Determine Whether The Series Converges Or Diverges. [infinity] 9 / N+ 4. N=1 A. Converges B. Diverges

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Understanding whether an infinite series converges or diverges is a fundamental concept in calculus and mathematical analysis. It plays a crucial role in various fields such as engineering, physics, and computer science, where infinite processes or sums are involved. In this article, we will analyze the series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$, examining whether it converges or diverges, and explore the methods used to determine the behavior of such series.

Introduction to Infinite Series

Before diving into the specific series, it's essential to grasp the basics of infinite series:

What Is an Infinite Series?

An infinite series is the sum of infinitely many terms, typically expressed as:

$$
\sum_{n=1}^{\infty} a_n
$$

where a_n represents the n-th term of the series.

Why Is It Important to Determine Convergence?

Not all infinite series sum to a finite value. Some diverge, meaning their partial sums grow without bound, while others converge to a specific finite value. Knowing whether a series converges or diverges helps in understanding the behavior of complex functions, evaluating limits, and approximating solutions in applied mathematics.

Understanding the Series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$

The series under consideration is:

$$
\sum_{n=1}^{\infty} \frac{9}{n + 4}
$$

This series resembles a harmonic-type series but with a shifted denominator. To analyze its convergence, we need to examine its behavior as n approaches infinity.

Initial Observations

- The general term $\frac{9}{n + 4}$ decreases as n increases because the denominator increases.
- For large n, $\frac{9}{n + 4}$ behaves similarly to $\frac{9}{n}$.

These observations suggest that the series might have similar properties to the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, which is known to diverge.

Methods to Determine Series Convergence

To rigorously determine whether the series converges or diverges, mathematicians use various convergence tests. The most relevant for this series are:

1. Comparison Test

Compare the series to a known benchmark series, such as the harmonic series.

2. Limit Comparison Test

Compare the limit of the ratio of the terms of the given series to those of a known divergent or convergent series.

3. Integral Test

Evaluate the improper integral of the function corresponding to the series' terms.

4. p-Series Test

Apply if the series can be expressed as a p-series $\sum \frac{1}{n^p}$.

Let's apply these methods to our series.

Applying the Comparison and Limit Comparison Tests

Comparison to the Harmonic Series

Since for large n:

$$
\frac{9}{n + 4} \sim \frac{9}{n}
$$

and the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, we suspect that our series also diverges.

Comparison:

$$
0 < \frac{9}{n + 4} < \frac{9}{n}
$$

for all $n \geq 1$. Because the harmonic series diverges, and our series' terms are comparable to $\frac{1}{n}$, the comparison test indicates divergence.

Limit Comparison Test:

Compute:

$$
\lim_{n \to \infty} \frac{\frac{9}{n + 4}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{9n}{n + 4} = \lim_{n \to \infty} \frac{9n}{n(1 + 4/n)} = \lim_{n \to \infty} \frac{9}{1 + 4/n} = 9
$$

Since this limit is a finite positive number (9), the limit comparison test confirms that $\sum_{n=1}^{\infty} \frac{9}{n + 4}$ diverges because the harmonic series diverges.

Applying the Integral Test

The integral test involves integrating the function $f(n) = \frac{9}{n + 4}$ over $[1, \infty)$:

$$
\int_{1}^{\infty} \frac{9}{x + 4} \, dx
$$

Compute the integral:

$$
\int \frac{9}{x + 4} \, dx = 9 \int \frac{1}{x + 4} \, dx = 9 \ln|x + 4| + C
$$

Evaluate the improper integral:

$$
\lim_{t \to \infty} 9 \ln|t + 4| - 9 \ln|1 + 4| = \lim_{t \to \infty} 9 \ln(t + 4) - 9 \ln 5
$$

As $t \to \infty$, $\ln(t + 4) \to \infty$, so the integral diverges to infinity. Therefore, by the integral test, the series diverges.

Conclusion: Does the Series Converge or Diverge?

Based on the comparison, limit comparison, and integral tests:

- The series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$ diverges.

Answer: B. Diverges

This conclusion aligns with the understanding that series resembling the harmonic series, even with finite shifts in the denominator, tend to diverge because their terms decrease too slowly to produce a finite sum.

