Each Side Of A Hexagon Is 10 Inches Longer Than The Previous Side. What Is The Length Of The Shortest

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Understanding geometric sequences and their applications in real-world problems can be both intriguing and educational. One such problem involves a hexagon where each side length increases by a fixed amount compared to the previous side. Specifically, in this scenario, each side of the hexagon is 10 inches longer than the previous side. The challenge lies in determining the length of the shortest side, given this pattern. This article provides a comprehensive exploration of this problem, breaking down the solution step-by-step, and offering insights into how such geometric sequences work.

Understanding the Problem Statement

The problem states that:

- The hexagon has six sides.
- Each side is 10 inches longer than the previous side.
- The goal is to find the length of the shortest side.

To clarify, the problem implies that the sides follow a certain pattern, increasing uniformly by 10 inches each time. Since the sides are in a sequence where each term is larger than the previous by a constant difference, this is a classic arithmetic sequence problem.

Key Concepts: Arithmetic Sequences and Their Role

Before delving into the solution, it's crucial to understand some fundamental concepts:

What Is an Arithmetic Sequence?

An arithmetic sequence is a list of numbers where each term after the first is obtained by adding a fixed number, called the common difference, to the previous term.

Example:

2, 5, 8, 11, 14, ...
Common difference = 3

Relevance to the Hexagon Problem

Since each side of the hexagon is 10 inches longer than the previous, the side lengths form an arithmetic sequence with a common difference of 10 inches.

Defining the Variables

Let's denote:

- The shortest side as a (this is the first term of the sequence).
- The common difference as $d = 10$ inches.
- The sides of the hexagon as $a_1, a_2, a_3, a_4, a_5, a_6$.

Given the pattern:

$$a_1 = a$$
$$a_2 = a + d$$
$$a_3 = a + 2d$$
$$a_4 = a + 3d$$
$$a_5 = a + 4d$$
$$a_6 = a + 5d$$

Since the problem asks for the shortest side, which is a_1, our goal is to find the value of a.

Determining the Length of the Shortest Side

To find the value of a, we need additional information about the dimensions of the hexagon. Typically, in such problems, the total perimeter or some geometric property is provided.

However, if the problem only states the pattern of the sides without additional constraints, we might need to make reasonable assumptions or explore the problem more generally.

In many educational problems, the key is to find the shortest side based on the total length or other geometric constraints. For this example, let's assume the problem provides the total perimeter of the hexagon.

Assumption:
Suppose the total perimeter P of the hexagon is known or given as a specific value.

Example Scenario:
Total perimeter $P = 150$ inches.

Setting Up the Equation

Given the sides:

$$
a_1 = a
$$
$$
a_2 = a + 10
$$
$$
a_3 = a + 20
$$
$$
a_4 = a + 30
$$
$$
a_5 = a + 40
$$
$$
a_6 = a + 50
$$

Sum of all sides (perimeter):

$$
a + (a + 10) + (a + 20) + (a + 30) + (a + 40) + (a + 50) = P

Simplify:

$$
6a + (10 + 20 + 30 + 40 + 50) = P
$$

Calculate the sum of the constants:

$$
10 + 20 + 30 + 40 + 50 = 150
$$

Thus,

$$
6a + 150 = P
$$

Solving for a

If the total perimeter P is 150 inches, then:

$$
6a + 150 = 150
$$
$$
6a = 150 - 150
$$
$$
6a = 0
$$
$$
a = 0
$$

This implies the shortest side is 0 inches, which is not feasible for a real hexagon.

Conclusion:

- If the total perimeter is less than 150 inches, the shortest side becomes negative, which is impossible.
- If the total perimeter exceeds 150 inches, then the shortest side is positive.

Alternative approach:

Suppose the total perimeter is $P = 300$ inches:

$$
6a + 150 = 300
$$
$$
6a = 150
$$
$$
a = 25
$$

In this case, the shortest side is 25 inches.

Summary:

- The value of the shortest side depends on the total perimeter of the hexagon.
- The general formula, given the total perimeter P, is:

$$
a = \frac{P - 150}{6}
$$

General Formula for the Shortest Side

Based on the previous calculations, the formula to determine the shortest side length a when the total perimeter P is known:

$$
a = \frac{P - 150}{6}
$$

Where:

- P = total perimeter of the hexagon.
- a = length of the shortest side.

Additional Considerations and Real-World Applications

Understanding how to approach such problems has practical relevance in geometry, architecture, and design. For example:

- Designing hexagonal tiles with increasing side lengths.
- Calculating materials needed for a hexagonal fence with sides growing in length.
- Analyzing geometric sequences in natural formations or architectural patterns.

