.

Calculating the Electron's Speed: From Lorentz Factor to Velocity

Given:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

and

$$\gamma = 1.5$$

we can solve for (v) :

$$1.5 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

which leads to:

$$\sqrt{1 - v^2/c^2} = \frac{1}{1.5} = \frac{2}{3}$$

Squaring both sides:

$$1 - \frac{v^2}{c^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Rearranged:

$$\frac{v^2}{c^2} = 1 - \frac{4}{9} = \frac{5}{9}$$

Taking the square root:

$$v = c \sqrt{\frac{5}{9}} = c \times \frac{\sqrt{5}}{3}$$

Numerically:

$$v \approx c \times 0.745$$

or

$$v \approx 0.745 c$$

This indicates that the electron must travel at approximately 74.5% of the speed of light for its kinetic energy to be half of its rest energy.

Implications and Significance of This Speed

Understanding this velocity has several important implications:

1. Relativistic Regimes

- At about 0.745c, electrons are well within the relativistic regime, where classical formulas no longer apply accurately.

- Particle accelerators commonly achieve these energies, making this calculation relevant for experimental physics.

2. Energy Considerations in Particle Physics

- Knowing the velocity corresponding to specific energy fractions helps in designing experiments and interpreting data.
- For example, in synchrotrons or linear accelerators, electrons are accelerated to speeds where their kinetic energy becomes a significant portion of their total energy.

3. Astrophysical Phenomena

- Cosmic rays often involve electrons moving at relativistic speeds; understanding their energy-speed relationship aids in modeling such phenomena.

Additional Insights: The Relationship Between Velocity and Energy

While the primary focus is on the specific speed where kinetic energy is half of rest energy, it's instructive to understand how the kinetic energy scales with velocity:

- Non-Relativistic Limit: For $(v \ll c)$, $(K \approx \frac{1}{2} m v^2)$, matching classical physics.
- Relativistic Limit: As $(v \rightarrow c)$, $(K \rightarrow \infty)$, requiring increasingly high energies to approach the speed of light.

Summary of Key Points

Parameter	Expression	Approximate Value at $(K = 0.5 E_0)$
(γ)	$(\frac{1}{\sqrt{1 - v^2/c^2}})$	1.5
(v)	$(c \times \frac{\sqrt{5}}{3})$	$\sim 0.745c$
Kinetic Energy (K)	$(\gamma - 1) m c^2$	$0.5 (E_0)$

Conclusion: The Electron's Speed at which Kinetic Energy Equals Half Rest Energy

Through a straightforward application of special relativity principles, we've determined that an electron must travel at approximately 74.5% of the speed of light for its kinetic energy to be equal to half of its rest energy. This velocity is a critical threshold in high-energy physics, bridging classical intuition with relativistic reality.

Understanding this relationship not only enhances fundamental physics knowledge but also equips researchers and engineers with essential insights for designing experiments, interpreting cosmic phenomena, and pushing the boundaries of particle acceleration technology. Whether in the context of cutting-edge laboratories or the vast expanse of the cosmos, grasping the dynamics of electrons at relativistic speeds remains a cornerstone of modern scientific exploration.

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