Additional Insights and Implications

Understanding the divergence of the series has practical implications:

- **Series in Approximate Calculations:** Recognizing divergence helps avoid incorrect assumptions about the sum's finiteness in numerical methods.
- **Mathematical Modeling:** When modeling phenomena with infinite processes, knowing whether the sum converges ensures the model's validity.
- **Advanced Series Analysis:** For more complex series, similar tests can be employed, emphasizing the importance of foundational convergence tests.

Summary

In summary:

- The series under investigation is $\sum_{n=1}^{\infty} \frac{9}{n + 4}$.
- Using comparison and integral tests, we find that this series diverges.
- The divergence stems from the fact that the terms behave similarly to $\frac{1}{n}$, whose harmonic series diverges.
- Therefore, the correct choice is:

B. Diverges

Final Thoughts

Determining whether an infinite series converges or diverges is a vital skill in mathematics. It requires analyzing the series' terms, applying appropriate convergence tests, and understanding the behavior of similar known series. While the series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$ diverges, the methods used to arrive at this conclusion are applicable to a vast range of series, making them essential tools in both theoretical and applied mathematics.

Remember: Always start by examining the terms' behavior as n approaches infinity. Use comparison or limit comparison tests for series similar to well-known divergent or convergent series, and employ the integral test when the terms are positive and decreasing. This systematic approach ensures accurate determination of a series' convergence status.

Frequently Asked Questions

Does the series ∑ (9 / (n + 4)) from n=1 to infinity converge or diverge?

The series diverges because it behaves like a constant multiple of the harmonic series, which diverges.

What test can be used to determine the convergence of ∑ (9 / (n + 4))?

The Comparison Test or Limit Comparison Test can be used, comparing it to the harmonic series ∑ 1/n.

Is the series ∑ (9 / (n + 4)) convergent or divergent as n approaches infinity?

It diverges because the terms do not approach zero fast enough, similar to the harmonic series.

What is the behavior of the general term 9 / (n + 4) as n approaches infinity?

The general term approaches zero, but that alone does not determine convergence; the series still diverges similar to the harmonic series.

Why does the series ∑ (9 / (n + 4)) diverge even though the terms tend to zero?

Because the terms decrease too slowly, like 1/n, and the harmonic series is known to diverge, so this series also diverges.

Additional Resources

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Understanding the behavior of infinite series is fundamental in advanced mathematics, especially in fields such as analysis, physics, and engineering. Among the many series studied, a common question posed is whether a given series converges (approaches a finite value) or diverges (grows without bound or oscillates indefinitely). In this article, we'll analyze the series:

$$
\sum_{n=1}^{\infty} \frac{9}{n + 4}
$$

and determine whether it converges or diverges. This problem, while seemingly straightforward, offers insights into the core concepts of series convergence tests and asymptotic behavior.

Understanding the Series: Basic Structure and Intuition

Before diving into convergence tests, it's vital to comprehend the structure of the series:

$$
\sum_{N=1}^{\infty} \frac{9}{N + 4}
$$

At first glance, this resembles a harmonic-type series since the numerator is constant (9) and the denominator grows linearly with N. As N increases, the terms $\frac{9}{N + 4}$ decrease toward zero, but the key question is whether they decrease fast enough to produce a finite sum.

Key observations:

- The terms are positive for all $N \geq 1$.
- The denominator is linear, growing without bound as N approaches infinity.
- The series resembles the harmonic series scaled by a constant factor.

Understanding the nature of these terms guides us toward applying appropriate convergence tests.

Applying the Comparison Test: A First Approach

One of the most straightforward tools to analyze such series is the Comparison Test. This test states:

> If $0 \leq a_N \leq b_N$ for all N beyond some N_0, and $\sum b_N$ converges, then $\sum a_N$ also converges. Conversely, if $\sum a_N$ diverges and $a_N \geq b_N \geq 0$, then $\sum b_N$ diverges.

In our case, consider the general term:

$$
a_N = \frac{9}{N + 4}
$$

Note that for all $N \geq 1$:

$$
a_N = \frac{9}{N + 4} \leq \frac{9}{N}
$$

because $N + 4 \geq N$.

Comparison to the harmonic series:

$$
\sum_{N=1}^{\infty} \frac{9}{N}
$$

which simplifies to:

$$
9 \sum_{N=1}^{\infty} \frac{1}{N}
$$

The harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ is well-known to diverge, meaning it grows without bound as N approaches infinity.