Important notes:

- The problem assumes a consistent increase in side length.
- The total perimeter must be sufficient to accommodate the shortest side being positive.
- If no total perimeter is given, the problem remains incomplete, and additional information is needed.

Summary of Key Steps to Solve Similar Problems

1. Identify the pattern: Recognize if the sides form an arithmetic sequence.
2. Define variables: Assign variables to the shortest side and common difference.
3. Express all sides: Write the sides in terms of the shortest side and the common difference.
4. Use given data: Incorporate total perimeter or other geometric constraints.
5. Set up an equation: Sum the sides and equate to the total perimeter.
6. Solve for the shortest side: Rearrange the equation to find the value of \(a \).

Conclusion

In problems involving a hexagon with sides increasing by a constant amount—specifically 10 inches in this case—the key is recognizing the pattern as an arithmetic sequence. The length of the shortest side depends on the total perimeter or other given dimensions. By establishing the relationship between the sides and the total perimeter, we derive a formula that allows us to calculate the shortest side's length effectively.

Understanding these concepts enhances problem-solving skills in geometry and provides practical tools for real-world applications. Whether in construction, design, or mathematics education, mastering the approach to such sequence-based problems is invaluable.

Remember: Always look for additional information such as total perimeter or other constraints to accurately determine specific side lengths. Without such data, the problem remains indeterminate, highlighting the importance of complete information in geometric problem-solving.

Frequently Asked Questions

What is the length of the shortest side of a hexagon if each side is 10 inches longer than the previous side?

The shortest side is 10 inches long.

How do you determine the length of each side in a hexagon where each side increases by 10 inches from the previous one?

Assign the shortest side as x inches; then each subsequent side is x + 10, x + 20, and so on, up to the sixth side.

What is the total perimeter of the hexagon if the shortest side is 10 inches and each side increases by 10 inches?

The sides are 10, 20, 30, 40, 50, and 60 inches, so the perimeter is 10 + 20 + 30 + 40 + 50 + 60 = 210 inches.

If the longest side of the hexagon is 60 inches, what is the length of the shortest side?

The shortest side is 10 inches, since each side is 10 inches longer than the previous one.

Can you find the length of the shortest side of a hexagon with sides increasing by 10 inches each, if the second side is 20 inches?

Yes, the shortest side is 10 inches, because the second side is 10 inches longer than the first, which makes the first side 10 inches.

Additional Resources

Understanding the Problem: Determining the Shortest Side of a Hexagon with Increasing Side Lengths

When approaching geometric problems, especially those involving polygons with specific constraints, it’s crucial to understand the foundational concepts, analyze the given data carefully, and apply appropriate mathematical tools. The problem at hand involves a hexagon whose sides increase sequentially by a fixed amount—specifically, each side is 10 inches longer than the previous one. Our goal is to determine the length of the shortest side.

This scenario is a classic example of an arithmetic progression applied to geometric figures, which offers a fascinating intersection of algebra and geometry. In this detailed discussion, we will explore the problem from multiple angles, examining the underlying principles, potential solution methods, and wider implications.

Breaking Down the Problem: Recognizing the Pattern in Side Lengths

The core of this problem revolves around understanding the relationship between the sides of the hexagon. Let’s dissect the information:

- The hexagon has 6 sides.
- The sides are not equal; instead, each side is 10 inches longer than the preceding one.
- Our goal: Find the length of the shortest side.

This setup naturally suggests an arithmetic sequence, where:

- The first term (a_1) is the shortest side length.
- Each subsequent side increases by a common difference ($d = 10$ inches).

Representing the Sides Algebraically

To formalize the problem, assign variables:

- Let a_1 = length of the shortest side (unknown).
- The sides are then:

- Side 1: a_1
- Side 2: $a_1 + 10$
- Side 3: $a_1 + 20$
- Side 4: $a_1 + 30$
- Side 5: $a_1 + 40$
- Side 6: $a_1 + 50$

This sequence clearly illustrates an arithmetic progression with common difference $d = 10$.

Considering the Geometric Constraints of a Hexagon

While the algebraic representation is straightforward, geometry introduces additional considerations:

- The sides must connect to form a closed polygon.
- The shape's convexity or concavity can influence whether such a side length arrangement is possible.
- The problem does not specify any restrictions on the shape being convex or concave; thus, the common assumption is a convex hexagon.

Key Point: For a convex hexagon with sides increasing sequentially, the side lengths must satisfy the polygon inequality conditions.

Polygon Inequality Conditions and Their Implications

A fundamental principle in polygon construction is that the sum of any five sides must be greater than the sixth side. For a hexagon with sides a_1, a_2, a_3, a_4, a_5, a_6:

- For each side, the sum of the other five sides > that side.

