Determine Whether Rolle's Theorem Can Be Applied To F On The Closed Interval [a, B]. (Select All That

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Introduction to Rolle's Theorem

Rolle's Theorem is a fundamental result in calculus that provides conditions under which a differentiable function attains a horizontal tangent within a closed interval. It is a special case of the Mean Value Theorem and has significant applications in analysis, physics, and engineering. Before applying Rolle's Theorem to a specific function \(f \) on an interval \([a, B]\), it is essential to verify that the function satisfies the theorem's hypotheses. This article explores these conditions in detail, discusses how to determine their satisfaction for a given function, and provides guidance on selecting all applicable conditions.

Understanding the Statement of Rolle's Theorem

Conditions Required for Rolle's Theorem

Rolle's Theorem states that if a function \(f \) satisfies the following three conditions on a closed interval \([a, B]\):

1. **Continuity on \([a, B]\):** \(f\) is continuous on the entire interval \([a, B]\).

2. **Differentiability on (a, B):** f is differentiable on the open interval (a, B).

3. **Equal Endpoint Values:** $f(a) = f(B)$.

then there exists at least one point $c \in (a, B)$ such that:

$$
f'(c) = 0
$$

This guarantees the existence of a point where the tangent to the graph of f is horizontal.

Step-by-Step Approach to Determine Applicability

To assess whether Rolle's Theorem applies to a particular function f over $[a, B]$, follow these steps:

1. Verify Continuity on $[a, B]$

- Check the domain of f: Is f continuous over the entire interval?
- Identify any points of discontinuity within $[a, B]$. If any exist, Rolle's Theorem does not apply.
- Typical sources of discontinuity include jumps, removable discontinuities, or asymptotes.

2. Verify Differentiability on (a, B)

- Determine if f is differentiable throughout the open interval.
- Focus on points where the function might not be differentiable, such as sharp corners, cusps, or points of non-smooth behavior.
- For functions with known derivatives, confirm that f' exists at all points in (a, B).

3. Check Endpoint Values $f(a)$ and $f(B)$

- Calculate the values of f at the endpoints a and B.
- Verify if $f(a) = f(B)$. If not, Rolle's Theorem cannot be applied.

Applying the Conditions to Specific Functions

To illustrate the process, consider various functions and analyze their suitability for Rolle's Theorem.

Example 1: Polynomial Function

Suppose $f(x) = x^3 - 3x + 1$ over $[a, B]$.

- Continuity: Polynomial functions are continuous everywhere, so f is continuous on $[a, B]$.
- Differentiability: Polynomial functions are differentiable everywhere; thus, f is differentiable on (a, B).
- Endpoint Values: Calculate $f(a)$ and $f(B)$. If they are equal, Rolle's Theorem applies; if not, it does not.

Example 2: Absolute Value Function

Consider $f(x) = |x|$ over $[-1, 1]$.

- Continuity: f is continuous everywhere.
- Differentiability: f is not differentiable at $x=0$, since the graph has a sharp corner there.
- Endpoint Values: $f(-1) = 1$, $f(1) = 1$, so $f(-1) = f(1)$.

Conclusion: Since f is not differentiable on the entire interval (due to the point at zero), Rolle's Theorem cannot be applied.

Selecting All Applicable Conditions

When evaluating whether Rolle's Theorem applies, it’s crucial to identify all conditions that hold true. In many cases, some conditions are easily verified, while others require careful examination.

- **Continuity on \([a, B]\):** Always check if the function is continuous. Discontinuities disqualify the theorem.
- **Differentiability on \((a, B)\):** Confirm the differentiability, especially at points where the function might have corners or cusps.
- **Equal Endpoint Values:** Ensure \(f(a) = f(B)\). If not, Rolle's Theorem does not hold.

Note: The theorem does not specify the behavior outside \((a, B)\) or whether the function is continuous or differentiable at the endpoints—only on the interval and at the endpoints for the values.

Special Cases and Additional Considerations

Some functions may satisfy most conditions but fail at specific points or due to particular properties. Here are some considerations:

Functions with Discontinuities or Non-differentiable Points

- If the function has finite discontinuities or sharp corners inside \([a, B]\), Rolle's Theorem does not apply.
- For example, functions with jump discontinuities or points where derivatives do not exist are disqualified.

Functions with Equal Endpoint Values but Not Continuous

- If \(f(a) = f(B)\), but \(f\) is not continuous on \([a, B]\), then Rolle's Theorem cannot be applied.

Functions with Endpoints Not in Domain

- Ensure \(a, B\) are within the domain of \(f\). If either endpoint is outside the domain, the theorem's conditions are invalid.