Since:

\[

$$
a_n \leq \frac{9}{n}
$$

and the larger series $\sum_{n=1}^{\infty} \frac{9}{n}$ diverges, the Comparison Test tells us nothing conclusively about the convergence of our original series. Instead, it suggests that our series behaves similarly to a harmonic series, which diverges.

Conclusion from comparison:

Because $\frac{9}{n + 4}$ behaves asymptotically like $\frac{9}{n}$, and the harmonic series diverges, our series cannot converge; it diverges as well.

The Limit Comparison Test: Confirming Divergence

While the comparison test provides an initial intuition, the Limit Comparison Test offers a more conclusive approach. The Limit Comparison Test states:

> For two series $\sum a_n$ and $\sum b_n$ with positive terms, if:

$$
\lim_{n \to \infty} \frac{a_n}{b_n} = c
$$

where c is a finite positive number, then both series either converge or diverge simultaneously.

Applying to our series:

Let:

$$
a_n = \frac{9}{n + 4}
$$
$$
b_n = \frac{1}{n}
$$

Calculate the limit:

$$
\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{9/(n + 4)}{1/n} = \lim_{n \to \infty} \frac{9}{n + 4} \times \frac{n}{1} = \lim_{n \to \infty} \frac{9n}{n + 4}
$$

Simplify:

$$
\lim_{n \to \infty} \frac{9n}{n + 4} = \lim_{n \to \infty} \frac{9n}{n(1 + 4/n)} = \lim_{n \to \infty} \frac{9}{1 + 4/n} = 9
$$

Since the limit is a finite positive number (9), and we know the harmonic series diverges, it follows that our original series diverges as well.

Final conclusion:

Based on the Limit Comparison Test, the series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$ diverges.

Integral Test: Alternative Confirmation

The Integral Test provides another perspective. It states:

> If $f(n) = \frac{9}{n + 4}$ is positive, continuous, and decreasing for $n \geq 1$, then the series $\sum_{n=1}^{\infty} a_n$ and the integral:

$$
\int_{1}^{\infty} f(x) \, dx
$$

either both converge or both diverge.

Applying the integral test:

Calculate:

$$
\int_{1}^{\infty} \frac{9}{x + 4} dx
$$

which simplifies to:

$$
9 \int_{1}^{\infty} \frac{1}{x + 4} dx
$$

The indefinite integral:

$$
\int \frac{1}{x + 4} dx = \ln|x + 4| + C
$$

Evaluate the improper integral:

$$
\lim_{t \to \infty} 9 \left[\ln|x + 4| \right]_{x=1}^{x=t} = 9 \left(\lim_{t \to \infty} \ln(t + 4) - \ln(5) \right)
$$

As $t \to \infty$, $\ln(t + 4) \to \infty$, so the integral diverges.

Conclusion:

Since the integral diverges, the series diverges as well.

Summary and Final Verdict

By applying multiple convergence tests—Comparison Test, Limit Comparison Test, and the Integral Test—it becomes clear that the series:

$$
\sum_{n=1}^{\infty} \frac{9}{n + 4}
$$

does not converge to a finite value. Instead, it diverges because its terms behave asymptotically like $\frac{9}{n}$, and the harmonic series is a classic example of divergence.

Therefore, the correct answer is:

B. Diverges

Implications and Broader Context

Understanding whether a series converges or diverges is not merely an academic exercise—it has practical implications across various scientific disciplines. For example:

- In physics, series convergence determines the stability of solutions in quantum mechanics or electromagnetism.
- In engineering, convergence affects the reliability of signal processing algorithms.
- In economics, infinite series are used to model long-term investments and growth, where divergence signals unbounded growth or instability.

In the case of the series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$, recognizing its divergence helps avoid incorrect assumptions about finite limits or stability in models relying on such series.

Conclusion

The analysis of the series $\sum_{n=1}^{\infty} \frac{9}{n + 4}$ reveals a fundamental characteristic: it diverges. Despite the terms decreasing to zero, they do not do so rapidly enough to produce a finite sum. This behavior aligns with the properties of the harmonic series, scaled by a constant factor, which is well-understood to diverge.

In mathematical practice, such insights reinforce the importance of applying multiple tests and understanding the asymptotic behavior of series terms. Recognizing divergence early saves time and guides mathematicians and scientists in choosing appropriate models and solutions.

Final answer:

B. Diverges

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