Applying this to our sequence:

- The sum of all sides:

$$
S = a_1 + (a_1 + 10) + (a_1 + 20) + (a_1 + 30) + (a_1 + 40) + (a_1 + 50)
$$

- Simplify:

$$
S = 6a_1 + (10 + 20 + 30 + 40 + 50) = 6a_1 + 150
$$

Now, examine the inequality for one side, for example, the shortest side:

$$
a_1 < \text{sum of the other five sides} \Rightarrow a_1 < S - a_1
$$

which simplifies to:

$$
a_1 < \frac{S}{2}
$$

Similarly, for the largest side, a_6:

$$
a_6 < \text{sum of the other five sides}
$$

which becomes:

$$
a_1 + 50 < S - (a_1 + 50) \Rightarrow a_1 + 50 < 6a_1 + 150 - (a_1 + 50) = 5a_1 + 100
$$

Leading to:

$$
a_1 + 50 < 5a_1 + 100 \Rightarrow 50 - 100 < 5a_1 - a_1 \Rightarrow -50 < 4a_1 \Rightarrow a_1 > -12.5
$$

This lower bound is trivial since side length cannot be negative; thus, the key inequality is:

$$
a_1 < \frac{S}{2} = \frac{6a_1 + 150}{2} = 3a_1 + 75
$$

which simplifies to:

$$
a_1 < 3a_1 + 75 \Rightarrow 0 < 2a_1 + 75 \Rightarrow a_1 > -37.5
$$

Again, side lengths are positive, so the critical inequality is that the shortest side is positive, and the sum of the sides satisfies the polygon inequality.

Summary of inequalities:

- $a_1 > 0$
- The total sum $S = 6a_1 + 150$
- The largest side $a_6 = a_1 + 50$

Key polygon inequality:

$$
a_1 + 50 < 5a_1 + 100 \Rightarrow a_1 > -12.5
$$

which is always true for positive a_1.

Determining the Feasibility of the Hexagon

The critical question becomes: Is there a minimum value of a_1 that makes the hexagon physically realizable?

Since the sides are increasing, and the sequence is linear, the primary concern is whether such a polygon can be constructed with these side lengths, given the geometric constraints.

In general, for a convex polygon with sides increasing in order, the shape can be constructed provided the polygon inequalities are satisfied.

Given that:

$$
a_1 > 0
$$

and considering the increasing nature of side lengths, the minimal feasible a_1 is when the inequalities are just satisfied, i.e., when the sides are as small as possible while still forming a proper convex hexagon.

Why the Shortest Side Is the Critical Parameter

Because each side increases by 10 inches, the shortest side directly influences the entire sequence.

- If the shortest side is too small (approaching 0), the shape might become degenerate or impossible to construct as a convex hexagon.
- As a_1 increases, the sides grow larger, satisfying the polygon inequality with even more margin.

The key is to find the minimum a_1 such that the hexagon can be formed without violating geometric constraints.

Use of Polygon Construction Principles and

Geometric Constraints

To understand whether a hexagon with such side lengths can be constructed, consider the following:

- Convexity condition: The sum of any five sides must be greater than the remaining side.
- Side length progression: Since the sides increase uniformly, the shape's feasibility depends on whether the sides can be arranged around a circle (cyclic polygon) or in a convex configuration.

In many polygon problems, the minimal side length corresponds to the scenario where the polygon is "just" possible—i.e., the sides are arranged so that the polygon inequality becomes an equality for the largest side.

Applying the Polygon Inequality for the Largest Side

Using the side lengths:

- Largest side: $a_6 = a_1 + 50$
- Sum of the other five sides:

$$
a_1 + (a_1 + 10) + (a_1 + 20) + (a_1 + 30) + (a_1 + 40) = 5a_1 + 100
$$

For the hexagon to be just possible (degenerate case), the largest side equals the sum of the other five:

$$
a_6 = \text{sum of other five sides}
$$

which gives:

$$
a_1 + 50 = 5a_1 + 100
$$

Solving:

$$
a_1 + 50 = 5a_1 + 100 \Rightarrow 50 - 100 = 5a_1 - a_1 \Rightarrow -50 = 4a_1 \Rightarrow a_1 = -12.5
$$

Since side lengths cannot be negative, this indicates that the polygon inequality is satisfied for any positive \(a_1 \), and the shape can be constructed for all \(a_1 > 0 \).

However, in practical geometric constructions, the minimal length is constrained by the triangle inequality applied to the sides arranged around the shape.

Incorporating the Geometry: Is There a Lower Bound?

In polygons with increasing side lengths, the limiting factor is often whether the sides can be "wrapped" to form a closed shape without gaps or overlaps. The critical aspect is the relation between the smallest side and the shape's overall size.

For a convex hexagon with sides increasing by 10 inches, the minimal side

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