Summary and Practical Tips

- Always verify the three core conditions: continuity on \([a, B]\), differentiability on \((a, B)\), and \(f(a) = f(B)\).
- Use the function's known properties or analyze its formula to check these conditions.
- Remember that satisfying all conditions guarantees the existence of at least one point \(c \in (a, B) \) where \(f'(c) = 0\).

Conclusion

In conclusion, determining whether Rolle's Theorem applies to a function \(f\) over a closed interval \([a, B]\) involves a systematic check of the theorem's three main conditions. By carefully analyzing the function's continuity, differentiability, and endpoint values, one can confidently identify all conditions that are satisfied. This process not only helps in applying Rolle's Theorem correctly but also deepens understanding of the function's behavior and the foundational principles of calculus. Always approach such problems methodically, and when in doubt, verify each condition explicitly to ensure accurate conclusions.

Frequently Asked Questions

What are the main conditions required to apply Rolle's Theorem to function f on the interval [a, b]?

The main conditions are that f is continuous on [a, b], differentiable on (a, b), and that f(a) equals f(b).

How can I determine if Rolle's Theorem applies to a specific function f on [a, b]?

Verify that f is continuous on [a, b], differentiable on (a, b), and check if f(a) equals f(b). If all are true, Rolle's Theorem can be applied.

Is it necessary for the function to be differentiable at the endpoints a and b for Rolle's Theorem to hold?

No, Rolle's Theorem only requires differentiability on the open interval (a, b); continuity on [a, b] and equal values at a and b are also needed.

What does it imply if a function f satisfies the conditions of Rolle's Theorem on [a, b]?

It implies that there exists at least one point c in (a, b) where the derivative f'(c) equals zero.

Can Rolle's Theorem be applied if f is not differentiable at some points within (a, b)?

No, differentiability within (a, b) is a requirement; if f is not differentiable at some points, Rolle's Theorem does not apply.

If f(a) ≠ f(b), can Rolle's Theorem be used to analyze the function on [a, b]?

No, one of the key conditions is that f(a) must equal f(b); if this is not the case, Rolle's Theorem cannot be applied directly.

How does the endpoint behavior of f affect the applicability of Rolle's Theorem?

The function must be continuous at the endpoints; any discontinuity at a or b prevents the application of Rolle's Theorem.

Additional Resources

Determine Whether Rolle's Theorem Can Be Applied To F On The Closed Interval [a, B]. (Select All That Apply)

When analyzing the behavior of a function within a specific interval, mathematicians often turn to fundamental theorems in calculus to identify key properties. One such essential theorem is Rolle's Theorem, which provides insight into the existence of stationary points under certain conditions. Determining whether Rolle's Theorem can be applied to F on the closed interval [a, B] is a critical step in understanding the function's behavior, especially when exploring the function's extrema or the roots within the interval. This guide offers a comprehensive breakdown, explaining how to evaluate the applicability of Rolle's Theorem, what conditions must be satisfied, and how to interpret the implications.

Understanding Rolle's Theorem: The Foundations

Before diving into application criteria, it’s important to grasp the core statement of Rolle's Theorem:

Rolle's Theorem states that if a function f satisfies the following three conditions on a closed interval $[a, b]$:

1. Continuity on $[a, b]$
2. Differentiability on (a, b)
3. Equal function values at the endpoints, i.e., $f(a) = f(b)$

then there exists at least one point $c \in (a, b)$ such that:

$$
f'(c) = 0
$$

This theorem essentially guarantees the existence of at least one stationary point within the interval where the derivative is zero, provided the conditions are met.

Key Conditions for Applying Rolle's Theorem

When asked to determine whether Rolle’s Theorem can be applied to a specific function F on the interval $[a, B]$, you should systematically verify the following conditions:

1. Continuity on the Closed Interval $[a, B]$

- Why it matters: Continuity ensures no breaks, jumps, or discontinuities in the function within the interval. Rolle's Theorem relies on the function being "whole" and smooth enough to analyze extrema and derivative behavior.

- How to verify:
- Check the function's definition at all points in $[a, B]$.
- Identify any points of discontinuity, such as asymptotes, jumps, or removable discontinuities.
- Utilize the function's formula or graph to confirm continuity.

2. Differentiability on the Open Interval (a, B)

- Why it matters: Differentiability ensures the function has a well-defined tangent (derivative) at each point within (a, B), which is essential for the existence of stationary points.

- How to verify:
- Examine the derivative of the function F where it exists.
- Check for points where the derivative might not exist, such as corners, cusps, vertical tangents, or discontinuities in the derivative.
- Ensure the function is smooth enough within the interval, excluding the endpoints.

