Determine The Value Of N So That The Vectors A And B Are Perpendicular: = + 5j + Nk And B = 2 - J + K

Determine The Value Of N So That The Vectors A And B Are Perpendicular: = + 5j + Nk And B = 2 - J + K

Introduction

Vectors play a fundamental role in various fields such as physics, engineering, computer graphics, and mathematics. One common problem encountered is determining when two vectors are perpendicular, also known as orthogonal. In this article, we will explore how to find the value of the scalar component N that makes two vectors perpendicular. The vectors in question are:

$$
\mathbf{A} = 0\mathbf{i} + 5\mathbf{j} + N\mathbf{k}
$$

$$
\mathbf{B} = 2\mathbf{i} - 1\mathbf{j} + 1\mathbf{k}
$$

Our goal is to determine the value of N such that the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular.

Understanding Perpendicular Vectors

What Does It Mean for Vectors to Be Perpendicular?

Two vectors are perpendicular if the angle between them is 90 degrees. Mathematically, this is expressed via the dot product:

$$
\mathbf{A} \cdot \mathbf{B} = 0
$$

The dot product of two vectors in three-dimensional space $\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$ and $\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$ is calculated as:

$$
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z
$$

Significance in Calculations

The dot product being zero is both a necessary and sufficient condition for the vectors to be orthogonal (perpendicular).

Step-by-Step Approach to Find N

1. Expressing the Vectors

Given:

$$
\mathbf{A} = 0\mathbf{i} + 5\mathbf{j} + N\mathbf{k}
$$
$$
\mathbf{B} = 2\mathbf{i} - 1\mathbf{j} + 1\mathbf{k}
$$

In component form:

$$
\mathbf{A} = (0, 5, N)
$$
$$
\mathbf{B} = (2, -1, 1)
$$

2. Computing the Dot Product

Applying the dot product formula:

$$
\mathbf{A} \cdot \mathbf{B} = (0)(2) + (5)(-1) + (N)(1)
$$

Simplify:

$$
\mathbf{A} \cdot \mathbf{B} = 0 - 5 + N
$$

Finding N for Orthogonality

Since for vectors to be perpendicular:

$$
\mathbf{A} \cdot \mathbf{B} = 0
$$

Set the dot product equal to zero:

$$
0 - 5 + N = 0
$$

Solve for N:

$$
N = 5
$$

Thus, the value of N that makes vectors $\mathbf{A}$ and $\mathbf{B}$ perpendicular is 5.

Significance and Applications

Understanding how to determine the perpendicularity of vectors is essential in multiple real-world applications:

- Physics: Calculating forces, velocities, and accelerations that are perpendicular.
- Engineering: Designing components and systems where orthogonal vectors ensure stability or specific behaviors.
- Computer Graphics: Ensuring that surface normals are perpendicular to surfaces for proper shading.
- Mathematics: Solving geometric problems involving angles and distances.

Additional Considerations

Verifying the Result

It’s always good practice to verify the result by substituting $N = 5$ back into the vectors and confirming the dot product:

$$
\mathbf{A} = (0, 5, 5)
$$

$$
\mathbf{A} \cdot \mathbf{B} = (0)(2) + (5)(-1) + (5)(1) = 0 - 5 + 5 = 0
$$

Since the dot product is zero, the vectors are indeed perpendicular when $N = 5$.

Visualizing the Vectors

Visualizing vectors helps to develop an intuitive understanding:

- $\mathbf{A}$ points somewhere in the 3D space with components along y and z.
- $\mathbf{B}$ points in a different direction.

When $N = 5$, the angle between the vectors is exactly 90°, confirming their orthogonality.

Summary

In this article, we have:

- Defined what it means for vectors to be perpendicular.
- Demonstrated how to compute the dot product.
- Applied the dot product condition to find the scalar N.
- Concluded that $N = 5$ makes vectors $\mathbf{A}$ and $\mathbf{B}$ perpendicular.

Understanding these principles is fundamental for solving a wide array of problems involving vectors in physics, engineering, and mathematics.

Final Thoughts

Determining the value of a component such as N to achieve perpendicularity is a straightforward but essential skill in vector analysis. It exemplifies how algebraic manipulation can solve geometric problems and highlights the importance of the dot product in vector calculus. Mastery of such techniques enhances problem-solving capabilities across scientific disciplines.

Keywords: vectors, perpendicular vectors, dot product, vector calculus, orthogonality, scalar component, vector analysis, N value, 3D vectors, physics, engineering, mathematics

Frequently Asked Questions

How do you determine the value of N so that vectors A and B are perpendicular?

To find N, set the dot product of vectors A and B to zero and solve for N, since perpendicular vectors have a dot product of zero.

What are the components of vectors A and B in the given problem?

Vector A has components (0, 5, N) and vector B has components (2, -1, 1).

How is the dot product of vectors A and B calculated?

The dot product is calculated as $(A_x)(B_x) + (A_y)(B_y) + (A_z)(B_z)$.

What is the equation to find N when vectors A and B are perpendicular?