3. Equal Function Values at the Endpoints: $F(a) = F(B)$

- Why it matters: The theorem requires that the function's values match at both endpoints; otherwise, the guarantee of a point with zero derivative does not hold.

- How to verify:
- Calculate $F(a)$ and $F(B)$.
- Confirm whether these values are equal.

Step-by-Step Approach to Determine Applicability

To systematically analyze whether Rolle’s Theorem can be applied, follow these steps:

Step 1: Analyze Continuity

- Check the function's formula or graph over $[a, B]$.
- Identify any points of discontinuity. For example:
- Rational functions with vertical asymptotes.
- Piecewise functions with jumps.
- If discontinuities exist within the interval, the theorem cannot be applied.

Step 2: Analyze Differentiability

- Differentiate the function where possible.
- Identify points where the derivative does not exist:
- Corners or cusps.
- Vertical tangent lines (where derivative approaches infinity).
- Confirm that differentiability holds over (a, B). If not, Rolle’s Theorem cannot be applied.

Step 3: Check Endpoints' Function Values

- Calculate $F(a)$ and $F(B)$.
- Confirm whether $F(a) = F(B)$.
- If yes, proceed to the next step.
- If no, Rolle's Theorem does not apply in this case.

Step 4: Summarize and Decide

- If all three conditions are satisfied, then Rolle's Theorem applies, guaranteeing at least one $c \in (a, B)$ where $F'(c) = 0$.
- If any condition fails, then the theorem cannot be applied directly.

Practical Examples

Example 1: Function $F(x) = x^2 - 4x + 3$ on $[1, 3]$

- Continuity: Polynomial functions are continuous everywhere.
- Differentiability: Polynomial functions are differentiable everywhere.
- Calculate $F(1)$ and $F(3)$:

$$
F(1) = 1 - 4 + 3 = 0
$$

$$
F(3) = 9 - 12 + 3 = 0
$$

- Values at endpoints are equal (0), so all conditions are satisfied.
- Conclusion: Rolle's Theorem applies. There exists at least one $c \in (1, 3)$ where $F'(c) = 0$.

Example 2: Function $F(x) = \sqrt{x}$ on $[0, 4]$

- Continuity: $\sqrt{x}$ is continuous on $[0, 4]$.
- Differentiability: $\sqrt{x}$ is differentiable on $(0, 4]$; at $x=0$, the derivative does not exist, but since we're considering the open interval $(0, 4)$, the differentiability condition is satisfied.
- Calculate $F(0)$ and $F(4)$:

$$
F(0) = 0
$$

\[

F(4) = 2

\]

- Values at endpoints are not equal.
- Conclusion: Rolle's Theorem does not apply because \(F(0) \neq F(4)\).

Additional Considerations and Common Pitfalls

While checking conditions seems straightforward, several nuances can impact the decision:

1. Endpoints and Differentiability

- The theorem requires differentiability on the open interval \((a, B)\), not necessarily at the endpoints. It’s acceptable for the function to be non-differentiable at \(a\) or \(B\), as long as the conditions are met within \((a, B)\).

2. Function Behavior at Endpoints

- When the function values at the endpoints are equal, it often indicates potential for the existence of a stationary point within the interval. But if the function is not continuous or not differentiable, Rolle's Theorem cannot be invoked.

3. Piecewise Functions

- For piecewise functions, verify each piece's continuity and differentiability. Discontinuities or corners at the endpoints or within the interval can disqualify the application.

4. Understanding the Implications

- Applying Rolle's Theorem confirms the existence of at least one stationary point where the derivative equals zero, which can be useful in optimization and root-finding problems.

When to Select All or Some Conditions

In some contexts, you might encounter questions asking which conditions are necessary or sufficient for Rolle’s Theorem. Generally:

- Necessary Conditions: Continuity on $[a, B]$ and differentiability on (a, B).
- Sufficient Condition: The endpoint values are equal.

If any necessary condition fails, Rolle's Theorem cannot be applied.

Summary: How to Determine Applicability

Step	Check	Conclusion
1	Is F continuous on $[a, B]$?	If no, Rolle’s Theorem does not apply
2	Is F differentiable on (a, B)?	If no, Rolle’s Theorem does not apply
3	Are $F(a)$ and $F(B)$ equal?	If no, Rolle’s Theorem does not apply
4	All conditions satisfied?	Then Rolle's Theorem applies, guaranteeing at least one c with $F'(c) = 0$

Final Thoughts

Determining whether Rolle's Theorem can be applied to \(F\) on the interval \([a, B]\) involves carefully verifying the three core conditions. This process not only confirms the theorem's applicability but also deepens understanding of the function's behavior

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