Set the dot product equal to zero: 02 + 5(-1) + N1 = 0, which simplifies to -5 + N = 0.

What is the value of N that makes vectors A and B perpendicular?

N equals 5, since solving -5 + N = 0 yields N = 5.

Additional Resources

Determine The Value Of N So That The Vectors A And B Are Perpendicular: = + 5j + Nk And B = 2 - J + K

In the world of vector mathematics, understanding the relationship between vectors often leads to insights spanning physics, engineering, computer graphics, and more. One fundamental concept is the perpendicularity of vectors, which indicates they are orthogonal and have a dot product of zero. Today, we delve into a classic problem: determining the value of N such that two given vectors are perpendicular. The vectors in question are A = 0i + 5j + Nk and B = 2i - j + k. This problem not only reinforces the core principles of vector algebra but also exemplifies how to approach similar questions systematically.

Understanding the Problem: Vectors and Perpendicularity

Before diving into calculations, it's essential to grasp what the problem entails. The vectors given are expressed in component form:

- Vector A: $\mathbf{A} = 0\mathbf{i} + 5\mathbf{j} + N\mathbf{k}$
- Vector B: $\mathbf{B} = 2\mathbf{i} - 1\mathbf{j} + 1\mathbf{k}$

In vector notation, these are:

$$
\mathbf{A} = (0, 5, N)
$$
$$
\mathbf{B} = (2, -1, 1)
$$

The question is: What is the value of N that makes vectors A and B perpendicular?

Perpendicular vectors, also known as orthogonal vectors, satisfy the condition:

$$
\mathbf{A} \cdot \mathbf{B} = 0
$$

where "·" denotes the dot product.

Recalling the Dot Product: The Key to Perpendicularity

The dot product of two vectors in three-dimensional space is calculated as:

$$
\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z
$$

Applying this to our vectors:

$$
\mathbf{A} \cdot \mathbf{B} = (0)(2) + (5)(-1) + (N)(1)
$$

which simplifies to:

$$
0 - 5 + N
$$

Since the vectors are perpendicular when their dot product equals zero:

$$
0 - 5 + N = 0
$$

Solving for N: The Mathematical Approach

To find the value of N, simply solve the equation:

$$
-5 + N = 0
$$

Adding 5 to both sides:

$$
N = 5
$$

This straightforward calculation demonstrates how the condition for perpendicularity directly relates to the components of the vectors.

Implications and Applications of Perpendicular Vectors

Understanding the specific value of N that makes vectors orthogonal is more than an academic exercise; it has practical applications across various fields.

Physics and Engineering

- Force Components: Engineers often decompose forces into perpendicular components to analyze structures or motion accurately.
- Electric and Magnetic Fields: Perpendicular vectors denote orthogonal field components, crucial in electromagnetic theory.

Computer Graphics and Animation

- Surface Normals: Calculating normal vectors perpendicular to surfaces is fundamental in rendering realistic images.
- Collision Detection: Orthogonal vectors are used to determine angles and orientations in 3D space.

Mathematics and Data Analysis

- Principal Component Analysis (PCA): Finds orthogonal axes that capture maximum variance.
- Machine Learning: Orthogonality can optimize feature spaces, avoiding redundancy.

Extending the Concept: Generalized Approach to Finding N

The specific problem of determining N can be generalized to a broader class of vector problems:

1. Identify the vectors in component form. Ensure that each vector's components are explicitly

defined.
2. Apply the dot product formula. Calculate the dot product explicitly.
3. Set the dot product to zero. This condition signifies perpendicularity.
4. Solve for the unknown component(s). Isolate the variable(s) and compute their values.

This approach is adaptable to higher dimensions or more complex vectors, making it a versatile technique in vector analysis.

Summary and Final Thoughts

Determining the value of N such that vectors A = $0i + 5j + Nk$ and B = $2i - j + k$ are perpendicular is a clear demonstration of fundamental vector principles. By leveraging the dot product, a simple algebraic equation was derived and solved, revealing that:

$$
\boxed{
N = 5
}
$$

This exercise underscores the importance of understanding vector operations and their geometric interpretations. Whether in academic settings, engineering design, or computer graphics, the ability to manipulate and analyze vectors is indispensable. The conceptual clarity gained from such problems enhances our capacity to tackle more complex multidimensional problems, emphasizing that sometimes, the key to unlocking a problem lies in fundamental mathematical principles.

In conclusion, the problem of finding N is not just an isolated exercise but a gateway to appreciating the elegance and utility of vector algebra in solving real-world challenges.

Determine The Value Of N So That The Vectors A And B Are Perpendicular 5j Nk And B 2 J K

Determine The Value Of N So That The Vectors A And B Are Perpendicular 5j Nk And B 2 J K

Related Articles

- Drag And Drop An Answer To Each Box To Correctly Explain The Derivation Of The Formula For The Volume
- Correlation Coefficient Between Return Of Stock A And B Is -1. Assume That You Can Borrow And Lend At
- Draw A Flowchart Or Write Pseudocode To Represent The Logic Of A Program That Allows The Userto Enter

Back to Home